Tax Policy in an Economic Federation With Proportional Membership Fees*

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Abstract
A significant part of the revenue in the EU budget is raised via a GNI-based resource. The purpose of this paper is to analyze how this way of raising funds to the central authority in an economic federation affects the tax policy implemented by the lower level jurisdictions. This question is analyzed both when labor is immobile, as well as mobile, between the jurisdictions. A key result is that if the government in a lower level jurisdiction acts as a Nash follower, then it has an incentive to implement a distortionary tax on labor whereas if the lower level government is able to act as a strategic leader within the federation, then the incentive to distort the labor market may be redundant.

Keywords: efficiency, optimal taxation, economic federation.
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1. Introduction

There is a growing literature that deals with optimal taxation and public expenditure in the context of economic federations. The bulk of this literature has some common features. First, most of these studies focus on centralized leadership where the central authority is able to commit to its policies whereas the lower level governments are not.¹ Second, the central authority is usually assumed to raise revenue either via lump sum fees imposed on the lower level governments, or by taxing some lower level tax base (such as labor income or the total sales of goods and services within a lower level jurisdiction). Although these assumptions are well suited to describe the institutional settings within most sovereign countries, these assumptions do not capture some key characteristics of the European Union (EU).

One feature of EU is that it is a political confederation where the governments in the member countries tend to precommit to their own policies after which the central authority redistributes income (e.g. via Structural and Cohesion Funds) conditional on the national policies.² From this perspective, the central authority in EU can be viewed as a Nash follower vis-a-vis the national governments while (at least the large member countries) can be viewed as Stackelberg leaders vis-a-vis the central authority. This type of hierarchical structure is usually referred to as a decentralized leadership.³

Another characteristic of EU is that the revenue available for the central authority comes from two main sources labelled own resources and other revenue⁴. The bulk of budgetary expenditure is financed from the system of own resources and there are three main categories of own resources; (i) traditional own resources which mainly consist of custom duties on imports from outside the EU and sugar levies, (ii) a VAT-based resource which is a standard percentage levied on a value-added-tax (VAT) base in each EU country and (iii) a GNI-based resource where a standard percentage is levied on the gross national income (GNI) of each EU country. Today the GNI-based resource is the dominant source of revenue and represents about 60% of total own resources.

These stylized facts indicate that a theoretical analysis of the incentive structure underlying public policy in the EU needs to incorporate (i) decentralized leadership in the sense that at

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² This has been emphasized by Caplan and Silva (2011).
⁴ Examples of other revenue are revenue accruing from the administrative operations of EU institutions such as proceeds from the sale of property, the supply of services and from bank interest. As such, other revenue is a minor source of income within the general EU budget.
least one lower level jurisdiction is able to act as a strategic leader vis-a-vis the federal level and (ii) a GNI-based source of income for the central authority. Doing this would make it possible to address several important questions. For example, if the fee paid by each lower level government to the federal level is proportional to that jurisdiction’s national income, what are the implications for the tax policy implemented in the lower level jurisdictions? How will this tax policy differ compared with a situation where the federal level uses lump sum fees to raise revenue? To what degree does the tax policy chosen by a lower level government depend on whether it is a Nash follower or whether the lower level government is able to act as a Stackelberg leader either vis-a-vis the central authority (vertical leadership) or vis-a-vis the other member countries (horizontal leadership)?

To my knowledge there are no earlier studies that simultaneously address decentralized leadership and proportional membership fees. This observation constitutes the point of departure for this paper where the purpose is to analyze how the optimal tax policy implemented by the lower level jurisdictions (countries) is affected when at least one of the lower level governments is able to act as a strategic leader vis-a-vis the central authority, and where the central authority raises revenue using a fee which is proportional to the national income in each member country. To analyze this issue, a model is set up where the income tax system at the national level is nonlinear. This is a reasonably realistic description of the tax instruments that many countries have at their disposal. It also means that the use of distortionary taxes is a consequence of optimization which allows for an analysis of how the national governments will respond to the different incentive structures implied by whether the federal level uses proportional fees or lump sum fees to raise revenue. To begin with, labor is assumed to be immobile between the jurisdictions but in the latter part of the paper, this assumption will be relaxed. In the final part the model is extended to include horizontal leadership where one member country is able to act as a strategic leader vis-a-vis the other member countries in the federation.

Only a few earlier studies have examined the incentive structure in a federation with decentralized leadership. Aronsson (2010) analyzes local public good provision and redistribution in an economic federation where the public sector is not able to observe the ability of the individual agents. Aronsson shows how federal ex-post redistribution modifies the provision of the local public good and the design of the optimal income tax. Caplan et al (2000) study federal lump sum redistribution and spillover effects of local public goods. With redistribution carried out by the central authority that is directed to the private sector, it is shown that decentralized leadership gives rise to an efficient outcome. Caplan and Silva
(2011) analyze decentralized provision of an impure public good where the central authority provides matching grants and redistributes income after the lower level jurisdictions have determined their contributions to the impure public good. Labor is assumed to be imperfectly mobile and it is shown that an equilibrium with positive contributions to the impure public good is Pareto efficient. Another issue that has been addressed in the context of decentralized leadership is environmental policy. Aronsson et al (2006) consider a model where the central authority sets emission targets to be implemented by the lower level jurisdictions. The governments in the lower level jurisdictions face a mixed tax problem and it is shown how the lower level governments use their policy instruments to influence the emission target imposed by the central authority. It is also shown that this provides an incentive for implementing a distortionary tax on labor and that the commodity tax does not satisfy the additivity property.

The outline of the paper is as follows. The private sector is described in Section 2 whereas the public sector and the tax policy with immobile labor is characterized in Section 3. In Section 4, the model is modified to allow for labor mobility while Section 5 concerns horizontal leadership. The paper is concluded in Section 6.

2. The Private Sector

Consider an economic federation made up of two countries, denoted 1 and 2, and a federal level. Each country $i$, $i = 1, 2$, is made up of $N_i$ identical consumers. To begin with, it is assumed that the consumers are immobile and cannot work in the other country. Each consumer in country $i$ has a utility function which is written

$$U_i = u(c_i, z_i) + \phi(g_i)$$

(1)

where $c_i$ is consumption and $z_i$ is leisure. The latter is given by $z_i = h - l_i$, where $h$ is a time endowment and $l_i$ the hours of work. As for $g_i$, it is a national public good which is provided by the government in country $i$, and the welfare effect of $g_i$ is confined within country $i$. The subutility functions in equation (1) satisfy

$$\frac{\partial u_i}{\partial c_i}, \frac{\partial u_i}{\partial z_i}, \frac{\partial \phi_i}{\partial g_i} > 0, \quad \frac{\partial^2 u_i}{\partial c_i^2}, \frac{\partial^2 u_i}{\partial z_i^2}, \frac{\partial^2 \phi_i}{\partial g_i^2} < 0$$

(2)

The budget constraint facing each consumer is written

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5 See also Köthenburger (2007).
6 See also Silva and Caplan (1997), and Caplan and Silva (1999).
7 Another issue that has been studied under decentralized leadership is tax competition (Köthenburger 2004).
where \( w_i \) is the wage and \( T_i(w_i l_i) \) is a labor income tax function imposed by the national government. The price of the consumption good is normalized to one which means that \( c_i \) may be interpreted as after tax income. Substituting the time constraint and the budget constraint into the utility function and maximizing w.r.t. \( l_i \) produces the standard first order condition

\[
(1 - T'_i) w_i \frac{\partial u(w_i l_i - T_i h - l_i)}{\partial c_i} - \frac{\partial u(w_i l_i - T_i h - l_i)}{\partial z_i} = 0
\]  

(4)

where \( T'_i = dT_i / d(w_i l_i) \) is the corresponding marginal labor income tax rate.

