Keeping up with the Joneses, the Smiths and the Tanakas: Optimal Taxation with Social Comparisons in a Multi-Country Economy

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Abstract
Recent empirical evidence suggests that between-country social comparisons have become more important over time. This paper analyzes optimal income taxation in a multi-country economy, where consumers derive utility from their relative consumption compared with both other domestic residents and people in other countries. The optimal tax policy in our framework reflects both correction for positional externalities and redistributive aspects of such correction due to the incentive constraint facing each government. If the national governments behave as Nash competitors to one another, the resulting tax policy only internalizes the externalities that are due to within-country comparisons, whereas the tax policy chosen by the leader country in a Stackelberg game also reflects between-country comparisons. We also derive a globally efficient tax structure in a cooperative framework. Nash competition typically implies lower marginal income tax rates than chosen by the leader country in a Stackelberg game, and cooperation typically leads to higher marginal income tax rates than the non-cooperative regimes.

Keywords: Optimal taxation, relative consumption, inter-jurisdictional comparison, asymmetric information, status, positional goods.

JEL Classification: D03, D62, D82, H21.

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1. Introduction

The globalization process has implied that information about people and their living conditions in other parts of the world has increased rapidly in recent decades. Indeed, the technological advancement of TV, Internet, and social media together with increased travelling have resulted in much better knowledge of the living conditions of others, and of some people in particular (such as the rich and famous), than was the case only a couple of decades ago. This suggests that people’s reference consumption is increasingly determined by consumption levels in other countries than their own. The present paper explores such between-country social comparisons and identifies the corresponding implications for optimal income tax policy, which, as far as we know, have not been addressed before.

A rapidly growing literature deals with optimal tax policy implications of relative comparison concerns based on one-country models; see, e.g., Boskin and Sheshinski (1978), Oswald (1983), Tuomala (1990), Persson (1995), Corneo and Jeanne (1997), Ljungqvist and Uhlig (2000), Ireland (2001), Dupor and Liu (2003), Abel (2005), Aronsson and Johansson-Stenman (2008, 2010), Wendner (2010), and Eckerstorfer and Wendner (2013). The present paper extends this literature to a multi-country framework where each national government can obviously not control the taxes and consumption levels in other countries. More specifically, it considers the policy implications of such a broader framework for social comparisons by analyzing optimal redistributive, nonlinear income taxation in a multi-country setting, where each individual derives utility from his/her relative consumption compared with both other domestic residents and people in other countries. Our approach and motivation are outlined in greater detail below.

Much of the happiness and questionnaire-based research dealing with individual well-being and relative consumption is silent about the role of cross-country comparisons, which is not surprising given the difficulties of measuring such effects. Yet arguments have recently been

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1 See, e.g., Easterlin (2001), Johansson-Stenman et al. (2002), Blanchflower and Oswald (2004), Ferrer-i-Carbonell (2005), Solnick and Hemenway (2005), Carlsson et al. (2007), Clark et al. (2008), Senik (2009) and Clark and Senik (2010). This literature typically assumes that relative consumption concerns are driven by within-country comparisons (based on various reference groups) or does not specify relative consumption in a
made suggesting that such comparisons are likely to have become more important over time (e.g., Friedman, 2005; Zhang et al., 2009; Becchetti et al., 2010; Clark and Senik, 2011). For example, Becchetti et al. (2010) examine the determinants of self-reported life-satisfaction using survey-data for countries in Western Europe from the early 1970s to 2002. To be able to assess the effects of cross-country comparisons and whether these effects have changed over time, the authors control for determinants of subjective well-being discussed in earlier literature such as relative income measures based on national comparisons (across education, age, and gender groups) as well as domestic GDP. Interestingly, the results show that the distance between the GDP of the individual’s own country and the GDP of the richest country in the data reduces individual life-satisfaction, and that the contribution to well-being of such cross-country comparisons increased over the study period. A possible interpretation is that the increased globalization through technological advancements in recent decades has meant that social comparisons between countries now have a greater influence on individual well-being than before.

