Self-Control Problems and Conspicuous Housing Consumption: Implications for Tax Policy

Thomas Aronsson and Andrea Mannberg

Department of Economics, Umeå School of Business and Economics, Umeå University, SE – 901 87 Umeå, Sweden

February 2013

Abstract

During the latest decades, household mortgage loans have increased substantially in many countries. We develop an OLG model where housing is a positional consumption good (such that housing choices are partly driven by relative consumption concerns), and where the consumers are also characterized by a preference for immediate gratification due to quasi-hyperbolic discounting. The purpose is to examine how a paternalistic government may reach its preferred resource allocation through a mix of taxes/subsidies on capital income and housing wealth. Our results show that the optimal policy typically implies a marginal savings-subsidy, while the marginal housing wealth may either be taxed or subsidized. Upward social comparisons imply a possible scenario where the housing wealth of the young generation is subsidized and the housing wealth of the middle-aged generation is taxed at the margin.

Keywords: Conspicuous consumption, self-control problem, housing, optimal taxation.

JEL Classification: D03, D62, H21, H23.

** The authors would like to thank Ronnie Schöb and Magnus Wikström for helpful comments and suggestions. Research grants from the Bank of Sweden Tercentenary Foundation, the Swedish Council for Working Life and Social Research, the Swedish Tax Agency, and the Wallander-Hedelius Fund are also gratefully acknowledged.
Introduction

The background of this paper is the increasingly high level of household indebtedness, primarily due to housing loans, that are currently observed in many countries. We develop a model where housing is a conspicuous consumption good, and where consumers also suffer from a self-control problem that may further exacerbate the tendency to over-consume housing. The model is used to analyze the potential for policy makers to mitigate these consequences through optimal taxation. This will be described in greater detail below.

During the past 15 years, household indebtedness has increased substantially in many OECD-countries. A large share of this increase is constituted by loans related to housing. Indeed, since the mid 1990s, British household mortgages have increased by a third (Kempson, 2002), and in Sweden, household debt-to-income ratio, of which 80 per cent accrues to loans on houses, has nearly doubled (Finnochiaro et al., 2011). Empirical evidence suggests that financial deregulation and innovation, which have increased the availability and lowered the cost of credit, may have played an important role in this development (e.g., Finnochiaro et al., 2011; Finnochario and Queijo von Heideken, 2007). Clearly, if the deregulation of financial markets has increased the access to credit for previously credit-constrained households, one would expect that rational households should now find it easier to follow their optimal consumption and saving path over the life-cycle and that this, in turn, should be beneficial for society as a whole. Yet, recent experiences suggest that housing-loans may on average be higher than optimal and, perhaps even more importantly, that household indebtedness may play a crucial role for the severity of financial crises.\(^1\)

So why do consumers spend so much on housing, and what are the policy implications of this phenomenon? If consumers are fully rational, there is reason to believe that over-consumption of housing is caused by some form of market failure, such that the optimal behavior of the individual is not the optimal behavior from the viewpoint of society. According to recent literature in behavioral economics, one likely reason for the large increase in indebtedness on housing, as made possible through financial deregulation, is that housing partly represents conspicuous consumption emanating from positional preferences. The idea that utility does not

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\(^1\) Indeed, the risk associated with household indebtedness has been known ever since Fisher (1933) developed his debt-deflation theory, which describes how a high level of household debt will exacerbate the effects of negative demand or supply shocks. Glick and Lansing (2010) note that many of the countries most affected by the recent financial crisis also experienced sharp increases in household indebtedness during the years before 2008. In the U.S., for example, the crisis was preceded by an increase in household indebtedness in to over 100 per cent of GDP. Such a high level had previously only occurred once; namely, in 1929.
only depend on absolute, but also relative consumption has a long, if debated, history in economics. Veblen, as early as 1899, noted the importance of using consumption as an instrument to display one’s relative position in society, and in 1949 Duesenberry developed a more formal theory of relative consumption. More recently, survey-based experiments and happiness research show that individual consumers derive well-being both from their own absolute consumption level (as in standard economic theory) and from their relative consumption by comparison with other people.\(^2\) Important findings in the survey-experimental literature are that more visible goods such as houses and cars are typically more positional than other, less visible goods (Alpizar et al., 2005; Solnick and Hemenway, 2005; Carlsson et al., 2007), and that the degree to which positional concerns matters for utility may be substantial: Alpizar et al. (2005) found that the marginal degree of positionality for housing is around 0.4-0.5 on average. This suggests that as much as 40-50 percent of the utility gain of an additional dollar spent on housing may be due to increased relative consumption.\(^3\) If individuals fail to realize that consumption of positional goods may in the end leave their rank in the social hierarchy unaltered, such consumption threatens to cause a “positional arms race” for social status (Frank, 1985, 2007, 2008) where individuals are driven by a desire to keep up with their referent others.

However, although positional preferences constitute a partial explanation as to why households are willing to spend increasing amounts of their future income on housing, there is also evidence suggesting that households are not always fully rational when making intertemporal choices. As such, the consumption and savings choice made today may not even be optimal from the individual household’s own long run perspective. This argument is based on findings in experimental literature indicating that consumer preferences might have a “present-bias”, which is meant to imply a time-inconsistent preference for immediate gratification. A mechanism that generates this type of bias is quasi-hyperbolic discounting, where the individual at any time \(t\) applies a higher utility discount rate to the tradeoff between periods \(t\) and \(t+1\) by comparison with the discount rates used for intertemporal tradeoffs in the more distant future (Phelps and Pollak, 1968; Laibson, 1997).\(^4\) If consumers are sophisticated hyperbolic discounters, then illiquid


\(^3\) In a survey of 2000 randomly selected Swedish citizens, the market research institute Synnovate found that one in three Swedes considers it to be important to have a home that is relatively more attractive than the homes of referent others (the results of this investigation were published in Dagens Nyheter, February 21, 2010).

\(^4\) A considerable amount of evidence based on experimental research points in this direction. See, e.g., Thaler (1981), Kirby (1997), Viscusi, Huber and Bell (2008) and Brown, Chua and Camerer (2009). See also Fredrick, Loewenstein and O’Donoghue (2002) for a review of empirical research on intertemporal choice. The recent interest in theories of hyperbolic discounting also emanates from research on savings behavior and wealth accumulation; see, e.g., Bernheim, Weinberg and Skinner (2001) and Mastrobouni and Weinberg (2009).
wealth (such as houses) may serve as a commitment device. The reason is simply that such consumers, who rationally anticipate their self-control problem to remain in the future, may act strategically to strengthen the incentives, as faced by their future selves, to save. In this case, therefore, the incentives to accumulate housing wealth due to positional concerns are exacerbated by a strategic incentive to accumulate housing (and other illiquid) wealth. Instead, if the consumers are naïve, this incentive does not arise, since naïve hyperbolic discounters do not act strategically vis-à-vis their future selves (as they expect the self-control problem to vanish in the future).\(^5\) Naïve consumers may, therefore, spend less on housing for this particular reason; however, they may also hold less liquid wealth to be able to enjoy the immediate consumption-benefits of more housing (and other goods). These arguments suggest that the incentives underlying choices of housing are possibly affected both by the desire for status (as reflected in relative consumption concerns) and by self-control problems due to present biased preferences. Therefore, the joint policy implications of conspicuous housing consumption and quasi-hyperbolic discounting are interesting to consider, irrespective of whether the consumers are better described by sophistication than naivety or vice versa.

This paper deals with the joint effects of positional concerns for housing and a self-control problem caused by quasi-hyperbolic discounting. The main purpose is to analyze the associated tax policy implications from the perspective of a paternalistic government, which does not share the consumer preference for immediate gratification, and which also aims at internalizing the externalities caused by positional concerns for housing. We develop an Overlapping Generations (OLG) model, where each consumer lives for three periods (the minimum number of periods required to be able to address quasi-hyperbolic discounting), and where housing is treated as a positional good. Each consumer derives utility from housing as well as from a non-positional and non-durable good. The set of tax instruments faced by the government in our framework consists of nonlinear taxes on capital income and housing wealth, respectively. Our analysis also distinguishes between naïve and sophisticated consumers. As we indicated above, earlier research shows that the behavioral implications of quasi-hyperbolic discounting depend on whether the consumers are naïve or sophisticated (see also O'Donoghue and Rabin, 1999; Diamond and

\(^5\) See, e.g., Angeletos et al. (2000). They compare the savings behavior of exponential discounters with that of sophisticated and naïve hyperbolic discounters, respectively, in the context of a numerical model. The results show that sophisticated hyperbolic discounters hold a smaller fraction of their assets in liquid form than exponential discounters, and also a smaller fraction than naïve hyperbolic discounters. Yet, both types of hyperbolic discounters tend to hold less liquid wealth (measured as a share of labor income) than exponential discounters.
Kőszegi, 2003), thus suggesting that the policy implications examined below will be sensitive to this distinction as well.\(^6\)