The production side in each country is made up of identical competitive firms which produce a homogenous good that can costlessly be transformed either into the private good or into the national public good. The number of firms is normalized to one and the firm uses labor in the production process. The production technology is linear so that output is given by \( Y_i = A_i l_i N_i \) where \( A_i \) is a constant. This implies that the wage will be constant at \( w_i = A_i \). In Section 4, the assumption of a linear production technology will be relaxed.

### 3. The Public Sector

Turning to the public sector, it is made up of the national governments in the two countries plus the federal level. The federal level collects revenue from the member countries to achieve redistribution between the two countries. Since I have an EU model in mind, the fee paid by the national government in each member country is proportional to that country's national income which in this simplified framework is given by \( Y_i = w_i l_i N_i \). This means that the federal budget constraint is given by

\[
0 = s_1 Y_1 + s_2 Y_2
\]  

(5)

where \( s_i \) is the federal tax rate levied on country \( i \)'s national income. Two types of interactions between the federal level and the member countries will be considered. One is a *noncooperative Nash equilibrium* where all decision makers in the public sector (both the federal level and the national governments) treat the choices made by the other agents in the public sector as given. The other is a *decentralized economic federation* where the federal level treats the choices made by the governments in the member countries as given whereas at
least one of the member countries is able to act as a strategic leader against the federal level. This means that the government in the leader country can use its policy instruments to influence the decisions made by the federal level (vertical leadership) but each national government treats the choices made by the other national government as exogenous. Later I will consider what happens if the leader country also recognizes that its tax policy may also influence the decisions in the other country (horizontal leadership).

3.1 Federal Policy

Let us start by characterizing the decisions made by the federal government. It is assumed to have a utilitarian objective function

\[ W_F = N_1 U_1 + N_2 U_2 \]  

(6)

The federal level takes into account that its choice of \( s_1 \) and \( s_2 \) will influence each national government’s ability to provide the national public good. As such, the federal level recognizes the national governments' budget constraints, which are given by

\[ g_i = N_i T_i(w_i l_i) - s_i Y_i \quad \text{for } i = 1,2 \]  

(7)

Since the federal level acts as a Nash follower vis-a-vis the lower level governments, the federal government treats \( T_i \) and \( Y_i \) as fixed\(^9\) but conditional on the observed levels of \( T_i \) and \( Y_i \), the federal government recognizes that the choice of \( s_i \) will influence the provision of \( g_i \) via equation (7). It is straightforward to show that in this setup the optimal choices of \( s_1 \) and \( s_2 \) will satisfy the following condition

\[ N_1 \frac{\partial \phi_1}{\partial g_1} = N_2 \frac{\partial \phi_2}{\partial g_2} \]  

(8)

Equation in (8) shows that the federal level will implement complete redistribution in the sense that the sum of marginal utilities of \( g_i \) are equalized between the countries in the federation. Since the federal level's policy is determined conditional on the choices made by

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\(^8\) An alternative would be to let the decisions at the federal level be the outcome of a bargain between the member countries. For the qualitative results to be derived below, the choice of functional form at the federal level is not important and to keep the analytical part as simple as possible, I use a utilitarian welfare function.

\(^9\) Since equation (4) implicitly defines labor supply as a function of the fixed wage rate and the parameters of the tax function, and since the federal level treats the national government’s choices of parameters in the tax function as exogenous, the national income \( (Y_i = w_i l_i N_i) \) is exogenously given from the perspective of the federal level.
the national governments, the federal first order conditions will define $s_1$ and $s_2$ as functions of the national governments’ decision variables. These functions will be referred to as federal reaction functions.

3.2 National Policies

The national government in country $i$ maximizes the utilitarian welfare function $N_iU_i$ subject to the budget constraint defined in equation (7). By using the private budget constraint in equation (3) to substitute for $T_i(w_i,l_i)$, equation (7) can be rewritten to read $g_i = N_i[(1-s_i)w_il_i - c_i]$. The Lagrangian corresponding to the national government’s maximization problem can then be formulated as follows

$$L_i = N_iu(c_i, h - l_i) + N_i\phi(g_i) + \gamma_i[N_i(1 - s_i)w_il_i - N_ic_i - g_i]$$

(9)

where $\gamma_i$ is the Lagrange multiplier associated with the national government's budget constraint. In the Appendix, the solution to the maximization problem is presented and the tax policy implemented by the national governments are summarized in the following proposition;

**Proposition 1:** Consider an economic federation made up of two countries and a the federal government where the latter raises revenue from the national governments by using fees that are proportional to the national income in each member country. Then, in a noncooperative Nash equilibrium where both the federal level and the national governments treat the choices made by the other public decision makers as given, the marginal tax on labor income in country $i=1,2$ will be positive and given by

$$T_i' = s_i$$

(10)

On the other hand, in a decentralized economic federation where the government in the leader country, denoted country $j$, acts as a strategic leader against the federal level, the marginal tax on labor income implemented by the government in the leader country will be zero, that is

$$T_j' = 0$$

(11)

To interpret these results let us, as a point of reference, first consider the outcome in a (standard) model where lump sum fees are used to raise revenue to the federal level. Here, in
the noncooperative Nash equilibrium (as it is defined in Proposition 1), the marginal tax rate would be set equal to zero which means that the outcome would be efficient. However, equation (10) shows that this is not the case in a noncooperative Nash equilibrium with proportional fees. To provide intuition for this result, let us take a look at the relevant income measures for the public and the private sector. The relevant income measure for the national government is the per capita net national income, 
\[ y^n_i = Y^n_i/N_i = (1 - s_i)w_i l_i, \]
whereas the relevant income measure for the private agent is the after tax income, 
\[ c_i = w_i l_i - T_i(w_i l_i). \]
The marginal effect of an increase in labor supply on these two income measures are given by 
\[ \frac{\partial y^n_i}{\partial l_i} = (1 - s_i)w_i \] and 
\[ \frac{\partial c_i}{\partial l_i} = (1 - T'_i)w_i, \]
respectively, and we immediately see that the effect on \( c_i \) will always exceed \( y^n_i \) as long as \( T'_i = 0 \). As such, an increase in the labor supply when \( T'_i = 0 \) will increase \( c_i \) by a larger amount than it does increase \( y^n_i \) because a fraction of the latter is paid as a fee to the federal level. From the national government’s perspective, this will induce the private agents to supply “too much” labor and to avoid this, the marginal effect of an increase in \( l_i \) on \( c_i \) must be realigned with the marginal effect of an increase in \( l_i \) on \( y^n_i \). This is achieved by setting the marginal tax rate \( T'_i \) equal to \( s_i \).