Yet, the policy implications of social comparisons between countries remain largely unexplored. To our knowledge, the only exception is Aronsson and Johansson-Stenman (forthcoming a), who address the optimal provision of national and global public goods in a two-country setting where each individual derives well-being from his/her relative private consumption through within- and between country comparisons, as well as from the relative consumption of national public goods through between-country comparisons. Yet, that study does not address optimal taxation but implicitly assumes that each government can raise sufficient revenue for public provision through lump-sum taxation, implying that both externality-correcting and redistributive roles of the tax system are ignored.

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2 See also James (1987) for an early discussion of how tastes (including positional concerns) are transferred from developed to developing countries.

3 Arguably, this interpretation presupposes that relative consumption concerns are not independent of access to social media. Indeed, in a recent survey of Europeans, Clark and Senik (2010) found that people without access to the Internet are less concerned with their relative consumption than people with such access.
The present study adds at least two important new dimensions. First, since all previous studies on tax policy and relative consumption that we are aware of are based on one-country model economies, the policy incentives associated with between-country comparisons, as well as those resulting from interaction between such comparisons and the (conventional) within-country comparison, still remain to be explored. Arguably, this is empirically relevant for the reasons mentioned above. Second, since between-country comparisons give rise to international externalities, the tax policies decided by national governments are no longer necessarily efficient at the global level. This leads to the question of tax policy coordination and cooperation among countries – an issue addressed in other areas of economics, although neglected so far in the study of tax policy under social interaction.

Section 2 presents the basic model of a multi-country economy, where individual utility depends on the individual’s own consumption of goods and leisure as well as on the individual’s relative consumption based on within-country and between-country comparisons, respectively. Section 3 deals with optimal income taxation for a baseline case where individuals are identical within each country (although not necessarily between countries). This model means that income taxation has no redistributive purpose and is motivated solely by the desire to internalize the positional externalities. As such, it generalizes results derived by, e.g., Persson (1995), Ljungqvist and Uhlig (2000), and Dupor and Liu (2003) to a multi-country setting. We start with the non-cooperative Nash solution, where each country takes the behavior of other countries as given. It is shown that each government will then fully internalize the positional externalities affecting people within its own country, but completely ignore the externalities affecting other countries. These externality-correcting taxes are expressed in terms of degrees of positionality, i.e., the degree to which relative consumption matters compared with absolute consumption.

However, while Nash competition is a common assumption in earlier literature on international externalities, it is not always the most realistic one since the ability to commit to public policy may differ among countries, e.g., due to differences in resources, size, and opportunities. Therefore, we also analyze a Stackelberg equilibrium where one country is acting as leader and the others as followers. We show that the policy incentives faced by the followers are analogous to those in the Nash equilibrium, whereas the leader will also take into account the externalities it causes to others, since such externalities will affect others’
behavior. In addition, if the preferences of the followers are characterized by a keeping-up-with-the-Joneses property, such that they prefer to consume more (and hence use less leisure) when the leader consumes more, ceteris paribus, then this constitutes a reason for the leader to increase the marginal income tax rate beyond the Nash equilibrium rate, and vice versa.

In Section 4, we analyze the potential for cooperative behavior. First we show, based on both the Nash equilibrium and the Stackelberg equilibrium, that there is scope for Pareto improvements through a small coordinated increase in the marginal income tax rates. Second, we consider a two-country framework where each government can pay the other country for increasing its marginal income tax rates. We then obtain a globally Pareto-efficient allocation implying that each government will fully internalize all positional externalities associated with private consumption, including those imposed on other countries. This is accomplished through a simple Pigouvian tax based on the sum of the marginal willingness to pay of all individuals, within as well as between the countries, to avoid the externality. In other words, the tax equals the globally aggregate marginal willingness to pay for the individual not to increase his/her consumption by one unit through increased labor supply.