The present study contributes to the literature in at least three ways. First and foremost, our paper is the first to combine relative consumption concerns for housing and present-biased preferences in the context of optimal taxation. This is of clear practical relevance by highlighting the potential role of the tax system as a mechanism to offset the incentives for over-consumption of housing described above, at least if the empirical evidence is taken seriously.\(^7\) It is also interesting from a theoretical point of view as it ties together two different areas of behavioral public economics. As such, we show that the optimal marginal tax rates attached to housing wealth typically vary between young and middle-aged consumers, and that this discrepancy is attributable both to positional concerns and to present-biased preferences. Second, in our paper, the positional good is a durable good (housing), whereas all earlier studies on public policy and relative consumption concerns that we are aware of instead analyze non-durable positional goods.\(^8\) Also this change of assumption has empirical relevance, since visible positional goods are typically durables. Third, as far as we know, the incentives to accumulate housing wealth are absent in earlier studies on public policy responses to quasi-hyperbolic discounting.\(^9\)

In Section 2, we present the model of consumer behavior as well as the decision-problem facing the government. A difficult question in the analysis of relative consumption concerns refers to the measurement of reference consumption at the individual level, and there is not much empirical evidence here. Veblen (1899) forcefully argues in favor of social comparisons upwards in the wealth distribution, such that individuals compare their own consumption with that of people who are wealthier or earn higher income than themselves.\(^10\) There is also some evidence

\(^6\) Empirical evidence suggests that naïveté is at least partly present in the population. For example, Mian and Sufi (2009) find that homeowners with a high credit card utilization rate and low credit scores have the strongest tendency to borrow against increases in home equity. In an experimental study, Hey and Lotito (2009) find behavioral patterns consistent with both naivety and sophistication, even if naivety seems to be a more common type of behavior. See also the overview article by DellaVigna (2009).

\(^7\) At first sight, a possible argument against our approach is that stricter regulation of banks may also be important; perhaps as important as the modifications of the tax system analyzed below. However, although this argument is legitimate, such regulation does not affect the basic incentives faced by consumers (if driven by concerns for relative consumption and/or self-control problems); it only constrains their behavior. To be able to analyze the potential role of the tax system, we abstract from regulative tools such as borrowing constraints throughout the paper.


\(^9\) Earlier studies in this area have been concerned with several different issues such as savings-subsidies (Laibson 1997), sin taxes (e.g., Gruber and Kőszegi, 2004; O'Donoghue and Rabin, 2003, 2006), health capital subsidies (Aronsson and Thunström, 2008) and public investment (Aronsson and Granlund, 2011).

\(^10\) Bowles and Park (2005) find that more inequality in society leads to longer work hours, which is consistent with (yet no formal evidence for) the idea of upward comparisons.
suggesting that individuals compare their own consumption with that of people who are similar to themselves; see, e.g., McBride (2001) who found that people’s well-being depends negatively on the income of other persons of roughly the same age as themselves. Our study tries to capture the ideas underlying both these approaches; yet with a focus on housing. In our benchmark model in Section 2, we assume that people compare their own housing consumption to that of other persons in their own generation. The implications for optimal taxation of capital income and housing wealth are analyzed in Section 3. In Section 4, we extend the analysis by assuming that individuals of the young generation (in addition to the within-generation comparison) also compare their own consumption of housing with that of the middle-aged generation, while the middle-aged only engage in within-generation comparisons. This is a variant of the upward comparison, given that the middle-aged have accumulated more housing wealth than the young. Section 5 summarizes and discusses the main results.

2. The Model

This section begins by presenting the objective and constraints faced by the consumers followed by a characterization of their consumption and savings behavior. We will then present the decision-problem faced by the government.

2.1 Consumers

Consider an OLG-model where each individual lives for three periods. All individuals of the same generation are identical and derive utility from the consumption of two goods: a durable good to be referred to as “housing” and a non-durable good. We assume that the individuals have positional preferences for housing and non-positional preferences for the non-durable good. Let \( h \) denote the consumption of housing and \( c \) the consumption of the non-durable good. An individual born in period \( t \), to be referred to as “generation \( t \) ” faces the following instantaneous utility functions when young, middle-aged and old, respectively:

\[
u_{0,t} = v_0(c_{0,t}, h_{0,t}, \Delta_{0,t}) = \bar{u}_0(c_{0,t}, h_{0,t}, \bar{h}_{0,t})
\]

\[\text{(1a)}\]

\[\text{11 Earlier studies typically assume that relative consumption concerns are governed by mean value comparisons (such that the reference consumption is the average consumption in the economy as a whole). Exceptions are Aronsson and Johansson-Stenman (2010) analyzing optimal income taxation under both within-generation and upward comparisons, and Micheletto (2011) addressing optimal income taxation under upward comparisons. In both these studies, the relative consumption concerns refer to the private consumption as a whole, which is modeled as a non-durable good.}\]
\[ u_{1,t+1} = v_1\left(c_{1,t+1}, h_{1,t+1}, \Delta_{1,t+1}\right) = \bar{u}_1\left(c_{1,t+1}, h_{1,t+1}, \bar{h}_{1,t+1}\right) \quad (1b) \]
\[ u_{2,t+2} = v_2\left(c_{2,t+2}\right) = \bar{u}_2\left(c_{2,t+2}\right). \quad (1c) \]

In equations (1), the subscripts 0, 1 and 2 refer to the individual’s age-group (0=young, 1=middle-aged and 2=old). The variables \( \bar{h}_{0,t} \) and \( \bar{h}_{1,t+1} \) are the measures of reference consumption faced by the young and middle-aged, respectively, of generation \( t \) (to be defined below), while \( \Delta_{0,t} = h_{0,t} - \bar{h}_{0,t} \) and \( \Delta_{1,t+1} = h_{1,t+1} - \bar{h}_{1,t+1} \) denote the relative consumption of housing for the young and middle-aged.\(^{12}\) Note that the individual only derives utility from housing during the first two periods of life; when old the individual sells the house and uses the income from this sale for non-durable consumption. We assume that the functions \( v_0(\cdot) \) and \( v_1(\cdot) \) are increasing in each argument and strictly concave. This means that the functions \( \bar{u}_0(\cdot) \) and \( \bar{u}_1(\cdot) \) are decreasing in \( \bar{h}_{0,t} \) and \( \bar{h}_{1,t+1} \), respectively.\(^{13}\) The functions \( \bar{u}_0(\cdot) \) and \( \bar{u}_1(\cdot) \) can be thought of as reduced forms, which allow us to shorten some of the notation.

As we indicated in the introduction, we assume to begin with that the relative consumption of housing is governed by within-generation comparisons, such that each individual compares his/her consumption with that of other people of the same generation, i.e.\(^{14}\)
\[ \bar{h}_{0,t} = h_{0,t} \quad (2a) \]
\[ \bar{h}_{1,t+1} = h_{1,t+1}. \quad (2b) \]

This assumption will be relaxed in Section 4, where we also allow for upward comparisons in terms of the distribution of housing wealth. Each individual is small relative to the economy as a whole and treats \( \bar{h}_{0,t} \) and \( \bar{h}_{1,t+1} \) as exogenous.

The lifetime utility function facing generation \( t \) is given by

\(^{12}\) Instead of measuring the relative consumption as the difference between the individual’s own consumption and the appropriate reference measure, one may use the quotient between them. Since it is of no significance for the qualitative results derived below which option is chosen, we use the technically more convenient difference comparison in what follows.

\(^{13}\) See also Oswald (1985) for a more general framework where relative consumption concerns may either lead to negative and positive externalities. The latter is interpretable in terms of “altruism”.

\(^{14}\) Since the individuals of the same generation are identical, we can interpret the right hand side of equation (2a) and (2b) in terms of the average consumption of housing among the young in period \( t \) and middle-aged in period \( t+1 \), respectively.
\[ U_{0,t} = u_{0,t} + \beta [u_{1,t+1}\theta + u_{2,t+2}\theta^2] \]  

(3)

where \( \theta = 1/(1 + \theta) \) is a conventional (exponential) utility discount factor with discount rate \( \theta \), while the parameter \( \beta \in (0,1) \) is a time-inconsistent preference for immediate gratification.