On the other hand, if the national government has the ability to act as a Stackelberg leader vis-a-vis the federal level, then Proposition 1 shows that the leader country has no incentive to implement a distortionary tax on labor. To explain this result, it is again sufficient to evaluate the marginal effects of an increase in \( l_j \) on \( y^n_j \) and \( c_j \). To evaluate the effect on \( y^n_j \), observe that when the national government in country \( j \) acts as a Stackelberg leader, then the national government takes into account that the federal tax rate is a function of \( c_j \) and \( l_j \); \( s_j = s(c_j, l_j) \).

If the marginal tax is zero, then the effect on \( y^n_j \) will be given by 
\[ \frac{\partial y^n_j}{\partial l_j} = w_j - s_j w_j - w_j l_j \left( \frac{\partial s_j}{\partial c_j} \frac{\partial c_j}{\partial l_j} + \frac{\partial s_j}{\partial l_j} \right) = w_j \]

The expression inside the parenthesis captures the net effect on the federal tax rate when \( T'_j = 0 \). To evaluate this effect I have used the comparative static properties of the federal reaction function (which are derived in the Appendix) together with \( \frac{\partial c_j}{\partial l_j} = w_j \). Equation (12) shows that although the direct effect on the net national income, \( w_j - s_j w_j \), is less than \( w_j \), the indirect effect on \( y^n_j \) via the corresponding reduction in \( s_j \) is positive and equal to \( s_j w_j \). Hence, \( \frac{\partial y^n_j}{\partial l_j} = w_j \). Since also the private marginal valuation of an increase in \( l_j \) is equal to \( w_j \) when \( T'_j = 0 \), it follows that the national government has no incentive to
implement a positive marginal tax on labor income. To explain why the federal level implements a lower federal fee when $l_j$ increases, observe first that an increase in $l_j$ will have a positive effect on $y_j = w_l l_j$. For a given level of $s_j$, this will have a positive effect on the federal tax revenue. This means that the federal level does not need to tax country $j$ as hard as before while still being able to raise more revenue. This gives room for a reduction in $s_j$.

### 3.2.1 Public Good Provision

Let us return to the noncooperative Nash equilibrium and pose the following question: does the fact that the national government in country $i$ implements a nonzero marginal tax on labor imply that the cost of raising tax revenue is sufficiently large so as to give rise to underprovision of the public good in the noncooperative Nash equilibrium? The answer is no because setting $T_i$ equal to $s_i$ corresponds to internalizing (from the national government’s point of view) for the fact that the private sector has a ‘wrong’ valuation of labor. Since externality correction per se does not cause the marginal cost of public funds in country $i$, $MCPF_i$, to exceed one, the provision of the national public good in country $i$ will satisfy the following efficiency condition

$$N_i \frac{\partial g_i}{\partial c_i} = 1$$

This is a standard optimality condition for public good provision and implies that the marginal rate of substitution between $g_i$ and $c_i$ is equal to the marginal rate of transformation between $g_i$ and $c_i$ (which is equal to one).

Turning to the decentralized economic federation framework, does the fact that the national government in the leader country $j$ implements a zero marginal tax on labor imply that the $MCPF_j$ is sufficiently low so as to give rise to an efficient provision of the public good? The answer is no because even though $T_j^\prime$ is set equal to zero, the government in the leader country observes that if it raises one additional dollar in tax revenue to increase the provision of national public good, the federal level will respond by implementing a larger fee. As such, only a fraction of the additional dollar raised in tax revenue will be spent on $g_j$. This causes the $MCPF_j$ to exceed one in the leader country which means that the national government will raise less tax revenue than otherwise. As a consequence, the national public good will be underprovided.

$^{10}$ The marginal cost of public funds in country $i$ is defined as $MCPF_i = \frac{y_i}{\partial u_i/\partial c_i}$. 
These results are summarized as follows:

**Proposition 2:** Consider an economic federation made up of two countries and a federal government where the latter raises revenue from the national governments by using fees that are proportional to the national income in each member country. Then, in a noncooperative Nash equilibrium where both the federal level and the national governments treat the choices made by the other public decision makers as given, the policy implemented by the national government in country \(i=1,2\) will be

\[
MCPF_i = 1, \quad N_i \frac{\partial f_i / \partial g_i}{\partial u_i / \partial c_i} = 1
\]

On the other hand, in a decentralized economic federation where the government in the leader country, denoted country \(j\), acts as a strategic leader against the federal level, the policy implemented by the government in the leader country will imply

\[
MCPF_j > 1, \quad N_j \frac{\partial f_j / \partial g_j}{\partial u_j / \partial c_j} > 1
\]

3.2.2 Revenue Taxes in the Noncooperative Equilibrium

It was shown in Proposition 1 that when the federal level uses proportional fees to raise revenue then, in the noncooperative Nash equilibrium, the national governments will implement a positive marginal tax on labor income to realign the private marginal value of labor with that of the national government. Can a tax on firm revenue achieve the same result? The answer is yes and to see this, recall that with a linear production technology, output in country \(i\) is given by \(Y_i = A_i l_i N_i\). Since the price of output is normalized to one, \(A_i l_i N_i\) will also correspond to the firm’s revenue. If the firm’s revenue is taxed at the rate \(\rho_i\), then the profit is given by \(\pi_i = (1 - \rho_i)A_i l_i N_i - w_i l_i N_i\). Since the wage in this context will be given by \(w_i = (1 - \rho_i)A_i\), private after tax income can be written as \(c_i = (1 - \rho_i)A_i l_i - T_i\). Then, if \(T_i\) is a lump sum tax, the private valuation of an increase in \(l_i\) will be given by \(\partial c_i / \partial l_i = (1 - \rho_i)A_i\). Since the per capita net national income will be given by \(y^n_i = (1 - \rho_i)A_i l_i\) it follows that \(\partial y^n_i / \partial l_i = (1 - \rho_i)A_i\). This implies that the national government can achieve \(\partial c_i / \partial l_i = \partial y^n_i / \partial l_i\) by setting the revenue tax equal to the proportional fee, \(\rho_i = s_i\).
3.2.3 Revenue Taxes in a Small Open Economy