In Section 5, we generalize the model used in Sections 3-4 to the more realistic case where there are also redistributional concerns within each country, and where the government has to rely on distortionary taxation for this redistribution due to asymmetric information. This generalization is clearly relevant from a practical policy perspective, and also because earlier literature shows that the optimal tax policy responses to relative consumption concerns in second-best economies may differ substantially from the policy responses typically derived in a full information context (see, e.g., Oswald, 1983; Tuomala, 1990; Ireland, 2001; Aronsson and Johansson-Stenman, 2008, 2010). In doing this, we use an extension of the two-type model developed by Stern (1982) and Stiglitz (1982), where the government in each country can use nonlinear income taxes but not tax leisure or ability directly. We show that the basic findings obtained in Sections 3-4 continue to hold under certain conditions, but that interactions between externality correction and redistribution through the self-selection constraint may also have important implications for optimal taxation. Section 6 provides some concluding remarks.
2. Preferences and Individual Behavior

In this section, we outline the basics of our model assuming that people have preferences for relative consumption both within and between countries. We have no ambition to explain why people derive utility from their relative consumption. An alternative approach would be to start from conventional preferences where instead relative consumption has a purely instrumental value; see, e.g., Cole et al. (1992, 1998) for interesting applications of such an approach and Cole et al. (1995) for thoughtful arguments in favor of it. Yet, while we certainly share the view that there are important instrumental reasons underlying why relative consumption matters, we see two main reasons for simply imposing such concerns directly into the utility function in the present paper. First, the fact that there has been an important evolutionary value to have more wealth than others provides an obvious reason for why selfish genes would prefer to belong to people with preferences for relative wealth and status (just as they would prefer to belong to people with preferences for having sex and against eating poisoned food); cf., Frank (1985), Samuelson (2004) and Rayo and Becker (2007). Second, the shortcut to ignore instrumental reasons in the model, and hence focus solely on effects through the utility function, makes the model comparable to much earlier literature on public policy and relative consumption as well as more tractable and suitable for analyzing the optimal tax problems at stake.

The model consists of a large number, $n$, of small countries with fixed populations. To begin with, we assume that the population in each country consists of a fixed number of identical individuals normalized to one. This assumption is relaxed in Section 5 below, where we introduce differences in ability (productivity) between individuals and assume that this ability is private information. Each individual in country $i$ derives utility from his/her absolute

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4 One might object that the evolutionary arguments are stronger for social comparisons within small groups than between countries, just as the evolutionary arguments for pro-social behavior are stronger within small groups. While agreeing in principle, we still have two counter-arguments: First, there is actually compelling evidence in favor of what Singer (1983) denotes the expanding circle with respect to pro-social behavior and ethics, i.e., that human beings over time tend to take into account consequences for larger and larger groups of people; see in particular Pinker (2012). Second, we are not biologically well adapted to the recent technological development implying, e.g., that we may emotionally perceive people on TV to be closer to us than most people who live in the same block.
consumption of goods, $c^i$, and use of leisure, $z^i$, and also from his/her relative consumption compared with other people. The latter is of two kinds: relative consumption compared with other people in the individual’s own country, $R^i$, and relative consumption compared with people in other countries, $S^i$. Relative consumption of the first kind can then be written as $R^i = r^i(c^i, \bar{c}^i)$, where $\bar{c}^i$ is average consumption in country $i$. Correspondingly, we can write relative consumption of the second kind, i.e., compared with the $n$-1 other countries, as a vector

$$S^i = s^i(c^i, \bar{c}^{-i}) = \{s^1(c^i, \bar{c}^1), ..., s^{i-1}(c^i, \bar{c}^{i-1}), s^{i+1}(c^i, \bar{c}^{i+1}), ..., s^n(c^i, \bar{c}^n)\},$$

where $\bar{c}^{-i}$ is a vector of average consumption levels in all countries except country $i$.