An individual of any generation \( t \) is assumed to buy housing when young at price \( P_t \), make an additional investment (e.g., extend the size of the home or renovate) when middle-aged at price \( P_{t+1} \) and, finally, sell the house when old at price \( P_{t+2} \), where \( I_{1,t+1} \) denotes the housing investment by the middle-aged generation \( t \). Furthermore, let \( y \) represent the labor income, \( s \) savings in liquid (non-housing) wealth and \( r \) the interest rate.\(^{15}\) Each young and middle-aged individual is assumed to supply one unit of labor inelastically. The budget constraint can then be written as

\[ c_{0,t} = y_{0,t} - s_{0,t} - P_t h_{0,t} - T_0(P_t h_{0,t}) \]  

(4a)

\[ c_{1,t+1} = y_{1,t+1} + (1 + r_{t+1})s_{0,t} - P_{t+1}I_{t+1} - T_1(s_{0,t}r_{t+1}, P_{t+1}h_{1,t+1}) \]  

- \( s_{1,t+1} \)  

(4b)

\[ c_{2,t+2} = (1 + r_{t+1})s_{1,t+1} - T_2(s_{1,t+1}r_{t+2}) + P_{t+2}h_{2,t+2}, \]  

(4c)

in which

\[ h_{1,t+1} = h_{0,t}(1 - \delta) + I_{1,t+1} \]  

(5a)

\[ h_{2,t+2} = h_{1,t+1}(1 - \delta) \]  

(5b)

and \( \delta \) is the rate of depreciation. The price on the non-durable good has been normalized to one. The variables \( T_0, T_1 \) and \( T_2 \) represent tax payments made during the three periods of life. Since we abstract from bequest motives (meaning that the consumer has no inherited wealth), the tax faced by the young consumer is based solely on the acquired housing wealth, whereas the tax paid by the middle-aged both depends on the capital income and housing wealth. The old consumer solely pays a tax based on the capital income. Note also that the old consumer makes no active decision in our framework; he/she just spends the remaining wealth for consumption of the non-durable good. For further use, we introduce the following short notations for marginal tax rates:

\(^{15}\) A table summarizing all notations can also be found in Appendix I.
As we indicated in the introduction, although consumers suffer from a self-control problem in our framework, it is not clear from empirical evidence whether consumer behavior is better described by naivety than sophistication or vice versa. Therefore, we consider both these possibilities in the analysis below. Since the consumer’s first order conditions under naivety are interpretable as technical special cases of the corresponding first order conditions under sophistication, we derive these conditions under the assumption that agents are sophisticated and then comment upon how the results are modified if agents are naive. To arrive at a time-consistent solution for the sophisticated agents, their decision-problems will be solved sequentially starting with the savings and investment behavior by the middle-aged.

**Decisions made by the middle-aged consumer**

The middle-aged generation $t$ chooses $s_{1,t+1}$ and $l_{1,t+1}$ to maximize $u_{1,t+1} + \beta u_{2,t+2} \Theta$ subject to the budget constraint given by equations (4b) and (4c), while treating $s_{0,t}$ and $h_{0,t}$ as exogenous. Note that this objective stands in conflict with equation (3) above, which is the reason as to why the dynamic inconsistency arises (the young consumer would like his/her middle-aged self to maximize $u_{1,t+1} + u_{2,t+2} \Theta$).

By using the functions $\bar{u}_0(\cdot)$ and $\bar{u}_1(\cdot)$ along with the following short notation:

$$\frac{\partial \bar{u}_{i,t+i}}{\partial c_{i,t+i}} = \bar{\mu}_c^i, \quad \text{for } i = 0, 1, 2, \quad (6a)$$

and

$$\frac{\partial \bar{u}_{i,t+i}}{\partial h_{i,t+i}} = \bar{\mu}_h^i, \quad \text{for } i = 0, 1, \quad (6b)$$

the first order conditions for $s_{1,t+1}$ and $l_{1,t+1}$ can be written as

$$s_{1,t+1}: \quad -\bar{\mu}_c^1 + \beta \Theta \bar{\mu}_c^2 (1 + r_{t+2}) - T'_{2,s} r_{t+2} = 0 \quad (7a)$$
Notice first that there is no incentive for a sophisticated middle-aged consumer to try to influence the incentives faced by his/her old self, since the old consumer is passive (and makes no intertemporal decisions) in our model. Therefore, the first order conditions for the middle-aged consumer are given by equations (7a) and (7b), irrespective of whether he/she is naïve or sophisticated.

By analogy to earlier comparable literature, equation (7a) shows that the self-control problem caused by quasi-hyperbolic discounting constitutes an incentive to save less than otherwise, since $\beta < 1$ means that the weight attached to future consumption is smaller than under conventional exponential discounting. Equation (7b) balances the marginal cost of the housing investment (the first term on the left hand side) with the marginal benefit (the second and third terms). The private marginal benefit of housing contains two parts, i.e. a direct welfare effect through the utility function for the middle-aged consumer as well as an indirect effect for his/her old self through the consumption value of housing wealth. Notice also that the private marginal benefit of the housing investment may either exceed, or fall short of, the social marginal benefit. There are two mechanisms involved here: (i) the private marginal utility of housing, $\bar{u}_{h1}^' = v_{h1}^' + v_{\Delta_1}^'$, exceeds the social marginal utility, $\bar{u}_{h1}^' + \bar{u}_{\Delta_1}^' = v_{h1}^'$, and (ii) the future private marginal consumption benefit is smaller than it would have been in the absence of the self-control problem since $\beta < 1$. The net effect determines whether the consumer has an incentive to consume more or less housing that preferred by the government.

By solving equation system (7a)-(7b), we can derive savings and investment functions, which can be written as

$$s_{1,t+1} = s_1(y_{1,t+1}, P_{t+1}, P_{t+2}, r_{t+1}, r_{t+2}, h_{1,t+1}, s_{0,t}, h_{0,t}) \quad (8a)$$

$$I_{1,t+1} = I_1(y_{1,t+1}, P_{t+1}, P_{t+2}, r_{t+1}, r_{t+2}, h_{1,t+1}, s_{0,t}, h_{0,t}) \quad (8b)$$

Notice that equations (8a) and (8b) are interpretable as reaction functions through which the young sophisticated consumer (by choosing $s_{0,t}$ and $h_{0,t}$) may influence the incentives faced by his/her middle-aged self. For a naïve consumer, on the other hand, there would be no such incentive, since the naïve consumer (erroneously) expects the self-control problem to be absent
in the future. Hence, in this case, the young consumer does not perceive the future change in objective function.

**Decisions made by the young consumer**

The young generation $t$ chooses $s_{0,t}$ and $h_{0,t}$ to maximize the life-time utility function given by equation (3) subject to the life-time budget constraint in equations (4a)-(4c). A sophisticated consumer also recognizes the reaction functions given by equations (8a) and (8b).

Define $U^{0}_{1,t+1} = u_{1,t+1} + u_{2,t+2}\Theta$ to be the objective that the young sophisticated consumer would like his/her middle-aged self to maximize, whereas the actual objective of the middle-aged consumer is given by $U_{1,t+1} = u_{1,t+1} + \beta u_{2,t+2}\Theta$. Therefore, by using equations (7a) and (7b), we can calculate the derivatives of the function $U^{0}_{1,t+1}$ such that

$$
\frac{\partial U^{0}_{1,t+1}}{\partial s_{1,t+1}} = \frac{1 - \beta}{\beta} \tilde{u}_{c}^t > 0 \quad (9a)
$$

$$
\frac{\partial U^{0}_{1,t+1}}{\partial l_{1,t+1}} = (1 - \beta)\Theta \tilde{u}_{c}^t P_{t+2}(1 - \delta) > 0. \quad (9b)
$$

For the sophisticated consumer, the first order conditions for $s_{0,t}$ and $h_{0,t}$ can then be written as

For the naive consumer, the first order conditions simplify to read

For the naive consumer, the first order conditions simplify to read

$$
\begin{align*}
\text{s}_{0,t}: & \quad 0 = -\tilde{u}_{c}^t + \beta \Theta \tilde{u}_{c}^t_c \left[ (1 + r_{t+1}) - T'_{1,s} r_{t+1} \right] \\
& \quad + \beta \Theta \left[ \frac{\partial U^{0}_{1,t+1}}{\partial s_{1,t+1}} \frac{\partial s_{1,t+1}}{\partial s_{0,t}} + \frac{\partial U^{0}_{1,t+1}}{\partial l_{1,t+1}} \frac{\partial l_{1,t+1}}{\partial s_{0,t}} \right] \quad (10a)
\end{align*}
$$

$$
\begin{align*}
\text{h}_{0,t}: & \quad 0 = -\tilde{u}_{c}^t + \beta \Theta \tilde{u}_{c}^t_c \left[ P_t + T'_{0,h} P_{t+1} \right] + \tilde{u}_{h}^t + \beta \left[ -\tilde{u}_{c}^t T'_{1,h} P_{t+1} + \tilde{u}_{h}^t \right] \Theta (1 - \delta) \\
& \quad + \beta \Theta \tilde{u}_{c}^t_c P_{t+2}(1 - \delta)^2 + \beta \Theta \left[ \frac{\partial U^{0}_{1,t+1}}{\partial s_{1,t+1}} \frac{\partial s_{1,t+1}}{\partial h_{0,t}} + \frac{\partial U^{0}_{1,t+1}}{\partial l_{1,t+1}} \frac{\partial l_{1,t+1}}{\partial h_{0,t}} \right]. \quad (10b)
\end{align*}
$$

With naive consumers, on the other hand, the incentive to influence the choices of one’s future self vanishes. Hence, with naive consumers the first order conditions simplify to read

$$
\begin{align*}
\text{s}_{0,t}: & \quad 0 = -\tilde{u}_{c}^t + \beta \Theta \tilde{u}_{c}^t_c \left[ (1 + r_{t+1}) - T'_{1,s} r_{t+1} \right] \quad (11a)
\end{align*}
$$

$$
\begin{align*}
\text{h}_{0,t}: & \quad 0 = -\tilde{u}_{c}^t + \beta \Theta \tilde{u}_{c}^t_c \left[ P_t + T'_{0,h} P_{t+1} \right] + \tilde{u}_{h}^t + \beta \left[ -\tilde{u}_{c}^t T'_{1,h} P_{t+1} + \tilde{u}_{h}^t \right] \Theta (1 - \delta) \\
& \quad + \beta \Theta \tilde{u}_{c}^t_c P_{t+2}(1 - \delta)^2. \quad (11b)
\end{align*}
$$
Equation (11a) implies that the first order condition for savings faced by a young naïve consumer takes exactly the same form, and has the same interpretation, as its counterpart for the middle-aged, i.e. equation (7a). Equations (7b) and (11b) also take similar forms; the major difference is that an increase in \( h_{0,t} \) influences the marginal utilities of non-durable consumption and housing both when young and middle-aged. This follows because \( h_{1,t+1} \) depends directly on \( h_{0,t} \) according to equation (6a). The additional complexity in case of sophistication follows because the young sophisticated consumer has an incentive to act strategically vis-à-vis his/her middle-aged self, which is seen from the two final terms on the right hand side in equation (10b). In general, although the right hand side of equation (9a) and (9b), respectively, is positive, none of the reaction function terms in equations (10a) and (10b) can be signed unambiguously.

