Can Labor Market Imperfections Cause Overprovision of Public Inputs?∗

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Abstract

This paper concerns provision of productive public inputs in the presence of unemployment. It is shown that if the government is able to implement optimal taxes on labor income and profit income, respectively, then the public input will be underprovided. On the other hand, if the government is not able to implement an optimal tax on labor income, e.g. because the labor income tax is determined at another level in the public sector (e.g. the municipal or the state level), then overprovision may occur. We derive an equation which links overprovision of the public input to (i) the employment rate and to (ii) the deviation of the actual labor income tax from the optimal level.

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1 Introduction

An important question in public economics is whether the provision of a public good in a second-best economy will exceed, or fall short of, the quantity provided in a first-best setting. In the literature, the main focus has been to analyze how the use of distortionary taxes,1 or the presence of labor market distortions,2 may influence the provision of a public good. However, these issues have received far less attention in the context of public inputs in the production.3 In particular, no previous study has made an attempt to relate the quantity provided of a public input to the rate of unemployment. This is somewhat surprising since the provision of a public input will be influenced by the presence of unemployment.4 The argument is that if the public input is complementary with labor in the production, then the government may have an incentive to provide more of that input to induce firms to hire more labor. The question is then whether this incentive will be sufficiently strong to cause overprovision compared with the first-best quantity?

The purpose of this paper is to analyze this issue. The labor market imperfection is assumed to arise because of trade union wage setting and we analyze two cases; one where the government faces no restriction in how it

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2See, for example, Bovenberg and van der Ploeg (1996), and Aronsson and Sjögren (2003).
3Exceptions are Martinez and Sanchez (2009a,b) who analyze the effects of distortionary taxes on the provision of a public input.
4See Aronsson and Wehke (2008) who characterize the optimal provision of public inputs in an economy where the labor market is dominated by trade unions, and Aronsson and Koskela (2008) who characterize public input provision in the presence of unemployment and oursourcing.
can use labor and profit income taxes to finance the provision of a public input, and another case where the government cannot implement the labor income tax that would be optimal from the government´s point of view. In the first case, we show that when the government is able to implement an optimal tax on labor income, then the public input will be underprovided compared with a first-best setting.

However, as has been pointed out in the literature on fiscal externalities\(^5\), in a multi-level system where there are different levels of governments, a policy-maker may not always be able to implement an income tax which is optimal. In Sweden, for example, income taxes are partly determined at the municipal level whereas decisions on major infrastructure projects are determined at the national level. In this context, the municipal level may implement income tax rates which are not optimal from the national government´s point of view. Therefore, we in this paper also address the following question: If the government cannot implement a labor income tax which is optimal from its point of view, will this constitute a motive to overprovide the public good? We show that the answer can be yes and we derive a formula which relates overprovision of the public input to the rate of employment and to the deviation of the actual labor income tax rate from its optimal level. Among the results, we find that overprovision of the public input can only occur if the implemented tax on labor income exceeds the optimal tax rate.

The outline of the paper is as follows. In section 2, we present the basic model while the main results are derived in Section 3. The paper is concluded in Section 4.

\(^5\)Early references in the field of vertical fiscal externalities are Hansson and Stuart (1987), and Johnson (1988).
2 The Basic Model

Consider an economy made up of firms, consumers, trade unions and a government. The firms are identical and their number is normalized to one. We assume, as in Barro (1990) and Barro and Sala-i-Martin (2001), that the production function is of Cobb-Douglas type

\[ F(N, K, G) = N^\alpha \cdot K^{1-\alpha} \cdot G^{1-\alpha} \]  

where \( \alpha \in (0, 1) \), \( N \) is labor, \( K \) a fixed factor and \( G \) a public input.\(^6\) This formulation of the technology means that the public input is factor augmenting. In the following, the fixed factor will be normalized to one, in which case we can write the profit as

\[ \Pi = F(N, G) - w \cdot N \]  

where \( w \) is the wage rate. The first-order condition for profit maximization implies \( w = F_N(N, G) \), which implicitly defines the following labor demand function

\[ N(w, G) = \alpha \frac{1}{\alpha} \cdot G \cdot w^{-\frac{1}{\alpha}} \]  

Substituting this labor demand function into equation (2) defines the profit as a function \( \Pi(w, G) \).