The utility function faced by the representative individual in country $i$ is given by

$$U^i = v^i\left(c^i, z^i, R^i, S^i\right) = v^i\left(c^i, z^i, r^i(c^i, \bar{c}^i), s^i(c^i, \bar{c}^{-i})\right) = u^i\left(c^i, z^i, \bar{c}^i, \bar{c}^{-i}\right), \quad (1)$$

where $v^i, u^i, r^i$ and all elements of $s^i$ are twice continuously differentiable. The function $v^i(\cdot)$ is assumed to be increasing in each argument and strictly quasi-concave, and describes the individual’s utility as a function of his/her own consumption and use of leisure, respectively, as well as of his/her relative consumption compared with others. The function $u^i(\cdot)$ is a convenient reduced form allowing us to shorten some of the notations below. For further use, we summarize the relationships between $u^i(\cdot)$ and $v^i(\cdot)$ as follows:

$$u^i_c = v^i_c + v^i_{r_c} r^i_c + v^i_{s_c} s^i_c,$$
$$u^i_z = v^i_z,$$
$$u^i_{r_c} = v^i_{r_c} r^i_c,$$
$$u^i_{s_{c^{-i}}} = v^i_{s_{c^{-i}}} s^i_{c^{-i}}.$$

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5 Following most previous comparable literature, we assume that leisure is completely non-positional, meaning that people only care about the absolute level of leisure. Aronsson and Johansson-Stenman (2013) analyze a model of optimal taxation where the consumers have positional preferences with respect to both private consumption and leisure.
where subscripts denote partial derivatives, i.e., \( u_i^c = \partial u^i / \partial c^i \), \( v_i^c = \partial v^i / \partial c^i \), \( r_i^c = \partial r^i / \partial c^i \), and \( s_i^c = \partial s^i / \partial c^i \), and similarly for the partial derivatives with respect to \( z^i \), \( \bar{c}^i \), and \( \bar{c}^{-i} \).

We also assume that \( r^i(\cdot) \) and \( s^i(\cdot) \) satisfy the criterion that the value of each function is unaffected if the individual’s own and others’ consumption are changed equally, i.e., \( r_{i^j}^c = -r_i^c \) and \( s_{i^k}^c = -s_i^c \) for all \( i, k \) such that \( i \neq k \). The first assumption is fairly innocuous and encompasses the most commonly used comparison forms, i.e., the difference comparison form where \( R^i = c^i - \bar{c}^i \), the ratio comparison case where \( R^i = c^i / \bar{c}^i \), and the flexible functional form suggested by Dupor and Liu (2003), which includes both the difference and the ratio forms as special cases. The second assumption, i.e., \( s_{i^k}^c = -s_i^c \), is stronger and essentially implies the difference comparison form such that \( s_i^c = c^i - \bar{c}^i \). Note also that people in different countries need not be identical regarding consumption levels or preferences.

The government in country \( i \) can tax private income (and hence consumption) by utilizing an income tax \( t^i \) and distribute back the revenues in lump-sum form, such that each individual receives a lump-sum payment, \( \tau^i \), regardless of behavior. For simplicity we assume a linear technology and perfect competition, implying zero profits, and that productivity is fixed with fixed before-tax wage rates \( w^i \). The individual budget constraint can then be written as

\[
w^i(\Omega - z^i)(1-t^i) + \tau^i = c^i,
\]  
(2)

Although a flexible functional form is always preferable to more restrictive formulations, it is of no great importance for the qualitative results whether the analysis is based on difference comparisons (such as in Akerlof, 1997; Corneo and Jeanne, 1997; Ljungqvist and Uhlig, 2000; Bowles and Park, 2005; and Carlsson et al., 2007) or ratio comparisons (such as in Boskin and Sheshinski, 1978; Layard, 1980; Abel, 2005; and Wendner and Goulder, 2008). Mujcic and Frijters (forthcoming) compare models based on difference comparisons, ratio comparisons and rank comparisons without being able to discriminate between them, whereas Corazzini et al. (2012) find that absolute differences, and not only rank, matter, suggesting that models based solely on rank comparisons are more restrictive than the other formulations. Aronsson and Johansson-Stenman (forthcoming b) show that the optimal tax policy implications of relative consumption concerns tend to be qualitatively similar regardless of whether these comparisons take the difference or ratio form.
where $\Omega$ is the total time available (i.e., 24 hours a day).