A plausible case on which some of our discussion of tax policy below is based is, nevertheless, that increases in \( s_{0,t} \) and \( h_{0,t} \) both imply increased savings when middle-aged, and that an increase in \( h_{0,t} \) also leads to a smaller housing investment when middle-aged. Some justification for this scenario is given in Appendix II, where we present comparative statics results for a quasi-linear utility function such that the instantaneous utility of the consumer’s old self is linear in the non-durable consumption. This additional assumption implies the following comparative statics of the reaction functions:

\[
\frac{\partial s_{1,t+1}}{\partial s_{0,t}} > 0, \quad \frac{\partial s_{1,t+1}}{\partial h_{0,t}} > 0, \quad \frac{\partial l_{1,t+1}}{\partial s_{0,t}} = 0, \quad \frac{\partial l_{1,t+1}}{\partial h_{0,t}} < 0.
\]

The intuition is that the more wealth that the young consumer builds up, the more resources will be available for the consumer’s middle-aged self (which leads to increased savings when middle-aged), and the more resources spent on housing when young, the less will be the need for additional investments when middle-aged (due to substitutability in the utility function). Notice also that the resources spent on housing by the young sophisticated consumer serves as a commitment device: by choosing a higher \( h_{0,t} \), the young consumer induces his/her middle-aged self to build up more liquid (non-housing) wealth through increased savings and a smaller additional housing investment.

**Production**

Since production is not an important aspect of our analysis, we simplify by assuming that output is produced by a linear technology. This assumption is common in earlier comparable literature, and means that the technology is such that producer and factor prices are exogenous.
2.2 The Government

The government is first mover vis-à-vis the private sector in our model. It also aims at correcting for the positional externalities presented above and correct for the self-control problem. Therefore, and by following earlier literature on optimal paternalism,\(^\text{16}\) we assume that the government does not share the consumer preference for immediate gratification (i.e. \(\beta = 1\) from the point of view of the government), although it respects all other aspects of consumer preferences. As such, the government attaches the following life-time utility to generation \(t\)

\[
W_{0,t} = u_{0,t} + u_{1,t+1}\theta + u_{2,t+2}\theta^2.
\]  

(12)

The (paternalistic) social welfare function is given by the discounted sum of life-time utilities that the government attaches to all future generations\(^\text{17}\)

\[
W_0 = \sum_{t} W_{0,t} \theta^t.
\]  

(13)

The resource constraint for the economy as a whole in period \(t\) can be written as

\[
\sum_{i=0}^{1} y_{i,t} + K_t(1 + r_t) - P_t(h_{0,t} + I_{1,t} - h_{2,t}) - \sum_{i=0}^{2} c_{i,t} - K_{t+1} = 0
\]  

(14)

where \(K\) is the capital stock defined such that \(s_{0,t-1} + s_{1,t-1} = K_t\) for all \(t\).

Therefore, the resource allocation preferred by the government can be derived by choosing \(c_{0,t}\), \(c_{1,t}\), \(c_{2,t}\), \(h_{0,t}\), \(I_{1,t}\) and \(K_t\) for all \(t\) to maximize the social welfare function in equation (13) subject to the resource constraint presented in (14), as well as subject to the expressions for reference consumption given by equations (2a) and (2b). The latter two restrictions follow because the government aims at internalizing the externalities generated by positional concerns, and will therefore recognize that the measures of reference consumption are endogenous. The Lagrangean corresponding to the this decision-problem is given by


\(^{17}\) This simple objective function for the government is motivated by our aim to address the tax policy implications of positional externalities and quasi-hyperbolic discounting in the simplest possible way. Equation (13) means that there are no reasons for distorting individual behavior other than externality correction and correction for the self-control problem.
\[ L = W_0 + \sum_t \gamma_t \left[ \sum_{i=0}^{2} y_{i,t} + K_t (1 + r_t) - P_t (h_{0,t} + l_{1,t} - h_{2,t}) - \sum_{i=0}^{2} c_{i,t} - K_{t+1} \right]. \] (15)

The first order conditions characterizing the optimal resource allocation for any generation \( t \) can then be written in terms of \( c_{0,t}, c_{1,t+1}, c_{2,t+2}, h_{0,t}, l_{1,t+1}, K_{t+1} \) and \( K_{t+2} \)

\[
\frac{\partial L}{\partial c_{0,t}} = \tilde{u}_{c_0} \Theta^t - \gamma_t = 0 \quad (16a)
\]

\[
\frac{\partial L}{\partial c_{1,t+1}} = \tilde{u}_{c_1} \Theta^{t+1} - \gamma_{t+1} = 0 \quad (16b)
\]

\[
\frac{\partial L}{\partial c_{2,t+2}} = \tilde{u}_{c_2} \Theta^{t+2} - \gamma_{t+2} = 0 \quad (16c)
\]

\[
\frac{\partial L}{\partial h_{0,t}} = \sum_{i=0}^{1} \tilde{u}_{h_i} \Theta^{t+i} (1 - \delta)^i - \gamma_t P_t + \gamma_{t+2} P_{t+2} (1 - \delta)^2 + \frac{\partial L}{\partial h_{0,t}} = 0 \quad (16d)
\]

\[
\frac{\partial L}{\partial l_{1,t+1}} = \tilde{u}_{h_{1}} \Theta^{t+1} - \gamma_{t+1} P_{t+1} + \gamma_{t+2} P_{t+2} (1 - \delta) + \frac{\partial L}{\partial h_{1,t+1}} = 0 \quad (16e)
\]

\[
\frac{\partial L}{\partial K_{t+1+t+i}} = -\gamma_{t+i} + \gamma_{t+1+i} (1 + r_{t+1+i}) = 0 \quad \text{for} \ i=0,1 \quad (16f)
\]

where (as before) \( \tilde{u}_{c_i} = \partial \tilde{u}_{i,t+i} / \partial c_{i,t+i} \) for \( i=0,1,2 \) and \( \tilde{u}_{h_i} = \partial \tilde{u}_{i,t+i} / \partial h_{i,t+i} \) for \( i=0,1 \). In equations (16d) and (16e), we have used \( \partial \tilde{h}_{0,t} / \partial h_{0,t} = 1 \) and \( \partial \tilde{h}_{1,t+1} / \partial h_{1,t+1} = 1 \) from equation (2a) and (2b), respectively.

The variables \( \partial L / \partial \tilde{h}_{0,t} \) and \( \partial L / \partial \tilde{h}_{1,t+1} \) represent the partial welfare effects of increases in \( \tilde{h}_{0,t} \) and \( \tilde{h}_{1,t+1} \). As such, these welfare effects depend on the strength of the relative consumption concerns for housing. To see this, we follow Johansson-Stenman et al. (2002), and define marginal degrees of positionality as follows by using equation (1):

\[
\alpha_{0,t} = \frac{\partial v_{0,t} / \partial \Delta_{0,t}}{\partial v_{0,t} / \partial h_{0,t} + \partial v_{0,t} / \partial \Delta_{0,t}}
\]
The variable $\alpha_{0,t}$ represents the marginal degree of housing positionality of the young generation $t$: it is interpretable as the fraction of the utility gain of an additional dollar spent on housing that is due to increased relative consumption. The variable $\alpha_{1,t+1}$ has an analogous interpretation for the middle-aged consumer in period $t+1$. Notice also that $\alpha_{0,t}$ and $\alpha_{1,t+1}$ reflect the marginal externality per unit of consumption of housing (caused by the young and the middle-aged of generation $t$), which is why these measures are useful from the point of view of public policy. By analogy to Aronsson and Johansson-Stenman (2010), we can then write the partial welfare effects of $h_{0,t}$ and $h_{1,t+1}$ in terms of the marginal degrees of positionality

$$\frac{\partial L}{\partial h_{0,t}} = -\alpha_{0,t} \frac{\partial u_{0,t}}{\partial h_{0,t}} \Theta^{t} < 0 \quad (17a)$$

$$\frac{\partial L}{\partial h_{1,t+1}} = -\alpha_{1,t+1} \frac{\partial u_{1,t+1}}{\partial h_{1,t+1}} \Theta^{t+1} < 0. \quad (17b)$$

Therefore, the more positional the consumers are, the larger is the welfare loss associated with the positional externality.