Turning to the consumption side of the economy, all consumers have identical preferences for consumption, \( c \), and these preferences are described by an isoelastic utility function

\[ u(c) = \begin{cases} \frac{c^{1-\eta} - 1}{1 - \eta} & \text{where } 0 < \eta < 1 \\ \ln(c) & \text{if } \eta = 1 \end{cases} \]  

There are three types of agents: (i) a firm-owner, (ii) employed workers and (iii) unemployed workers, and these agent types are distinguished by

\(^6\)If the exponent on \( G \) would be smaller (larger) than \( 1-\alpha \), then diminishing (increasing) returns to \( K \) and \( G \) would apply in a dynamic context (see Barro and Sala-i-Martin 2001).
the superindices "f", "e" and "u". Beginning with the firm-owner, he is endowed with the fixed factor. The firm-owner does not work and he receives the profit income in return for providing the fixed factor to the firm. The firm-owner’s net income is given by \( \Pi - T^f \), where \( T^f \) is a lump-sum tax. Turning to the labor force, it is made up of \( M \) workers out of whom \( N \) are employed and \( M - N \) are unemployed. A worker who is not employed receives an exogenous alternative income, \( b \), which can be thought of as the income earned if he emigrates to another jurisdiction or if he works on the black market. As for an employed worker, he supplies one unit of labor inelastically in return for the after-tax wage \( \omega = (1 - t) \cdot w \), where \( t \) is the labor income tax rate.

All workers are assumed to belong to a trade union which.\(^7\) Following e.g. Oswald (1993), and Aronsson and Sjögren (2004), we assume that there is a known lay-off ordering among the workers and the objective of the union is to maximize the utility of the union member with median seniority, conditional on that this person does not become unemployed. This means that the trade union’s objective function concides with the utility function of the median union member, \( u(\omega) \).

The wage rate is determined in a bargain between the trade union and the firm. This bargain will be characterized within a right-to-manage framework.\(^8\) If no contract is signed, the union members become unemployed and obtain the fall-back utility \( u(b) \), whereas the firm makes zero profit. If we define \( \Phi = u(\omega) - u(b) \) and \( \Pi(w,G) - 0 \) to be the rents from bargaining, the outcome of the bargain will be the wage rate that maximizes the Nash product \( \Phi^\beta \cdot \Pi^{1-\beta} \), where \( \beta \in (0,1) \) is the bargaining power of the trade union. At an interior solution, the first-order condition can be written as

\[
\beta \cdot \Pi(w,G) \cdot (1 - t) \cdot u'(\omega) + (1 - \beta) \cdot [u(\omega) - u(b)] \cdot \Pi_w(w,G) = 0 \quad (6)
\]

Equation (6) implies that the after-tax wage will satisfy \( \omega > b \) and by using

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\(^7\)Since the number of firms is normalized to one, so is also the number of trade unions.

\(^8\)See Oswald (1985).
the functional forms of the production function and the utility function, we can solve for the bargained wage as a function

\[ w = \frac{A \cdot b}{1 - t} \]  

(7)

where \( A \) is a positive constant.9

3 Optimal Policy

The government maximizes a utilitarian social welfare function

\[ W = N \cdot u^c + (M - N) \cdot u^u + u^f \]  

(8)

and the budget constraint is given by

\[ t \cdot w \cdot N + T^f - G = 0 \]  

(9)

In the unrestricted optimal tax problem, the policy instruments are \( t, T^f \) and \( G \), and the Lagrangian corresponding to the government’s problem can be written as

\[ \mathcal{L} = N \cdot u^c + (M - N) \cdot u^u + u^f + \gamma \cdot (t \cdot w \cdot N + T^f - G) \]  

(10)

where \( \gamma \) is the Lagrange multiplier associated with the government’s budget constraint.

Let us first consider the optimal policy for the public input in a first-best setting. Within the context of this model, this corresponds to the policy chosen when there is no unemployment. Substituting \( N = M \) into equation (10) and maximizing w.r.t. to the policy instruments, it is straightforward

9If \( \eta \neq 1 \), then

\[ A = \left[ 1 - (1 - \eta) \cdot \frac{\beta}{1 - \beta} \cdot \frac{1 - \alpha}{\alpha} \right]^{-1/\eta} \]

whereas if \( \eta = 1 \), then

\[ A = e^{\frac{\beta}{1-\eta} \cdot \frac{1-\alpha}{\alpha}} \]
to show that the solution to this problem produces the standard condition for an optimal provision of the public input

$$F_G (M, G^*) = 1$$

(11)

where $G^*$ denotes the first-best quantity. Note that conditional on $M$, there is a unique level of $G^*$ which satisfies equation (11).