A drawback of using the revenue tax is that it will not be an effective instrument in a small open economy framework if we were to introduce mobile capital into the model. To show this, let us modify the basic model as follows. Assume that the firm uses capital, $K_i$, and labor, $L_i$, in the production process. The production function is written $F(K_i, L_i)$ and it is increasing and concave in each argument and characterized by constant returns to scale. Capital is hired on the international capital market outside the federation at the exogenously given world market interest rate, $\bar{r}$, while labor is hired on the domestic labor market. Normalizing the production function w.r.t. $L_i$ allows us to define output per unit of labor as $f(k_i) = F(K_i, L_i)/L_i$, where $k_i = K_i/L_i$ is the capital stock per unit of labor. It is assumed that the national government in each country may levy a tax, $\rho_i$, on the firm's revenues which means that the profit is given by

$$\pi_i = [(1 - \rho_i)f(k_i) - w_i - \bar{r}k_i]L_i$$ (16)

Under constant returns to scale, the profit is zero and the following necessary conditions apply

$$\bar{r} = (1 - \rho_i)f'(k_i)$$ (17)
$$w_i = (1 - \rho_i)f(k_i) - \bar{r}k_i$$ (18)

Now, observe that equation (17) can be rewritten to read $f'(k_i) = \bar{r}/(1 - \rho_i)$ which means that the revenue tax de facto becomes an implicit tax on capital. However, as long as the world market interest rate, $\bar{r}$, is treated as exogenous by country $i$, capital will be infinitely elastic. In this situation it is straightforward to show that the government in country $i$ will tax capital at a zero rate which means that $\rho_i$ will be set equal to zero. Hence a revenue tax will not be a suitable instrument to achieve the desired realignment of private and public valuation of labor in a small open economy. Only a tax on labor income can achieve this.

4. Labor Mobility

Up until now it has been assumed that workers are immobile between countries. However, in EU agents have the possibility to work in other member countries without having to emigrate from the country of origin. To capture this, the model is extended in the following direction. The production function in country $i$ is modified to read $Y_i = F_i(L_i)$, where $L_i$ is labor and $F_i(L_i)$ is an increasing and strictly concave function of $L_i$. I allow the technologies to differ between countries which means that the production function $F_i(L_i)$ is country
specific. The assumption of decreasing returns to scale means that there will be a positive profit, \( \pi_i \), which is given by

\[
\pi_i = F_i(L_i) - w_i L_i
\]  

(19)

The first order condition w.r.t. labor is \( w_i = F'_i(L_i) \) and it implicitly defines the demand for labor as a function \( L^*_i(w_i) \).

In this part of the model the number of consumers is normalized to one. The profit in country \( i \) accrues to the consumer who resides in country \( i \). This implies that the consumer now earns both labor income and profit income. Income is taxed at the source which means that if the consumer partially works in the other country \( j \) \((j = 1,2 \text{ and } j \neq i)\) then the part of the consumer's labor income that is earned in country \( j \) is also taxed in country \( j \). To model this, I parameterize the tax functions facing the resident in country \( i \), and who works in both country \( i \) and country \( j \), as follows

\[
T'_i(w_i l^i_i, \pi_i) = t_i w_i l^i_i + T_i
\]  

(20)

\[
T'_j(w_j l^j_j, \pi_i) = t_j w_j l^j_j
\]  

(21)

where \( l^i_i \) is the part of total labor supply, \( l_i \), which is supplied at home and \( l^j_j = l_i - l^i_i \) is the part of total labor supply that is supplied abroad. Equation (20) is the tax function facing the agent in his/her home country. Here \( t_i \) is the (marginal) tax on labor income and \( T_i \) is the lump sum part of the tax function. This formulation of the tax function means that the government has the ability to tax the profit at 100% or more (i.e. to set \( T_i > \pi_i \) if it is optimal to do so). Equation (21) shows the tax function facing the agent in the other country \( j \) where the (marginal) tax rate is \( t_j \). As can be seen, it is assumed that no lump sum tax is levied on a nondomestic worker. The budget constraint facing the consumer in country \( i \) can now be written as

\[
c_i = (1 - t_i) w_i l_i + (1 - t_j) w_j l^j_i + \pi_i - T_i
\]  

(22)

Substituting this equation and \( l^j_i = l_i - l^j_i \) into the subutility function \( u(c_i, h - l_i) \) and maximizing w.r.t. \( l_i \) and \( l^j_i \) produces the following necessary conditions (at an interior optimum).
\[(1 - t_i)w_i \frac{\partial u(c_i, h - l_i)}{\partial c_i} = \frac{\partial u(c_i, h - l_i)}{\partial z_i}\]  \hspace{1cm} (23)

\[(1 - t_i)w_i = (1 - t_j)w_j\]  \hspace{1cm} (24)

Equation (23) corresponds to a standard first order condition for the total labor supply whereas equation (24) implies that the after tax wage rates must be equalized between countries. If this condition is not satisfied, all workers in the federation would prefer to work in the country where the after tax wage is highest. However, if the production functions satisfy

\[\lim_{l_i \to 0} w_i = F'(L_i) \to \infty, \quad \lim_{l_j \to 0} w_j = F'(L_j) \to \infty\]  \hspace{1cm} (25)

then corner solutions can be ruled out.

If we substitute \(l_i^I = l_i - l_i^I\) and equation (24) into the private budget constraint, and use the short notation \(w_i^n = (1 - t_i)w_i\) to denote the after tax wage in country \(i\), equation (22) reduces to \(c_i = w_i^n l_i + \pi_i - T_i\). If we then substitute this expression into equation (23), the resulting expression implicitly defines the total labor supply chosen by the agent living in country \(i\) as a function \(l_i = l(w_i^n, \pi_i - T_i)\). To simplify the calculations it is assumed that labor supply is only a function of the net wage,\(^{11}\) that is \(l_i = l(w_i^n)\) where \(dl_i/dw_i^n > 0\).

In equilibrium, the aggregate supply of labor in the federation, \(l(w_i^n) + l(w_j^n)\), will be equal to the aggregate demand, \(L_i^d + L_j^d\). Combining this equilibrium condition with \(w_i^n = w_i^n, w_i = F_i'(L_i), w_j = F_j'(L_j)\) and equation (24) produces the following set of equations

\begin{align*}
0 &= (1 - t_i)F_i'(L_i) - (1 - t_j)F_j'(L_j) \quad (26) \\
0 &= L_i + L_j - l_i[(1 - t_i)F_i'(L_i)] - l_j[(1 - t_j)F_j'(L_j)] \quad (27)
\end{align*}

These two equations implicitly define the equilibrium levels of employment in the two countries, \(L_i\) and \(L_j\), as functions of the marginal tax rates

\[L_i \left( \tilde{t}_i, \tilde{t}_j \right), \quad L_j \left( \tilde{t}_i, \tilde{t}_j \right)\]  \hspace{1cm} (28)

\(^{11}\) This would arise if, for example the subutility function \(u(c_i, h - l_i)\) is quasilinear or if the subutility function could be written as \(u(c_i - e(l_i))\), where \(e(l_i)\) is an increasing and convex function of \(l_i\).
where the signs above $t_i$ and $t_j$, respectively, indicate the sign of the corresponding partial derivative.