Although the measures of reference consumption facing the representative consumer in country $i$, i.e., $\bar{c}^i$ and $\bar{c}^{-i}$, are endogenous in our model, we assume that each individual treats them as exogenous. This reflects the idea that each individual is small relative to the economy as a whole, which is the conventional assumption in models with externalities. The individual first order condition regarding the consumption-leisure tradeoff then becomes

$$u_i'w[1-t'] = u_i',$$

where (as before) subscripts denote partial derivatives.

### 2.1 Degrees of positionality

The optimal tax policy presented below depends on the extent to which relative consumption matters at the individual level (and not just on whether or not it matters). Following Johansson-Stenman et al. (2002) and Aronsson and Johansson-Stenman (2008), we introduce the concept of “degrees of positionality” as reflections of the extent to which relative consumption matters for utility. Yet, since we have several countries, we will have different measures for the extent to which relative consumption matters within the country and the extent to which relative consumption matters between countries.

Let us define the degree of domestic positionality as

$$\alpha_i^j = \frac{v_i^j r_i^j}{v_i^j + v_i^j r_i^j + v_i^j s_i^j},$$

where $v_i^j$ and $s_i^j$ are vectors such that

$$v_i^j \equiv \{ v_i^{j1}, ..., v_i^{j\mu-1}, v_i^{j\mu+1}, ..., v_i^{jm} \}$$

and

$$s_i^j \equiv \{ s_i^{j1}, ..., s_i^{j\mu-1}, s_i^{j\mu+1}, ..., s_i^{jm} \}^T,$$
while \(v^i_s \equiv \partial v^i / \partial S^k\), \(s^i_c \equiv \partial s^i / \partial c^i\), and \(v^i_s s^i_c = \sum_k v^i_s s^i_c\). The variable \(\alpha^i\) reflects the fraction of the overall utility increase from the last dollar consumed that is due to the increased relative consumption compared with other people in the individual’s own country.

Similarly, we can define the partial degree of foreign positionality as

\[
\beta^{ik} = \frac{v^i_s s^i_c}{v^i_c + v^i_k r^i + v^i_s s^i_c},
\]

which reflects the fraction of the overall utility increase from the last dollar consumed by the representative consumer in country \(i\) that is due to the increased relative consumption compared with people in country \(k\). Note that comparisons with the consumption levels in some countries (e.g., neighbors) may of course be more important than with those in other countries. We can then define the overall degree of foreign positionality as

\[
\beta^i = \sum_{k,l} \beta^{ik} = \frac{v^i_s s^i_c}{v^i_c + v^i_k r^i + v^i_s s^i_c}.
\]

As such, \(\beta^i\) reflects the fraction of the utility increase from the last dollar consumed that is due to the increased relative consumption compared with people in other countries. Note that \(\beta^i\) thus reflects the net effect of all relative consumption comparisons with the other countries. The total degree of positionality is then correspondingly defined as

\[
\rho^i = \alpha^i + \beta^i,
\]

meaning that \(\rho^i\) reflects the fraction of the utility increase from the last dollar consumed that is due to increased relative consumption of any kind, i.e., including comparisons with people both within and outside the individual’s own country.

3. Optimal Tax Policy and Noncooperative Behavior
We start in subsection 3.1 by considering the policy implications of a Nash equilibrium such that each national government treats the decisions made in the other countries as exogenous. In subsection 3.2, we consider a Stackelberg equilibrium, where one of the countries is acting as leader and the others as followers.

3.1 Nash competition

The decision-problem of the government in country $i$ implies maximization of $U^i$, where the externalities that each domestic resident imposes on other domestic residents are taken into account, while the externalities imposed on other countries remain uninternalized. As such, the government in country $i$ recognizes that $c^i$ is endogenous, while it treats $c^{-i}$ as exogenous. The public budget constraint is given by

$$w'(\Omega - z') = c', \quad (7)$$

implying the Lagrangean

$$L' = u' \left(c^i, z^i, c^{-i}, \tilde{c}^{-i}\right) + \gamma' \left[w'(\Omega - z') - c'\right]. \quad (8)$$