The marginal tax rates that would implement the resource allocation preferred by the government in a decentralized setting can then be derived by comparison of the social first order conditions given by equations (16a)-(16f) with the private first order conditions for savings and housing consumption. This is the issue to which we turn next.

3. Optimal Taxation in the Benchmark Model

The purpose of this section is to characterize the optimal tax policy from the point of view of the paternalistic government, whose preferred resource allocation was discussed in the previous section. Our basic question is how this first best allocation can be implemented via taxation of capital income and housing wealth. We begin by analyzing the optimal taxation of savings and housing wealth implemented for the middle-aged, where the distinction between naivety and sophistication is of no importance for the qualitative results, and then continue with the optimal tax policy implemented for the young. The distinction between age-groups is also useful because the consumers make different behavioral mistakes during different phases of the life-cycle;
something that will be even clearer in the next section, where we assume that the positional concerns are also driven by upward comparisons.

3.1 Optimal Taxation of the Middle-Aged

To simplify the notation, let

$$MRS_{h_{1},c_1} = \frac{\partial u_{1,t+1}/\partial h_{1,t+1}}{\partial u_{1,t+1}/\partial c_{1,t+1}} > 0$$

denote the marginal rate of substitution between housing and the non-durable good for the middle-aged consumer. We have derived the following result:

**Proposition 1.** To implement the preferred resource allocation, the government should increase the savings by the middle-aged through a marginal savings subsidy. The housing wealth of the middle-aged should either be subsidized or taxed at the margin, depending on the relative magnitude of the self-control problem and the positional externality generated by the middle-aged. The optimal marginal tax rates are given by

$$T_{2,s}^{t} = \frac{\beta - 1 + r_{t+2}}{\beta} < 0 \quad (18a)$$

$$T_{1,h}^{t} = \frac{\beta - 1}{1 + r_{t+2}} \frac{P_{t+2}(1 - \delta)}{P_{t+1}} + \frac{\alpha_{1,t+1}}{P_{t+1}} MRS_{h_{1},c_1} \quad (18b)$$

Proof: see Appendix III.

Note first that the marginal tax structure in Proposition 1 applies irrespective of whether the consumers are sophisticated or naïve, since the middle-aged (whose behavior is targeted here) have no incentives to strategically influence their old selves. Equation (18a) differs in a fundamental way from the optimal capital income tax policy under relative consumption concerns derived by Aronsson and Johansson-Stenman (2010), who found that differences in the marginal degree of positionality over time determine whether savings should be subsidized or taxed at the margin. Instead, the savings subsidy derived here only reflects the self-control problem: the smaller $\beta$ is, the larger will the savings subsidy required to implement the resource allocation preferred by the paternalistic government be, ceteris paribus. The intuition behind the difference in results is that, whereas Aronsson and Johansson-Stenman assumed that the private consumption as a whole is a positional good (they made no distinction between positional and non-positional goods), we focus on positional preferences for housing, and the tax on housing wealth is then the appropriate instrument for internalizing the associated externality. As such, the
principle of targeting applies, meaning that the savings subsidy takes the same general form as in
earlier comparable literature on public policy responses to quasi-hyperbolic discounting without
any positional concerns (e.g., Aronsson and Thunström, 2008).

The marginal tax/subsidy on housing wealth in equation (18b) reflects two different policy
incentives. As explained in the analysis of consumer behavior in Section 2, $\beta < 1$ means that the
consumer attaches a lower weight on the future marginal benefit of housing (where housing
creates consumption possibilities for the middle-aged consumer’s old self) than he/she would
have done, had the self-control problem been absent. This means a tendency to under-invest in
housing, which the first term on the right hand side of equation (18b) serves to correct for. As
such, this term is negative as it reflects a motive to subsidize housing. The second term on the
right hand side is positive, as it serves to correct for the positional externality that each middle-
aged consumer imposes on other people of the same generation through his/her investment in
housing. The more positional people are on average, as measured by the marginal degree of
positionality, the more likely it is that the right hand side of equation (18b) is positive. In the
absence of any positional concerns, i.e. if $\alpha_{1,t+1} = 0$, it would be optimal to subsidize housing
wealth to counteract the tendency to underinvest in housing due to the self-control problem.
Similarly, if we were to replace the paternalistic government by a welfarist government that does
not attempt to correct for the self-control problem (such that equation (12) is replaced by
equation (3)), then $T_{1,h}' = \alpha_{1,t+1} MRS_{h_1,c_1}/P_{t+1} > 0$.

3.2 Optimal Taxation of the Young

To shorten the notation as much as possible also in this case, we introduce the following
marginal rates of substitution between $h_{0,t}$ and $c_{0,t}$, between $c_{1,t+1}$ and $c_{0,t}$, between $h_{1,t+1}$ and
$c_{0,t}$, and between $c_{2,t+2}$ and $c_{0,t}$, respectively, i.e.

\[
MRS_{h_0,c_0} = \frac{\partial u_{0,t}}{\partial h_{0,t}} \frac{\partial h_{0,t}}{\partial c_{0,t}}, \quad MRS_{c_1,c_0} = \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} \frac{\partial c_{1,t+1}}{\partial c_{0,t}} \Theta,
\]

\[
MRS_{h_1,c_0} = \frac{\partial u_{1,t+1}}{\partial h_{1,t+1}} \frac{\partial h_{1,t+1}}{\partial c_{0,t}} \Theta, \quad MRS_{c_2,c_0} = \frac{\partial u_{2,t+2}}{\partial c_{2,t+2}} \frac{\partial c_{2,t+2}}{\partial c_{0,t}} \Theta^2,
\]

each of which is positive by the assumptions made earlier. For the same reason, we also define
the following measure of “future marginal benefit of housing” in period $t$:
\[ R_{0,t} = MRS_{h_{t},c_{0}}(1 - \delta) + MRS_{c_{t},c_{0}}P_{t+2}(1 - \delta)^2 > 0. \]

Proposition 2 characterizes the marginal tax rates for the young under the assumption that the consumers are naïve, where we have used \( MRS_{c_{t},c_{0}} = 1/(1 + r_{t+1}) \) from the first order conditions of the social decision-problem. The case where the consumers are sophisticated is discussed below.

**Proposition 2.** If the consumers are naïve, the allocation preferred by the government implies the following marginal tax policy implemented for the young generation:

\[
T_{1,s} = \frac{\beta - 1 + r_{t+1}}{r_{t+1}} < 0 \tag{19a}
\]

\[
T_{0,h} = -\frac{\beta(1 - \delta)}{(1 + r_{t+1})} \frac{P_{t+1}P_{1,h}}{P_{t}} + \frac{\beta - 1}{P_{t}} R_{0,t} + \frac{C_{0,t}}{P_{t}} MRS_{h_{0},c_{0}} \tag{19b}
\]

where \( T_{1,h} \) is given in equation (18b).

The proof of Proposition 2 follows from the proof of Proposition 3 below. Equations (19a) and (19b) closely resemble their counterparts for the middle-aged. Clearly, equation (19a) implies that the paternalistic government subsidizes the savings also for the young consumer. In addition, the savings subsidy implemented for the young naïve consumer in period \( t \) is the same as that implemented for the middle-aged living in the same period, which can be seen by evaluating equation (18a) for generation \( t-1 \) instead of for generation \( t \). Turning to the tax formula for housing wealth, the second and third terms on the right hand side of equation (19b) are analogous to their counterparts for the middle-aged; there is an incentive to subsidize housing wealth due to that the self-control problem induces the consumer to under-value the future marginal benefit of \( h_{t} \) (the second term), and an incentive to tax housing wealth because the consumers generate positional externalities (the third term). The strength of the latter effect is again dependent on the marginal degree of positionality; here measured for the young consumer.

However, an interesting difference between equations (18b) and (19b) arises because the housing stock bought when young, \( h_{0,t} \), directly affects the consumer's housing stock when middle-aged, \( h_{1,t+1} \), as described in equation (6a). As such, an increase in \( h_{0,t} \) directly affects the marginal tax/subsidy on housing wealth facing the consumer's middle-aged self, which is captured by the first term on the right hand side of equation (19b). Since housing is a durable good, this relationship creates a complementarity in housing tax policy over time. If \( T_{1,h} > 0 \), it follows that
the marginal tax paid when middle-aged weakens the incentive to invest in housing when young; therefore, the government may implement the preferred resource allocation by taxing the housing wealth of the young generation at a lower marginal rate (or subsidize it at a higher marginal rate) than would be possible in the absence of this tax-complementarity. The interpretation is analogous if $T'_{1,h} < 0$. Note also that the effect of $T'_{1,h}$ on $T'_{0,h}$ depends on the “degree of hyperbolic discounting”: the less present-biased the preferences are (i.e. the closer to one $\beta$ is), the greater is the complementarity between taxes on durables over the individual life-cycle. To see the intuition behind this result, consider the extreme case of $\beta = 0$, i.e., when the young generation only cares about current consumption. In that case, young individuals ignore future taxes in their optimization problem, and the complementarity therefore vanishes.