Let us now turn to the public sector. In an open economy framework, the federal fee can either be levied on country $i$'s gross domestic product (GDP) which in this context is given by $GDP_i = F_i(L_i)$, or on country $i$'s gross national income (GNI) which is equal to GDP plus the net income from earnings paid to/from abroad. In our framework, the gross national income in country $i$ will be given by

$$GNI_i = F_i(L_i) + w^n_j(l_i - L_i)$$  \hspace{1cm} (29)

Since the qualitative results to be derived below will differ depending on whether the federal tax base is $GNI$ or $GDP$, I will consider both cases. This implies that when the federal fee is levied on $GNI$ then the federal budget constraint will be given by $0 = s_i GNI_i + s_j GNI_j$. I will refer to this as Case 1. On the other hand, when the federal fee is levied on GDP, then the federal budget constraint is written $0 = s_i GDP_i + s_j GDP_j$ and this will be referred to as Case 2. In each case, the solution to the federal level's maximization problem implicitly defines $s_i$ and $s_j$ as functions of the national governments' decision variables, that is

$$s_i = s_i(t_i, T_i, t_j, T_j)$$  \hspace{1cm} (30)

$$s_i = s_i(t_i, T_i, t_j, T_j)$$  \hspace{1cm} (31)

The comparative static properties of these reaction functions will, of course, differ between Case 1 and Case 2.

Turning to the national governments, I assume that domestic and nondomestic workers face the same (marginal) labor income tax rate. The outcome when the government can tax domestic and foreign workers at different rates will be analyzed in the next section. The objective of the government in country $i$ is to maximize $U_i$ subject to the following restrictions

$$c_i = (1 - s_i)w_i l_i + \pi_i - T_i$$  \hspace{1cm} (32)

$$\pi_i = F_i(L_i) - w_i L_i$$  \hspace{1cm} (33)

$$g_i = t_i w_i l_i + T_i - s_i GNI_i$$  \hspace{1cm} (Case 1)  \hspace{1cm} (34)

$$g_i = t_i w_i l_i + T_i - s_i GDP_i$$  \hspace{1cm} (Case 2)  \hspace{1cm} (35)
where I have used $L_i = l_i^I + l_i^J$ to rewrite the national government's budget constraint. Each national government recognizes the functions $w_i = F_i^I(L_i)$, $w_j = F_j^J(L_j)$, $L_i(t_i, t_j)$ and $L_{ij}(t_i, t_j)$ but treats the choices made by the other national government as exogenous. If the national government acts as a Stackelberg leader vis-a-vis the federal level, the national government also takes the federal reaction function $s_i = s_i(t_i, T_i, t_j, T_j)$ into account. Define the equilibrium after tax wage, $w_j^n(t_i, t_j)$, and the equilibrium tax base, $I_j(t_i, t_j)$, in the other country, as

$$w_j^n(t_i, t_j) = (1 - t_j)F_j^J[L_j(t_i, t_j)]$$

$$I_j(t_i, t_j) = F_j^J[L_j(t_i, t_j)]L_j(t_i, t_j)$$

Equation (36) implies that the net wage in the other country is decreasing in $t_i$, that is

$$\frac{\partial w_j^n}{\partial t_i} = (1 - t_j)F_j^{II} \frac{\partial L_j}{\partial t_i} < 0$$

Let us also define

$$a_i = -\frac{1}{w_i(\partial t_i/\partial t_i)} > 0$$

The main results can now be summarized as follows;

**Proposition 3:** Consider an economic federation made up of two countries and a federal government, and where labor is perfectly mobile between the member countries. The marginal tax rates implemented by the national governments in the member countries can then be summarized as follows:

(i) If country $i$ acts as a **Nash follower** and the federal fee is proportional to GNI, then the optimal marginal tax rate is given by

$$t_i = a_i(l_i - L_i) \frac{\partial w_j^n}{\partial t_i} - \frac{a_i \varepsilon_l e_i^\gamma}{1 - s_i} \frac{\partial w_j^n}{\partial t_i}$$

where $\varepsilon_l^\gamma > 0$ is the elasticity of labor supply w.r.t. $w_i^n$. 


(ii) If country $i$ acts as a **Nash follower** and the federal fee is proportional to GDP, then the optimal marginal tax rate is given by

$$t_i = a_i (l_i - L_i) \frac{\partial w^n}{\partial \tau_i} + s_i$$  \hspace{1cm} (41)

(iii) If country $i$ acts as a **Stackelberg leader** then, regardless of whether the federal fee is proportional to GNI or GDP, the optimal marginal tax rate is given by

$$t_i = a_i (l_i - L_i) \frac{\partial w^n}{\partial \tau_i} + t_j a_i \frac{\partial l_j}{\partial \tau_i}$$  \hspace{1cm} (42)

The first term on the RHS looks the same in each tax formula and reflects that the national government in country $i$ can use the domestic marginal labor income tax to influence the after tax wage in the other country. This type of mechanism is well understood from earlier literature and in this context it implies that if the country is a net exporter of labor ($l_i > L_i$), then an increase of the after tax wage in the other country, $w^n_j$, will lead to a higher national income in country $i$. An increase in $w^n_j$ can be accomplished by subsidizing domestic labor. On the margin this reduces the export of labor to country $j$ which contributes to increase $w^n_j$.

If country $i$ instead is a net importer of labor ($l_i < L_i$), then the argument for a positive marginal tax rate is analogous.

The second term on the RHS in each of the tax formulas in Proposition 3 are novel and reflect motives that are distinctly associated with whether the national government acts as a Nash follower in either Case 1 or in Case 2, or if the national government acts as a Stackelberg leader, against the federal level. Beginning with the second term in part (i), it is positive (including the minus sign) and reflects a motive for the government in country $i$ to influence the domestic labor supply which only appears when the government in country $i$ acts as a Nash follower and the federal fee is proportional to $GNI_i$. Since the fee paid by the government in country $i$ is given by $s_i GNI_i$, this reflects that the national government on the margin has an incentive to reduce $GNI_i$ (compared with when $s_i = 0$) in order to pay a smaller fee. This can be accomplished by reducing the labor supply by implementing a slightly larger marginal tax on labor than otherwise. This leads both to a reduction in total labor supply, $l_i$, as well as to a reallocation of labor from country $i$ to the other country $j$. Since the increase in $l_j^i$ has a negative effect on the net wage in the other country, $w^n_j$, while...
the reduction in domestic labor supply, \( l_i^d = l_i - l_i^f \), has a negative effect on domestic output, it follows that \( GNI_i \) will be reduced which leads to a lower membership fee, \( s_i GNI_i \).

As for the second term on the RHS in equation (41), recall from Proposition 1 that when the federal fee is levied on GDP,\(^\text{12}\) then the marginal tax on labor income is set such that \( t_i = s_i \). The mechanism underlying that result is also at work here which explains the appearance of \( s_i \) in equation (41).