The corresponding first order conditions are given by

$$u'_c + u'_{c^i} = \gamma', \quad (9)$$

$$u'_c = \gamma' w'. \quad (10)$$

By using equations (9) and (10) and the private first order condition for labor supply given by equation (3), we obtain the following result:

**Proposition 1.** The marginal income tax rate facing the representative consumer in an arbitrary country $i$ in Nash equilibrium is given by

$$t^i = \alpha'.$$
Proof: Combining equations (9) and (10) gives
\[ u_{it}^i = w_i^t (u_c^i + u_{it}^i). \] (11)
Using \( u_{it}^i w_i^t = u_i^t w_i^t - u_i^t \) from equation (3), substituting into equation (11), and solving for \( t_i^t \) yields
\[ t_i^t = -\frac{u_c^i}{u_i^t}. \] (12)
Finally, using equations (1) and (4), we can rewrite equation (12) in terms of the degree of domestic positionality. Since \( u_c^i = v_c^i + v_{it}^i r_c^i + v_{it}^i s_c^i \) and \( u_i^t = v_i^i r_i^t \), we have
\[ t_i^t = -\frac{v_i^i r_i^t}{v_c^i + v_{it}^i r_c^i + v_{it}^i s_c^i} = \alpha_i^t, \] (13)
where we have used that \( r_i^t = -r_i^t \). QED

Hence, the optimal tax is simply given by the sum of people’s marginal willingness to pay for an individual to reduce his/her consumption, where the sum of the marginal willingness to pay is measured within the own country. Each government will fully internalize the positional externalities within the country, but not at all internalize the positional externalities inferred on other countries. And a tax that fully internalizes the positional externalities within the country, in turn, equals the degree of domestic positionality as defined by equation (4). Yet, it should be clear that the tax formula in Proposition 1 does not implement a global welfare optimum, since transnational positional externalities are ignored.

3.2 Country i is a Stackelberg leader

Assume now instead that country i is a Stackelberg leader in relation to country k, which is a Stackelberg follower, and that it plays the Nash game with all other countries. If the government in country k is a Stackelberg follower, it clearly behaves as in the Nash equilibrium. Yet, the optimization problem for country i is modified, since i will take into account welfare effects on i caused by the changed actions in k that choices by i induce. As a consequence, the government in country i will not take the consumption in country k as given.

7 As such, the corrective tax derived here resembles Nash equilibrium tax formulas in the literature on environmental policy (e.g., van der Ploeg and de Zeeuw, 1992; Aronsson and Löfgren, 2000).

8 The assumption that it plays the Stackelberg game with only one follower country is made for convenience; it is straightforward to allow several countries to be Stackelberg followers.
as it did in subsection 3.1 above, but rather let it be a function of its own consumption, such that \( c^k = c^k (\bar{c}^i) \). Then we can instead write the Lagrangean as

\[
L = u'(c', z', \bar{c}^i, \bar{c}^k (\bar{c}^i), \bar{c}^{i-k}) + \gamma'[w'(\Omega - z') - c'],
\]

where \( \bar{c}^{i-k} \) denotes a vector of average consumption levels in all other countries than \( i \) and \( k \), which are still treated as exogenous by the government in country \( i \). The first order conditions become

\[
u^i_c + u^{i, c}_c + u^{i, c}_z \frac{\partial \bar{c}^k}{\partial \bar{c}^i} = \gamma',
\]

\[
u^i_c = \gamma' w'.
\]

Before we analyze the tax policy implemented by the Stackelberg leader in any detail, the relationship between \( \bar{c}^i \) and \( \bar{c}^k \) in equation (15) needs to be addressed, since the incentive for country \( i \) to exercise leadership through tax policy depends on how country \( k \) (the Stackelberg follower) responds to an increase in \( \bar{c}^i \), ceteris paribus. The following characterization will then be used:

**Definition 1.** The consumption in country \( k \) is characterized by a cross-country keeping-up-with-the-Joneses (staying-away-from-the-Joneses) property with respect to the consumption in country \( i \) if

\[
\frac{\partial \bar{c}^k}{\partial \bar{c}^i} > 0 \text{ or } < 0.
\]