An interesting implication of Proposition 2 is that the optimal tax policy may feature a marginal tax on the housing wealth faced by young consumers even in the absence of any relative consumption concerns, i.e. if $\alpha_{0,t} = \alpha_{1,t+1} = 0$. Clearly, in this case, Proposition 1 implies a marginal subsidy attached to the housing wealth of the middle-aged such that $T'_{1,h} < 0$ which, in turn, means that the first term on the right hand side of equation (19b) is positive. This is a consequence of the tax-complementary discussed above, which is here meant to imply that the marginal subsidy implemented for the middle-aged may lead to over-consumption of housing among the young.

If the consumers are sophisticated, Proposition 2 no longer gives a complete characterization of the marginal tax/subsidy rates facing the young generation, although the tax policy implemented for the middle-aged in Proposition 2 continues to apply. To present the marginal tax structure under sophistication in the simplest possible way, let $\hat{T}'_{1,s}$ and $\hat{T}'_{0,h}$ be short notations for the marginal tax/subsidy rates that apply under naivety, which are given by the right hand side expressions of equations (19a) and (19b). We can then characterize the marginal tax structure as in Proposition 3.

**Proposition 3.** If the consumers are sophisticated, the resource allocation preferred by the government implies the following marginal tax/subsidy rates implemented for the young generation $t$:

$$T'_{1,s} = \hat{T}'_{1,s} + \frac{(1 - \beta)}{\beta} \frac{1}{r_{t+1}} \frac{\partial s_{1,t+1}}{\partial s_{0,t}} + \frac{(1 - \beta)}{1 + r_{t+2}} \frac{P_{t+2}(1 - \delta)}{r_{t+1}} \frac{\partial I_{1,t+1}}{\partial s_{0,t}}$$

(20a)
\[ T'_{0,h} = \tilde{T}'_{0,h} + \frac{(1 - \beta)}{p_t} \frac{1}{(1 + r_{t+1})} \frac{\partial s_{1,t+1}}{\partial h_{0,t}} + \beta \frac{(1 - \beta)}{p_t} \frac{P_{t+2}}{\prod_{i=1}^{2}(1 + r_{t+i})} \frac{\partial l_{1,t+1}}{\partial h_{0,t}} \]  

(20b)

Proof: see Appendix III.

In equation (20b), we have used \( MRS_{c_2c_0} = \frac{1}{\prod_{i=1}^{2}(1 + r_{t+i})} \) from the social first order conditions. Without any restrictions on the functional form of the utility function, equations (20a) and (20b) cannot in general be signed. However, as we show in appendix II, a quasi-linear utility function, where the instantaneous utility faced by the old consumer is linear in consumption of the non-durable good, offers some guidance. In this case, equation (20a) reduces to

\[ T'_{1,s} = \frac{(1 - \beta)}{\beta} \frac{1}{r_{t+1}} \frac{\partial s_{1,t+1}}{\partial s_{0,t}} > T'_{1,s} \]  

(20a')

The intuition behind equation (20a') is straightforward. Sophisticated young individuals in period \( t \) correctly predict the present-biased preference of their middle-aged selves in period \( t+1 \) and thus incorporate the reactions of their future selves in their consumption choices. Since present-biased preferences induce middle-aged individuals to under-save, and since saving in period \( t \) increases the incentives to save in period \( t+1 \), sophisticated young individuals have incentives to engage in precautionary saving. This, in turn, reduces the need to subsidize the saving of the young generation.

Concerning the tax of housing wealth implemented for young individuals, the same quasi-linear utility function implies that we can identify two opposing effects such that

\[ T'_{0,h} = \tilde{T}'_{0,h} + \frac{(1 - \beta)}{p_t} \frac{1}{(1 + r_{t+1})} \frac{\partial s_{1,t+1}}{\partial h_{0,t}} + \beta \frac{(1 - \beta)}{p_t} \frac{P_{t+2}}{\prod_{i=1}^{2}(1 + r_{t+i})} \frac{\partial l_{1,t+1}}{\partial h_{0,t}}. \]  

(20b')

On the one hand, sophisticated individuals correctly predict that their middle-aged selves will under-save due to the appearance of present-biased preferences. Since the investment in housing as young increases the incentives to save for the middle-aged, young individuals have incentives to over-invest in housing. This implies that the tendency to over-consume housing (due to

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18 Equation (20a') is analogous to the formula for the savings-subsidy to sophisticated hyperbolic discounters derived by Aronsson and Sjögren (2011) in a different context; their model neither contains housing wealth nor relative consumption concerns. Furthermore, since hyperbolic discounting is the only imperfection in their model, they did not have to assume a quasi-linear utility function to be able to sign the comparative statics of the reaction function for savings.
positional concerns) is amplified, and thus a stronger motive for taxing the housing wealth faced by the young consumer. In other words, and given that the incentives faced by the middle-aged are corrected through the policy summarized in Proposition 1, the strategic action among sophisticated young consumers to induce their future selves to save more gives rise to an extra motive for taxing their housing wealth as represented by the second term on the right hand side of equation (20b'). The intuition is, therefore, that this extra motive for taxation targets the commitment strategy to over-invest in housing.

On the other hand, in addition to an under-saving in financial assets, present-biased preferences also imply that middle-aged individuals under-invest in housing according to equation (7b). Hence, sophisticated young individuals also have incentives to reduce their own consumption of the durable good in order to stimulate investments as middle-aged. For the government, this constitutes a reason to increase $h_{0,t}$ which, in turn, can be accomplished through a subsidy (or lower tax) on the housing wealth for sophisticated young consumers as reflected in the third term on the right hand side of equation (20b'). As such, the commitment strategy chosen by the young sophisticated consumer gives rise to an ambiguous policy incentive. In a simpler model without the additional housing investment, however, the third term on the right hand side of equation (20b') vanishes: in that case, the strategic incentive faced by sophisticated consumers would imply a motive for taxing their housing wealth at a higher marginal rate, ceteris paribus.

4. Optimal Taxation with upward comparisons by the young

In the above, we have assumed that individuals have positional preferences for housing in the sense of valuing their own consumption relative to that of other people in their own generation. As a consequence, although within-generation comparisons are in accordance with some empirical evidence (see the introduction), we have so far neglected the implications of other types of social comparisons. We now consider the case when young individuals also compare their own consumption level of durable goods with that of the middle-aged living in the same period. Since the middle-aged in our model have made an additional investment in housing (and presumably have more housing wealth than the young), this is interpretable as an upward comparison. Let us then define the following reference measures:

$$\tilde{h}_{0,t} = \sigma h_{0,t} + (1 - \sigma)h_{1,t} \quad (21a)$$

$$\tilde{h}_{1,t+1} = h_{1,t+1}, \quad (21b)$$
where $\omega; 0 \leq \omega \leq 1$, is the weight that each young individual attaches to the consumption of others within their own generation. As can be seen in equation (21b), we maintain the assumption that middle-aged individuals only compare their consumption level to that of other middle-aged individuals. Notice that all private first order conditions remain intact, since each private agent treats the measures of reference consumption as exogenous. The social first order conditions for $c_{0,t}, c_{1,t+1}, c_{2,t+2}, K_{t+1}$ and $K_{t+2}$ remain as in equations (16a)-(16c) and (16f) above, whereas the social first order conditions for $h_{0,t}$ and $I_{1,t+1}$ are now represented by equations (22a) and (22b), i.e.

$$
\frac{\partial L}{\partial h_{0,t}} = \sum_{i=0}^{1} \tilde{u}_{t}^{i} \Theta_{t+1}^{i}(1-\delta)^{i} - y_{t}P_{t} + y_{t+2}P_{t+2}(1-\delta)^{2} + \frac{\partial L}{\partial h_{0,t}}\omega
$$

(22a)

$$
\frac{\partial L}{\partial I_{1,t+1}} = \tilde{u}_{t}^{1} \Theta_{t+1}^{1} - y_{t+1}P_{t+1} + y_{t+2}P_{t+2}(1-\delta) + \frac{\partial L}{\partial h_{1,t+1}}
$$

$$
+ \frac{\partial L}{\partial h_{0,t+1}}(1-\omega) = 0
$$

(22b)

where the partial welfare effects of increased reference consumption, i.e. the two partial derivatives $\partial L/\partial h_{0,t}$ and $\partial L/\partial h_{1,t+1}$, are again given in equations (17a) and (17b).

4.1 Tax policy implemented for the middle-aged

When the young generation holds positional preferences based on within-generation and upward comparisons, the marginal tax structure for the middle-aged is characterized as described in proposition 4.

**Proposition 4. If the positional preferences are characterized by both within-generation and upward comparisons according to equations (21a) and (21b), the government may implement its preferred resource allocation by a marginal savings-subsidy to the middle-aged in accordance with equation (18a), whereas the policy rule for the marginal tax/subsidy on the housing wealth of the middle-aged is given by**
\[
T_{1,h}^t = \frac{\beta - 1}{1 + \gamma_{t+2}} \frac{P_{t+2}(1 - \delta)}{P_{t+1}} + \frac{\alpha_{1,t+1}}{P_{t+1}} \text{MRS}_{h_1,c_1} \\
+ \frac{\alpha_{0,t+1}}{P_{t+1}} \frac{\partial u_{0,t+1}}{\partial h_{0,t+1}} + \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} (1 - \omega). \tag{23}
\]

Proof. See Appendix III

The intuition behind the first part of Proposition 4 is that the tendency among the middle-aged to under-save in liquid assets is solely a consequence of hyperbolic discounting. Hence, since upward comparisons do not change the effect of present-biased preferences, the structure of the marginal subsidy on saving remains unaltered.