Turning to the marginal labor income tax implemented by the national government when it acts as a Stackelberg leader vis-a-vis the federal level, recall from Proposition 1 that in the absence of labor mobility, the marginal tax on labor income is set equal to zero. However, when labor is mobile, equation (42) shows that labor mobility provides the Stackelberg leader with two distinct motives for choosing a nonzero marginal tax. The first is to use the tax on labor as a tool to influence the wage in the other country. This is captured by the first term on the RHS of equation (42). The second motive is to use the tax on labor as a tool to influence the tax base of labor in the other country, \( l_j = w_j L_j \). The reason for why the government in country \( i \) has an incentive to do this is that conditional on \( t_j \) and \( T_j \), an increase in \( l_j \) leads to more tax revenue in country \( j \) which, all else equal, provides the federal level with an incentive to redistribute resources from country \( j \) to country \( i \) via a lower federal tax rate, \( s_i \).

Since this effect works via the reaction function \( s_i = s_i(t_i, T_i, t_j, T_j) \), it will only appear in the marginal income tax formula when the national government is able to act as a Stackelberg leader vis-a-vis the federal level. Therefore I will refer to the second term on the RHS of equation (42) as the \textit{leadership effect}. If \( \frac{\partial l_j}{\partial t_i} > 0 \), the leadership effect provides the national government with an incentive to set \( t_i \) higher than otherwise but if \( \frac{\partial l_j}{\partial t_i} < 0 \), the opposite argument applies. It can be shown that the sign of \( \frac{\partial l_j}{\partial t_i} \) is determined by the following rule

\[
\text{sign} \frac{\partial l_j}{\partial t_i} = \text{sign} \left( 1 + \frac{1}{\varepsilon^d_j} \right) \quad (43)
\]

where \( \varepsilon^d_j = \frac{\partial l_i^d}{\partial w_j} \frac{w_j}{l_j} \) is the elasticity of labor demand. This result can be summarized as follows;

\[^{12}\text{In Section 2.1.3., GDP was equal to } w_i l_i.\]
**Corollary 1:** If the national government in country $i$ is able to act as a Stackelberg leader vis-a-vis the federal level, then the **leadership effect** will provide the national government with an incentive to implement a positive marginal tax on labor income if the labor demand elasticity in the other country $j$, exceeds one in absolute value ($\varepsilon_j > 0$). If, on the other hand, the labor demand elasticity in the other country is less than one in absolute value ($\varepsilon_j < 0$), then the leadership effect will provide the national government in country $i$ with an incentive to implement a negative marginal tax on labor income.

Finally, observe that if the federal level would use lump sum fees instead of proportional fees to collect revenue from the national governments, then $s_i = 0$ in which case equations (40) and (41) both reduce to $t_i = \alpha_i(l_i - L_i) \frac{\partial w_i^n}{\partial t_i}$. This shows that a national government which acts as a Nash follower will have an incentive to choose a nonzero marginal income tax rate also under lump sum fees as long as the national government in country $i$ can influence the after tax wage in the other country. Turning to the Stackelberg leader it follows that since the federal tax rate, $s_i$, does not appear directly in equation (42), this equation would be the same also if the federal level would use a lump sum fee to raise revenue. The latter result can be summarized as follows;

**Proposition 4:** The incentives underlying the optimal tax policy implemented by a Stackelberg leader are independent of how the federal level collects its revenue.

### 4.2 Different Tax Rates

Let us now allow the government in country $i$ to tax domestic and nondomestic workers at different rates. This case is only relevant when the country is a net importer of labor in which case the inequalities $l_i < L_i$ and $l_j > L_j$ hold. Let $t_i^d$ denote the tax rate facing a domestic worker in country $i$ whereas $t_i^j$ is the tax rate facing a worker from country $j$ working in country $i$.\(^\text{13}\) To simplify the algebra, I assume that the technology in country $i$ is linear so that $w_i$ is fixed\(^\text{14}\) while output in the other country is given by the production function $F_j(L_j)$. In this case the equalization of net wages implies

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\(^\text{13}\) Observe that for country $j$, which is a net exporter of labor and therefore does not have any workers from country $i$ working in country $j$, only one tax, $t_j$, is necessary.

\(^\text{14}\) This assumption does not affect any of the qualitative results derived below.
\[(1 - t_i^j)w_i = (1 - t_j)F'_j(L_j) \tag{44}\]

Equation (44) implicitly defines the equilibrium level of \(L_j\) as a function \(L_j(t_i^j, t_j)\) which is increasing in \(t_i^j\) but decreasing in \(t_j\). By substituting \(L_j(t_i^j, t_j)\) into the equilibrium condition that aggregate supply will equal the aggregate use of labor summed over the two countries, we obtain

\[L_i = l_i[(1 - t_i^j)w_i] + l_f[(1 - t_f)F'_f(L_f)] - L_j(t_i^j, t_j) \tag{45}\]

This equation defines \(L_i\) as a function \(L_i(t_i^j, t_i^f, t_f)\) which is decreasing in \(t_i^j\) and \(t_i^f\) but increasing in \(t_f\). If we in this situation solve the federal level's maximization problem, the federal reaction functions can be written as functions

\[s_i = s_i(t_i^j, t_i^f, t_f, t_j) \tag{46}\]
\[s_i = s_i(t_i^j, t_i^f, t_f, t_j) \tag{47}\]

Finally, solving the maximization problem facing the national government in country \(i\), the following results can be derived;

**Proposition 5:** Consider an economic federation where labor is perfectly mobile between the member countries and where domestic and nondomestic workers can be taxed at different rates. When country \(i\) is a net importer of labor, the optimal tax policy implemented by the government in country \(i\) can be summarized as follows:

(i) If country \(i\) acts as a Nash follower and the federal fee is proportional to GNI, then the optimal tax policy features

\[t_i^j = a_i(L_i - L_i) \frac{\partial w_j}{\partial t_i} \tag{48}\]
\[t_i^j = s_i \tag{49}\]

(ii) If country \(i\) acts as a Nash follower and the federal fee is proportional to GDP, then the optimal tax policy features
(iii) If country \( i \) acts as a Stackelberg leader then, regardless of whether the federal fee is proportional to GNI or GDP, the optimal tax policy features

\[
t_i^j = a_i(l_i - L_i) \frac{\partial w^n}{\partial t_i} + s_i \quad (50)
\]
\[
t_i^j = s_i \quad (51)
\]

Comparing these results with those presented in Proposition 3, it follows that the marginal tax rates imposed on the nondomestic workers that are presented in equations (48), (50) and (52) basically replicate the marginal tax rates presented in equations (40) - (42). The intuition is straightforward. As explained in the previous section, there are potentially three reasons for implementing nonzero marginal tax rates when labor is mobile; (i) to influence the after tax wage in the other country, \( w^j \), (ii) adapt to the federal fee system and (iii) if country \( i \) is a Stackelberg leader vis-a-vis the federal level, also to influence the tax base in the other country. When the national government can tax domestic and nondomestic workers at different rates, the tax on nondomestic labor, \( t_i^j \), will be a more direct instrument to exercise control over \( w^j \) and \( l_j \) than the tax on domestic workers, \( t_i^j \). As a consequence, equations (50) and (52) are identical to equations (41) and (42), and the only reason for why equation (48) differs from equation (40) is that \( t_i^j \) will not influence the labor supply of a domestic worker.\(^\text{15}\)

On the other hand, the only reason for implementing a nonzero marginal tax on domestic workers is to adjust to the federal fee system, as can be seen in equations (49) and (51). However, when the government acts as a strategic leader, this is not necessary as discussed above.