### 3.1 Unrestricted Income Taxation

Next, we turn to the case when there is unemployment in equilibrium. Let us begin with the optimal tax policy. Differentiating the Lagrangian w.r.t. $t$ and $T^f$, and then using these first-order conditions to solve for the optimal labor income tax rate, $t^*$, we can derive the following tax formula (see the Appendix)

$$t^* = -\frac{u(\omega) - u(b)}{\gamma \cdot w}$$

(12)

Since the bargained after-tax wage satisfies $\omega > b$, the optimal tax policy involves subsidizing labor income. The reason is that since the bargained wage defined in equation (7) is increasing in $t$, the government implements a subsidy on labor income to induce the trade union to opt for a lower wage. The subsidy is financed via the lump-sum tax on profit income.

Conditional on this tax policy, let us now consider the provision of the public input. Differentiating the Lagrangian defined in equation (10) w.r.t. $G$, and then using the first-order condition for $T^f$ together with $w = F_N$, produces

$$0 = F_G (N (w, G^\circ), G^\circ) - 1 + N_G (w, G^\circ) \cdot \left[ \frac{u(\omega) - u(b)}{\gamma} + t \cdot w \right]$$

(13)

where $G^\circ$ denotes the quantity of the public input chosen in the presence of unemployment. Equation (13) shows that two incentives influence the provision of the public input in this case; a "pure motive" captured by the term $F_G - 1$ and an "employment motive" reflected by the remaining expression on the right hand side (RHS) in equation (13). Beginning with
the "pure motive", observe first that since $N$ and $G$ are complements in production, it follows that $F_G(N,G) < F_G(M,G)$ whenever $N < M$. This means that if there is unemployment in equilibrium, then the "pure motive" induces the government to provide less of the public input. The "employment motive", on the other hand, will induce the government to provide more of the public input as long as $N_G > 0$, and as long as the expression inside square brackets in equation (13) is positive. However, from equation (12) it follows that if the government is able to implement the optimal tax $t^*$, then the expression inside square brackets is zero in which case equation (13) reduces to $F_G(N,G^o) = 1$. In this situation, only the "pure motive" for providing the public input remains which leads to underprovision. This result is summarized as follows;

**Proposition:** Consider an economy where the wage bargain between firms and trade unions create unemployment, and where the government is free to choose the tax on labor income optimally. In this situation, the public input will be underprovided compared with the first-best outcome.

### 3.2 Restricted Income Taxation

Let us now consider the situation discussed in the introduction where the government cannot implement $t^*$. Instead the tax on labor income is fixed at a level $\bar{t}$, which is treated as given by the government. In this case, equation (13) will be given by

$$0 = F_G(N(w, G^o), G^o) - 1 + N_G(w, G^o) \cdot w \cdot (\bar{t} - t^*)$$

(14)

where $t^*$ is the "desired" optimal labor income tax which is defined in equation (12).

As long as $\bar{t} \neq t^*$, the "employment motive" discussed above will no longer be redundant. If $\bar{t} > t^*$, the "employment motive" induces the government to provide more of the public input and the question is when this
"employment motive" is sufficiently strong to cause overprovision of the public input in the sense that $G^o > G^*$. To analyze this, let us use that the first-best quantity $G^*$ is implicitly determined by equation (11). Substituting $F_G(M,G^*) = 1$ into equation (14) and using that

$$F_G(N,G^*) = (1 - \alpha) \cdot N^\alpha \cdot G^{-\alpha}$$

(15)

$$N_G(w,G) = \frac{N}{G}$$

(16)

$$w = F_N(N,G) = \alpha \cdot N^{\alpha-1} \cdot G^{1-\alpha}$$

(17)

produces

$$\left( \frac{G^o}{G^*} \right)^{\alpha} = \left[ 1 + \frac{\alpha}{1-\alpha} \cdot (\bar{t} - t^*) \right] \cdot \left( \frac{N}{M} \right)^{\alpha}$$

(18)

If there is overprovision of the public input, then $G^o/G^* > 1$. As such, equation (18) implies that employment rates $(N/M)$ associated with overprovision will satisfy the following inequality

$$\frac{N}{M} > \left[ 1 + \frac{\alpha}{1-\alpha} \cdot (\bar{t} - t^*) \right]^{-\frac{1}{\alpha}}$$