Concerning the tax/subsidy on housing wealth, upward comparisons imply that consumption of durable goods by the middle-aged not only imposes an externality on other middle-aged individuals, but also affects the reference level consumption faced by young individuals in the same time period. The term \(\alpha_{0,t+1} \frac{\partial u_{0,t+1}}{\partial h_{0,t+1}} + \frac{\partial u_{1,t+1}}{\partial c_{1,t+1}} (1 - \omega)\) reflects this effect, and it represents the extra tax placed on the middle-aged in order to correct for the extra positional externality imposed by the middle-aged on the young. The more positional people in the young generation are on average, as measured by the marginal degree of positionality \(\alpha_{0,t+1}\), and the higher the weight placed on the consumption of durables by the middle-aged in the reference measure faced by the young, the more likely it is that the right hand side of equation (23) is positive. As such, upward comparisons clearly strengthen the policy incentive to tax the housing wealth of the middle-aged. The intuition is, of course, that the larger the weight \((1 - \omega)\) that the young attach to the housing consumption of the middle-aged when forming the reference measure, the larger is the negative externality that the middle-aged impose on the young. Therefore, the housing tax policy presented in Proposition 4 may differ substantially from that of the benchmark model analyzed in Proposition 1.

4.2 Tax policy implemented for the young

Let us then turn to the optimal tax policy faced by the young. Consider first the case of naïve consumers, where the tax policy is characterized as follows:

**Proposition 5.** If the positional preferences are characterized by both within-generation and upward comparisons according to equations (21a) and (21b), and if the consumers are naïve, the government may implement its
preferred resource allocation by a marginal savings-subsidy to the young in accordance with equation (19a), whereas the policy rule for the marginal tax/subsidy on the housing wealth of the young is given by

$$T_{0,h}^* = -\beta(1 - \delta)p_{t+1} T_{1,h}^* + \frac{\beta - 1}{P_t} R_{0,t} + \omega \frac{\alpha_{0,t}}{P_t} MRS_{h_0,c_0}$$ (24)

where $T_{1,h}^*$ is defined in equation (23).

Proof. See Appendix III

Again, adding upward comparisons does not affect the incentive faced by the policy maker to subsidize the savings by the young; as such, the savings-subsidy is governed by the same incentives as implicit in equation (19a) above. However, as can be seen in equation (24), upward comparisons imply that the housing tax (subsidy) on the young is reduced (increased) through the weight-factor, $\omega$. The intuition behind this result is that, when young individuals place some weight on the average consumption of the middle-aged, their consumption does not generate as large externalities on other young individuals as in the case without upward comparisons. In addition, since the middle-aged only hold positional preferences related to consumption within their own generation, the consumption of young individuals does not produce any externalities in terms of affecting the utility of the middle-aged generation within the same time period. Indeed, the greater weight the young generation attaches to the consumption of the older generation, the less externalities do their own housing consumption produce. In the extreme case of $\omega = 0$, when each young individual only compares his/her own housing consumption with that of the middle-aged, no within-generation externality among the young is produced, and the government thus only has an incentive to correct for behavioral mistakes caused by hyperbolic discounting.

Therefore, if the within-generation comparison among the young is weak such that $\omega$ is small, Propositions 4 and 5 together imply a possible scenario where the housing wealth of the young is subsidized and the housing wealth of the middle-aged taxed at the margin; at least if the consumers are naïve.

Turning finally to the case where the consumers are sophisticated, the additional tax components, which are due to that sophisticated young consumers act strategically vis-à-vis their future selves, are the same as in Proposition 3 above. This is so because the incentive to act strategically is solely due to that a sophisticated young consumer realizes that his/her middle-aged self is subject to the same self-control problem as the one currently present; it has nothing to do with whether the positional concerns are governed by within-generation comparisons, upward comparisons or
a mix between these two possibilities. Therefore, Proposition 3 continues to apply here as well with the only modification that the variable $\tilde{h}_{0,h}$ on the right hand side of equation (20b) is now defined in terms of equation (24) instead of in terms of equation (19b).

6. Summary and Discussion

During the latest decades, household indebtedness has increased substantially in many countries along with the deregulation of the financial markets. Furthermore, a large share of these loans is typically related to housing. Arguments based on the literature on relative consumption comparisons and dynamic consumer choices, respectively, can be made as to why this behavior may neither be optimal for society as a whole nor for the consumers themselves in a longer time-perspective. We develop an OLG model where housing is (in part) a positional good, and where the consumers are also characterized by a preference for immediate gratification due to quasi-hyperbolic discounting. Each consumer lives for three periods and makes savings-decisions and housing-investments both when young (in the first period) and middle-aged (in the second). The main purpose is to analyze the associated tax policy incentives from the point of view of a paternalistic government, which does not share the consumer preference for immediate gratification, and which also aims at internalizing the externalities caused by positional concerns for housing. The policy instruments facing the government in our model are nonlinear taxes/subsidies on capital income and housing wealth, respectively.

Our results show that savings should in general be subsidized at the margin. The savings-subsidy result differs from a corresponding result derived by Aronsson and Johansson-Stenman (2010), who found that positional concerns may either lead to higher or lower marginal capital income tax rates depending on whether the marginal degree of positionality increases or decreases over the individual life-cycle. In their framework, private consumption as a whole constitutes a positional good, whereas the present study distinguishes between a positional good (housing) and a non-positional good, which explains why we find that the tax/subsidy on housing wealth is used to internalize the positional consumption externality.

In our benchmark model where the positional concerns are driven solely by within-generation comparisons, housing wealth should either be taxed or subsidized depending on (among other things) whether the tax incentive caused by conspicuous housing consumption dominates, or is dominated by the tendency to under-value future housing wealth created by hyperbolic discounting. However, if we also allow for upward comparisons such that the young compare their housing consumption with that of the middle-aged (who have made a second housing-
investment), the policy incentive to tax the housing wealth held by the young becomes weaker, while the incentive to tax the housing wealth held by the middle-aged becomes stronger. In that case, a possible scenario is that the government may implement its preferred resource allocation through a policy mix where the housing wealth of the young is subsidized and the housing wealth of the middle-aged taxed at the margin; at least if the consumers are naïve. Under sophistication, another motive for the government to influence the accumulation of housing wealth also arises, as young sophisticated consumers use housing wealth as a commitment device. This may, or may not, strengthen the motive for taxing the housing wealth held by young sophisticated consumers, depending on whether the incentive faced by young consumers to act strategically leads to over-investment (to stimulate future savings) or under-investment (to stimulate future investments) in housing wealth.

Future research may take several directions and we briefly discuss two of them here. First, we have assumed a perfect capital market and made no distinction between housing loans and other aspects of saving/borrowing. In reality, there are sometimes such differences, and housing loans may be governed by different conditions than other types of loans. Although our result show that a first best optimum for a paternalistic government can be implemented by the two tax instruments discussed above without adding any explicit restrictions on housing loans, it would, nevertheless, be interesting to address the welfare consequences of such restrictions in future research. For example, the sub-prime housing loans that partially caused the 2008 financial crisis were characterized by teaser rates (i.e., very low or negative interest payments during the first year(s) of the loan contract). With these types of house loans, hyperbolic discounting will have implications beyond those dealt with in our study. Second, the model presented in this paper does not consider uncertainty of future house prices. If the purchase of a house is associated with the potential risk of a future financial loss, new implications of present biased preferences may arise. However, adding uncertainty to the current model would mainly imply that the mechanisms of interest to the current analysis become less clear. In addition, there is little evidence that the housing market should be expected to be more uncertain than e.g. markets for financial assets. Hence, although uncertainty related to the housing market is interesting in its own right, it should perhaps be analyzed in a more simplistic framework than the one presented here.