5. Horizontal Leadership

So far it has been assumed that strategic leadership can only be exercised vertically. Let us now instead consider horizontal strategic leadership where one country recognizes that its tax policy may influence the decisions made by the national government in the other country. To isolate the pure effect of horizontal leadership, let us assume that vertical leadership vis-a-vis

\(^{15}\) The second term on the RHS of equation (40) reflects a labor supply effect as discussed earlier.
the federal level cannot be exercised by the leader country. As such, it will treat the federal decision variables as exogenous in this scenario. It is also reasonable to assume that a basic reason for why one country is able to exercise influence over the decisions taken by the government in the other country is that the leader country is “large” in the sense that the government in the follower country treats all variables associated with the leader country, including the after tax wage, as exogenous.

Let us begin by characterizing the tax policy implemented by the follower country, $i$. Since the government in country $i$ treats $w_f^j$ as fixed and therefore perceives that $\partial w_f^j / \partial t_i = 0$, the following results are immediately available by using the results from Propositions 3 and 5;

**Corollary 2:** Consider an economic federation where labor is perfectly mobile between the member countries and where country $i$ is a small Nash follower in the sense that the government in country $i$ treats all public decision variables, as well as the wage, in the leader country as exogenous. The optimal tax policy implemented by the government in the follower country can then be summarized as follows:

(i) If domestic and nondomestic workers are taxed at the same rate, and if the federal level uses a fee that is proportional to country $i$'s GNI to raise revenue, then the national government in the follower country will implement a zero marginal tax on labor income, that is $t_i = 0$ (follows from part (i) in Proposition 3 when $\partial w_f^i / \partial t_i = 0$).

(ii) If domestic and nondomestic workers are taxed at the same rate, and if the federal level uses a fee that is proportional to country $i$'s GDP to raise revenue, then the national government in the follower country will implement a positive marginal tax on labor income which is given by $t_i = s_i$ (follows from part (ii) in Proposition 3 when $\partial w_f^i / \partial t_i = 0$).

(iii) If domestic and nondomestic workers are taxed at different rates, and if the federal level uses a fee that is proportional to country $i$'s GNI to raise revenue, then the national government in the follower country will implement the following tax policy (follows from part (i) in Proposition 5 when $\partial w_f^i / \partial t_i = 0$)

$$t_i^j = 0, \quad t_i^j = s_i$$ (54)
(iv) If domestic and nondomestic workers are taxed at different rates, and if the federal level uses a fee that is proportional to country i's GDP to raise revenue, then the national government in the follower country will implement the following tax policy (follows from part (ii) in Proposition 5 when \( \partial w_i^t / \partial t_i = 0 \))

\[
t_i^f = s_i, \quad t_i^f = s_i
\]  

Corollary 2 shows that the marginal tax rates implemented by the follower country will either be zero or equal to the federal fee, \( s_i \). Then, as long as the leader country does not exercise vertical leadership and therefore treats \( s_i \) as exogenous, the conclusion is that the Stackelberg leader will not be able to exercise any influence over the follower country’s marginal tax rates. Furthermore, since the labor supply functions in this model are assumed to be independent of the lump sum parts of the national governments' tax functions, there is no incentive for the leader country to influence the lump sum taxes in the follower country. This can be summarized as follows:

**Proposition 6:** Consider an economic federation where labor is perfectly mobile between the member countries. If country \( i \) is able to act as a horizontal Stackelberg leader then it will implement a zero marginal tax rate on labor.

### 6. Conclusions

This paper concerns optimal income tax policy in an economic federation where the federal level uses membership fees that are proportional to either GDP or GNI to raise revenue from the lower level jurisdictions. The paper extends the analyses of previous studies by incorporating proportional membership fees into a framework where at least one of the lower level jurisdictions is able to exert leadership either vertically and/or horizontally. I would like to emphasize three broad conclusions:

(i) If the lower level jurisdictions act as Nash followers then the presence of proportional membership fees will provide the national governments with an incentive to implement a distortionary marginal tax on labor income. By doing so, the national government realigns the private valuation of labor with that of the public sector.

(ii) If a country in the economic federation is able to act as a Stackelberg leader-vis-a-vis the federal level (vertical leadership), then the national government need not implement a
distortionary marginal tax on labor income to realign the private valuation of labor with that of the public sector. Then, the only motive for implementing a nonzero marginal tax on labor depends on whether this policy instrument may influence the allocation of labor between countries such that it affects the foreign after tax wage or the foreign tax base.

(iii) If a country in the economic federation is able to act as a Stackelberg leader vis-a-vis the other member countries (horizontal leadership), then the results in this paper indicate that the motive to implement a distortionary tax on labor income in that country is eliminated.

Appendix

Proof of Propositions 1 and 2:  
When labor is immobile the national government's Lagrangian is given by equation (9). As is standard in the theory of optimal nonlinear taxation, $c_i$ and $l_i$ are treated as government decision variables and the first order conditions become ($N_i$ is divided away in A.1 and A.2):

\[
\begin{align*}
\frac{\partial L_i}{\partial c_i} &= \left( \frac{\partial u_i}{\partial c_i} - y_i \right) - y_i w_i l_i \frac{\partial s_i}{\partial c_i} = 0 \quad (A.1) \\
\frac{\partial L_i}{\partial l_i} &= y_i (1 - s_i) w_i - \frac{\partial u_i}{\partial z_i} - y_i w_i l_i \frac{\partial s_i}{\partial l_i} = 0 \quad (A.2) \\
\frac{\partial L_i}{\partial g_i} &= N_i \frac{\partial \phi_i}{\partial g_i} - y_i = 0 \quad (A.3)
\end{align*}
\]

In the noncooperative Nash equilibrium, where the federal fee is treated as exogenous, $\partial s_i / \partial c_i = 0$ and $\partial s_i / \partial l_i = 0$. Using (A.1) to replace $y_i$ in (A.2) and comparing the resulting expression with the private first order condition in equation (4) produces equation (10) in Proposition 1. In addition (A.1), and (A.1) and (A.3) can be rearranged/combined to produce the results in equation (14) in Proposition 2.