(19)

Three observations can be made from this analysis. First, the inequality in equation (19) cannot be satisfied if $\bar{t} < t^*$. As such, overprovision can never occur if the implemented labor income tax $(\bar{t})$ is below the "desired" labor income tax $(t^*)$. Second, conditional on the output elasticity of labor $(\alpha)$, the larger $\bar{t}$ is relative to $t^*$, the smaller will be the employment rates that are associated with overprovision of the public input. For example, if $\alpha = 0.5$ and $\bar{t} - t^* = 0.05$, then overprovision will occur if $N/M$ exceeds 0.90803 whereas if $\alpha = 0.5$ while $\bar{t} - t^* = 0.10$, then overprovision will occur if $N/M$ exceeds 0.82645. Third, the expression on the RHS in equation (19) is generally decreasing in $\alpha$.\textsuperscript{10} As such, for a given deviation in the labor

\textsuperscript{10}The derivative of the RHS in equation (19) is given by

$$\frac{1}{\alpha^2} \cdot \ln \left[ 1 + \frac{\alpha}{1-\alpha} \cdot (\bar{t} - t^*) \right] \cdot \left[ 1 + \frac{\alpha}{1-\alpha} \cdot (\bar{t} - t^*) \right]^{-\frac{1}{\alpha}}$$

$$- \frac{1}{\alpha} \cdot \left[ \frac{1}{(1-\alpha)^2} \cdot (\bar{t} - t^*) \right] \cdot \left[ 1 + \frac{\alpha}{1-\alpha} \cdot (\bar{t} - t^*) \right]^{-\frac{1}{\alpha} - 1}$$
income tax, \( \bar{t} - t^* \), overprovision will be associated with lower employment rates, the smaller is \( \alpha \). In particular, if \( \alpha \) approaches one, then the RHS in equation (19) approaches zero. In this situation there will be overprovision of the public input for almost any level of \( N/M \) (except when \( N/M \) is very small).

These results can be summarized as follows;

**Proposition 2:** Consider an economy where the wage bargain between firms and trade unions create unemployment and where the production technology is of Cobb-Douglas type. If the employment level in this setting satisfies the inequality

\[
N > \left[ 1 + \frac{\alpha}{1 - \alpha} \cdot (\bar{t} - t^*) \right]^{-\frac{1}{\alpha}} \cdot M
\]

then the public input will be overprovided compared with the first-best quantity.

4 Concluding Remarks

In this paper, we analyze if unemployment may induce the government to overprovide a public input in the production. We show that if the government is able to implement optimal taxes on labor income and profit income, respectively, then the public input will be underprovided. On the other hand, if the government is not able to implement an optimal tax on labor income, then overprovision may occur. In this case, we derive an equation which links overprovision of the public input to (i) the rate of employment and (ii) the deviation of the actual labor income tax from the optimal level. An avenue for future research would be to analyze to what degree this result will hold in the presence of distortionary taxes. It would also be interesting

Since the expression in the first row contains the natural log, this expression is for most values of \( \alpha \) smaller in absolute value than the expression in the second row which is negative.
to study whether the result derived in the paper is reflected in empirical data.

5 Appendix

Differentiating equation (10) w.r.t $b, G, t$ and $T^f$ produces

$$\frac{\partial L}{\partial T^f} = \gamma - u'(\Pi - T^f) = 0 \quad (A.1)$$

$$\frac{\partial L}{\partial t} = w \cdot N \cdot [\gamma - u'(\omega)] + \frac{\partial L}{\partial w} \frac{\partial w}{\partial t} = 0 \quad (A.2)$$

$$\frac{\partial L}{\partial G} = 0 = \Pi_G \cdot u'(\Pi - T^f) + N_G \cdot [u(\omega) - u(b)]$$

$$+ \gamma \cdot [t \cdot w \cdot N_G - 1] \quad (A.3)$$

where

$$\frac{\partial L}{\partial w} = N_w \cdot [u(\omega) - u(b)] + N \cdot (1 - t) \cdot u'(\omega)$$

$$- N \cdot u(\pi - T^f) + \gamma \cdot [t \cdot N + t \cdot w \cdot N_w] \quad (A.4)$$

To derive the tax formula in equation (12), we substitute (A.1) and (A.4) into (A.2), and use the properties of (7), and then solve for $t^*$.

References