Appendix I
<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{j,t}$</td>
<td>Consumption of the non-durable good by age-group $j$ in period $t$</td>
</tr>
<tr>
<td>$h_{0,t}$</td>
<td>Consumption of durable good by the young in period $t$</td>
</tr>
<tr>
<td>$I_{1,t}$</td>
<td>Investment in durable good by the middle-aged in period $t$</td>
</tr>
<tr>
<td>$h_{j,t}$</td>
<td>Stock of durable good held by the individual of age-group $j$ in period $t$</td>
</tr>
<tr>
<td>$\bar{h}_{j,t}$</td>
<td>Reference level of consumption of the durable good for age-group $j$ in time period $t$</td>
</tr>
<tr>
<td>$\Delta_{j,t}$</td>
<td>$h_{j,t+i} - \bar{h}_{j,t+i}$: Relative consumption of the durable good by age-group $j$ in time period $t$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate of durable goods</td>
</tr>
<tr>
<td>$P_{t}$</td>
<td>Price of durable good in period $t$</td>
</tr>
<tr>
<td>$r_{t}$</td>
<td>Interest rate in period $t$</td>
</tr>
<tr>
<td>$\Theta^{t}$</td>
<td>Exponential discount factor for time period $t$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Quasi-hyperbolic discount factor</td>
</tr>
<tr>
<td>$\gamma_{j,t}$</td>
<td>Exogenous income of age-group $j$ in period $t$</td>
</tr>
<tr>
<td>$s_{j,t}$</td>
<td>Saving in liquid wealth by age-group $j$ in period $t$</td>
</tr>
<tr>
<td>$T_{j}$</td>
<td>Tax payments by age-group $j$</td>
</tr>
</tbody>
</table>

**Appendix II**

Suppose that the instantaneous utility function faced by the middle-aged is additively separable in the non-durable good, and that the instantaneous utility faced by the old is linear, such that
\[ u_{1,t+1} = \tilde{u}_1(c_{1,t+1}, h_{1,t+1}, \bar{h}_{1,t+1}) = \phi(c_{1,t+1}) + \psi(h_{1,t+1}, \bar{h}_{1,t+1}) \]  \quad (A1)

\[ u_{2,t+2} = \tilde{u}_2(c_{2,t+2}) = c_{2,t+2} \]  \quad (A2)

which means that the remaining life-time utility faced by the middle-aged, \( u_{1,t+1} + \beta u_{2,t+2} \theta \), is quasi-linear. The right hand side of equation (A1) is increasing in \( c_{1,t+1} \) and \( h_{1,t+1} \) as well as strictly concave. By choosing \( s_{1,t+1} \) and \( I_{1,t+1} \) to maximize this life-time utility function for the middle-aged subject to the budget constraint given in equations (5b) and (5c), while treating the marginal tax/subsidy rates \( T'_{2,s} \) and \( T'_{1,h} \) as constant in the resulting first order conditions, we can derive the following qualitative comparative statics with respect to \( s_{0,t} \) and \( h_{0,t} \):

\[
\frac{\partial s_{1,t+1}}{\partial s_{0,t}} > 0, \quad (A3a)
\]

\[
\frac{\partial I_{1,t+1}}{\partial s_{0,t}} = 0, \quad (A3b)
\]

\[
\frac{\partial s_{1,t+1}}{\partial h_{0,t}} > 0, \quad (A3c)
\]

\[
\frac{\partial I_{1,t+1}}{\partial h_{0,t}} < 0. \quad (A3d)
\]

In the opposite extreme case, where the middle-aged consumer’s life-time utility in linear in \( c_{1,t+1} \) and nonlinear in \( c_{2,t+2} \), it follows that \( s_{0,t} \) will vanish from the first order conditions for \( s_{1,t+1} \) and \( I_{1,t+1} \). In that case, we also have \( \partial I_{1,t+1}/\partial h_{0,t} < 0 \), whereas the other three comparative statics derivatives will be zero.

Appendix III

Proof of Proposition 1

Consider first the expression for \( T'_{2,s} \). By using equations (16b), (16c) and (16e), we can derive the following savings condition from the government’s decision problem:

\[-\tilde{u}'_{c_1} + \tilde{u}'_{c_2} \theta (1 + r_{t+2}) = 0. \quad (A4)\]

For equations (7a) and (A4) to hold simultaneously, \( T'_{2,s} \) must be such that
\[(1 - \beta)\bar{u}'_{c_2}(1 + r_{t+2}) + \beta \bar{u}'_{c_2} T'_{2,s} r_{t+2} = 0.\]  \hspace{1cm} (A5)

Solving equation (A5) for \(T'_{2,s}\) and rearranging gives equation (18a).

Continuing with the derivation of the expression for \(T'_1\), note first that the social first order condition for \(I_{1,t+1}\) can be written as

\[\bar{u}'_{h_1} - \bar{u}'_{c_1,1} P_{t+1} + \bar{u}'_{c_2} \Theta P_{t+2} (1 - \delta) - \alpha_{t+1} \bar{u}'_{h_1} = 0,\]  \hspace{1cm} (A6)

where we have used equations (16b), (16c), (16e) and (16f). For equations (7b) and (A6) to hold simultaneously, it follows by comparison that \(T'_1\) must solve the following equation:

\[\bar{u}'_{c_1} T'_{1,h} P_{t+1} + (1 - \beta) \bar{u}'_{c_2} \Theta P_{t+2} (1 - \delta) - \alpha_{t+1} \bar{u}'_{h_1} = 0.\]  \hspace{1cm} (A7)

Solving equation (A7) for \(T'_1\), while using equation (A1), gives equation (18b).

**Proof of Propositions 2 and 3**

Consider first \(T'_1\). In order for the government to reach the desired resource allocation, both equations (16a)-(16b) and the private first order condition for saving, equation (10a), need to be fulfilled. Note that, from equations (16a)-(16b) and equation (16f), the following must hold.

\[-\bar{u}'_{c_0} + \Theta \bar{u}'_{c_1} (1 + r_{t+1}) = 0\]  \hspace{1cm} (A8)

For equations (A8) and (10a) to hold simultaneously, \(T'_{1,s}\) must be set such that

\[(\beta - 1) \bar{u}'_{c_1} (1 + r_{t+1}) + (1 - \beta) \Theta \bar{u}'_{c_1} \frac{\partial s_{1,t+1}}{\partial s_{0,t}} \]  \hspace{1cm} (A9)

\[+ \beta (1 - \beta) \Theta^2 \bar{u}'_{c_2} P_{t+2} (1 - \delta) \frac{\partial I_{1,t+1}}{\partial s_{0,t}} - \beta \Theta \bar{u}'_{c_1} T'_{1,s} r_{t+1} = 0\]

Solving equation (A9) and rearranging terms gives equation (20a).

Turning to the marginal durables’ tax for the young generation, note that the social first order condition for \(h_{0,t}\) can be written

\[\bar{u}'_{h_0} + \Theta \bar{u}'_{h_1} (1 - \delta) - \bar{u}'_{c_0} P_t + \bar{u}'_{c_2} \Theta^2 P_{t+2} (1 - \delta)^2 - \alpha_{0,t} \frac{\partial u_{0,t}}{\partial h_{0,t}} = 0\]  \hspace{1cm} (A10)
where we have used equations (16a), (16c), (16d) and (16f). For equations (10b) and (A10) to hold simultaneously, it follows by comparison that $T'_{0,h}$ must solve the following equation:

$$
(\beta - 1)\partial u'_{c_2} P_{t+2}(1 - \delta)^2 + \alpha_{0,t}\hat{u}'_{h_0} - \beta \hat{u}'_{c_1} T''_{1,h} P_{t+1} \Theta (1 - \delta)
+ \beta \hat{u}'_{h_1} \Theta (1 - \delta) + (1 - \beta) \Theta \left[\hat{u}'_{c_1} (1 + \tau_{t+1})\right] \frac{\partial s_{t+1}}{\partial h_{0,t}}
+ \beta (1 - \beta) \Theta^2 \hat{u}'_{c_2} P_{t+2}(1 - \delta) \frac{\partial I_{t+1}}{\partial h_{0,t}} - \hat{u}'_{c_2} T''_{0,h} P_{t} = 0
$$

(A11)

Solving equation (A11) for $T'_{0,h}$, while using equation (A5), gives equation (20b).

Proof of Proposition 4

Notice first that since neither the private first order conditions for saving and housing investment by the middle-aged, nor the social first order conditions for non-durable consumption and the capital stock are affected by introducing upward comparisons, the savings subsidy implemented for the middle-aged can also in this case be derived by solving equation (A5) for $T'_{2,s}$.

Then, for equations (7b) and (22b) to hold simultaneously, it follows by comparison that $T'_{1,h}$ must solve the following equation:

$$
-u'_{c_1} T'_{1,h} P_{t+1} + (\beta - 1)\partial u'_{c_2} P_{t+2}(1 - \delta) + \alpha_{1,t+1} u'_{h_1}
+ (1 - \sigma) \alpha_{0,t+1} u'_{h_{0,t+1}} = 0
$$

(A12)

Therefore, solving equation (A12) for $T'_{1,h}$ gives equation (23).

Proof of Proposition 5

Since neither the private first order conditions for saving and housing investment by the young nor the social first order conditions for non-durable consumption and the capital stock are affected by introducing upward comparisons, the savings subsidy implemented for the young can also in this case be derived by solving equation (A9) for $T'_{1,s}$.

Then, notice that equation (22a) can be written as

$$
\hat{u}'_{h_0} + \Theta \hat{u}'_{h_1} (1 - \delta) - \hat{u}'_{c_2} P_{t} + \hat{u}'_{c_2} \Theta^2 P_{t+2}(1 - \delta)^2 - \alpha_{0,t} \Theta \frac{\partial u_{0,t}}{\partial h_{0,t}} = 0.
$$

(A13)
For equations (11b) and (A13) to hold simultaneously, it follows that $T'_{0,h}$ must solve the following equation:

\[
(\beta - 1) \theta \bar{u}_c P_{t+2} (1 - \delta)^2 + \alpha_{0,t} \sigma \bar{u}_{h_0} - \beta \bar{u}_{c_1} T'_{1,h} P_{t+1} \Theta (1 - \delta) + \beta \bar{u}_{h_1} \Theta (1 - \delta) - \bar{u}_{c_0} T'_{0,h} P_t = 0
\]

(A14)

Solving equation (A14) for $T'_{0,h}$ and rearranging gives equation (24).■

References