Turning to the Stackelberg leader, multiply (A.1) by $(1 - s_i) w_i$ and add the resulting expression to (A.2) (let us change notation from $i$ to $j$ to be consistent with the notation in the paper). This produces

\[
0 = (1 - s_j) w_j \frac{\partial u_j}{\partial c_j} - \frac{\partial u_j}{\partial z_j} - y_j w_j l_j \frac{\partial s_j}{\partial l_j}
\]

where

\[
\frac{\partial s_j}{\partial l_j} = \frac{\partial s_j}{\partial l_j} + (1 - s_j) w_j \frac{\partial s_j}{\partial c_j}
\]

To proceed we need to evaluate the comparative static properties of the federal reaction function; $s_j$. By solving the federal level’s maximization problem it is straightforward to derive the following comparative static results
By combining (A.4) – (A.8) we can derive $T'_j = 0$ which is the result presented in equation (11) in Proposition 1. In addition, by substituting (A.6) into (A.1) one can derive

$$MCPF_j = \frac{N_i\phi''_i + N_j\phi''_j}{N_i\phi''_i} > 1 \quad \text{(A.8)}$$

which is the result in the first part in equation (15). Furthermore, by combining (A.3) with (A.8) produces

$$\frac{N_j}{N_i} \frac{\partial \phi_j/\partial g_j}{\partial u_j/\partial c_j} = \frac{N_i\phi''_i + N_j\phi''_j}{N_i\phi''_i} > 1 \quad \text{(A.9)}$$

which is the result in the second part in equation (15).

Proof of Proposition 3:

When the federal fee is levied on GNI, the Lagrangian corresponding to the national government’s problem can be written as

$$L_i = u(c_i, h - l_i) + \phi(g_i) + \gamma_i [ (t_i - s_i)w_i l_i - s_i w_j^n (l_i - L_i) + T_i - g_i ]$$

$$+ \mu_i [(1 - t_i)w_i l_i - T_i - c_i]$$

where $\mu_i$ is the shadow price associated with the private budget constraint. The first order conditions become

$$\frac{\partial L_i}{\partial c_i} = \frac{\partial u_i}{\partial c_i} - \mu_i = 0 \quad \text{(A.10)}$$

$$\frac{\partial L_i}{\partial T_i} = \gamma_i - \mu_i - \gamma_i GNI_l \frac{\partial s_l}{\partial T_i} = 0 \quad \text{(A.11)}$$

$$\frac{\partial L_i}{\partial t_i} = \gamma_i \left[ w_i L_i + (t_i - s_i)w_i \frac{\partial L_i}{\partial t_i} - s_i (l_i - L_i) \frac{\partial w_j^n}{\partial t_i} + s_i w_j^n \frac{\partial l_i}{\partial t_i} + s_i w_j^n \frac{\partial L_i}{\partial l_i} \right]$$

$$- \mu_i w_i l_i - \gamma_i GNI_l \frac{\partial s_l}{\partial t_i} = 0 \quad \text{(A.12)}$$

$$\frac{\partial L_i}{\partial g_i} = \frac{\partial \phi_i}{\partial g_i} - \gamma_i = 0 \quad \text{(A.13)}$$

Beginning with the noncooperative Nash equilibrium, use that (A.11) implies $\mu_i = \gamma_i$ and substitute this into (A.12). Using the definition of the labor supply elasticity, together with $\frac{\partial w_j^n}{\partial t_i} = -w_i$, produces the tax formula presented in equation (40).

Turning to the Stackelberg leader, multiply (A.11) by $w_i l_i$, subtract the resulting expression from (A.12) and then divide by $\gamma_i$. This produces
\[ 0 = w_i(L_i - l_i) + (t_i - s_i)w_i \frac{\partial L_i}{\partial t_i} - s_i(l_i - L_i) \frac{\partial w_i}{\partial t_i} + s_i w_j^n \frac{\partial L_i}{\partial t_i} - GNI \frac{\partial \bar{s}_i}{\partial t_i} \]

where

\[ \frac{\partial \bar{s}_i}{\partial t_i} = \frac{\partial s_i}{\partial t_i} - \frac{\partial s_i}{\partial T_i} w_i l_i \]

By solving the federal government’s problem, one can show that

\[ \frac{\partial \bar{s}_i}{\partial t_i} \]

Substituting this expression into the tax formula above, multiplying by \( (1 - s_i)w_i(L_i - l_i) \frac{\partial L_i}{\partial t_i} - s_i(l_i - L_i) \frac{\partial w_i}{\partial t_i} + s_i w_j^n \frac{\partial L_i}{\partial t_i} - GNI \frac{\partial \bar{s}_i}{\partial t_i} \)

and using

that \( \frac{\partial w_j}{\partial t_i} = -w_i \), produces equation (42).

When the federal fee is levied on GDP, the Lagrangian corresponding to the national government’s problem can be written as

\[ L_i = u[c_i, h - l_i[(1 - t_i)w_i]] + \phi(g_i) + \gamma_i [(t_i - s_i)w_i L_i(t_i, t_j) + T_i - g_i] + \mu_i [(1 - t_i)w_i l_i[(1 - t_i)w_i] - T_i - c_i] \]

The first order conditions become

\[ \frac{\partial L_i}{\partial c_i} = \frac{\partial u_i}{\partial c_i} - \mu_i = 0 \] (A.14)

\[ \frac{\partial L_i}{\partial t_i} = \gamma_i - \mu_i - \gamma_i w_i L_i \frac{\partial s_i}{\partial t_i} = 0 \] (A.15)

\[ \frac{\partial L_i}{\partial t_i} = \gamma_i \left[w_i L_i + (t_i - s_i)w_i \frac{\partial L_i}{\partial t_i} - \mu_i w_i l_i - \gamma_i w_i L_i \frac{\partial s_i}{\partial t_i} \right] = 0 \] (A.16)

\[ \frac{\partial L_i}{\partial g_i} = \frac{\partial \phi_i}{\partial g_i} - \gamma_i = 0 \] (A.17)

Beginning with the noncooperative Nash equilibrium, use that (A.15) implies \( \mu_i = \gamma_i \) and substitute this into (A.16). Using the definition of the labor supply elasticity, together with \( \frac{\partial w}{\partial t_i} = -w_i \), produces the tax formula presented in equation (41).

Turning to the Stackelberg leader, multiply (A.15) by \( w_i l_i \) and subtract the resulting expression from (A.16). This produces
\[ 0 = L_i - l_i + (t_i - s_i) \frac{\partial L_i}{\partial t_i} - L_i \frac{\partial \tilde{s}_i}{\partial t_i} \]

By solving the federal government’s problem, one can show that
\[ \frac{\partial \tilde{s}_i}{\partial t_i} = \frac{w_i L_i \phi''_i \left( w_i L_i - w_i l_i + (t_i - s_i) w_i \frac{\partial L_i}{\partial t_i} \right)}{(w_i L_i)^2 \left( \phi''_j + \phi''_i \right)} - \frac{w_i L_i \phi''_j \left( s_i w_i \frac{\partial L_i}{\partial t_i} + t_j w_j \frac{\partial L_j}{\partial t_i} + t_j L_j \frac{\partial w_j}{\partial t_i} \right)}{(w_i L_i)^2 \left( \phi''_j + \phi''_i \right)} \]

Substituting this expression into the tax formula above, multiplying by \((w_i L_i)^2 \left( \phi''_j + \phi''_i \right)\) and using that \(\frac{\partial w_j}{\partial t_i} = -w_i\). Solving for the tax rate produces equation (42).

References