Let \( SMRS_{i, z}^{k} = (u^k_c + u^k_z) / u^i_z \) denote the social marginal rate of substitution between private consumption and leisure from the point of view of country \( k \), whose government treats \( \bar{c}^i \) as exogenous. In other words, \( SMRS_{i, z}^{k} \) reflects the marginal rate of substitution between private consumption and leisure in country \( k \) for a given relative consumption within the country (but not between countries). We can then derive the following result:
**Lemma 1.** \( \frac{\partial c^k}{\partial c^i} > 0 \) \( (< 0) \) iff \( \frac{\partial SMRS^k_{cz}}{\partial c^i} > 0 \) \( (< 0) \).

Proof: See Appendix.

Using Definition 1 and Lemma 1, we are now ready to analyze the optimal tax policy implicit in the Stackelberg game equilibrium:

**Proposition 2.** The optimal income tax formula in country \( k \), where the government is a Stackelberg follower, is the same as in the Nash equilibrium. The optimal marginal income tax in country \( i \), where the government is a Stackelberg leader vis-à-vis country \( k \), is given by

\[
i^i = \alpha^i + \frac{\partial c^k}{\partial c^i} \beta^k.
\]

Therefore, the optimal marginal income tax rate facing the Stackelberg leader is larger (smaller) than the optimal rate implied by the Nash equilibrium formula if the utility function in country \( k \) is such that

\[
\frac{\partial SMRS^k_{cz}}{\partial c^i} > 0 \quad (< 0),
\]

meaning that the consumption in country \( k \) is characterized by a cross-country keeping-up-with-the-Joneses (staying-away-from-the-Joneses) property with respect to the consumption in country \( i \).

**Proof:** Starting with the tax formula, we combine equations (15) and (16) to derive

\[
u_c^i = \psi^i \left[ u_c^i + u_c^k + u_c^i \frac{\partial c^k}{\partial c^i} \right].
\]

(17)

Next, combining equations (3) and (17) and solving for \( t^i \), gives

\[
t^i = - \frac{u_c^i + u_c^i \frac{\partial c^k}{\partial c^i}}{u_c^i}.
\]

(18)
Finally, since $u_i^t = v_i^t + v_r^t r_i^t + v_s^t s_i^t$ and $u_{c+i}^t = v_s^t s_{c+i}$ as defined above, we obtain

$$t^i = -\frac{v_i^t + v_r^t r_i^t + v_s^t s_i^t}{\frac{\partial \tilde{c}_{c+i}^k}{\partial c^i}} = \alpha^i + \frac{\partial \tilde{c}_{c+i}^k}{\partial c^i} \beta^i,$$

where we have used $r_i^t = -r_i^t$ and $s_i^t = -s_i^t$. The second part follows immediately from combing equation (19) with Lemma 1. QED

Thus, the optimal marginal income tax rate in country $i$, the Stackelberg leader, is larger than the rate corresponding to optimal taxation in the Nash equilibrium if consumption becomes more valuable relative to leisure on the margin in country $k$ due to a consumption increase in country $i$. Intuitively, if increased consumption in country $i$ induces people to consume more in country $k$, and hence causes larger negative externalities on country $i$, this constitutes a reason to reduce the consumption in country $i$, and hence to increase the marginal income tax.

4. Cooperative Solutions

4.1 The scope for a Pareto-improving tax reform

We showed in Section 3 that each government in the Nash equilibrium will only internalize the positional externalities caused in their own country. The same applies in the Stackelberg case, where the optimum conditions are the same for the followers, while the leader will also add a component related to induced consumption changes in other countries due to transnational keeping-up-with-the-Joneses effects. Thus, there is scope for Pareto-improving tax reforms:

**Proposition 3.** Based on either the Nash equilibrium or the Stackelberg game equilibrium, there is scope for Pareto-improving tax reforms through small increases in the marginal income tax rates.

**Proof:** The welfare effect in country $i$ if country $k$ increases its marginal income tax rate is given by

$$u_i^t, \frac{\partial \tilde{c}_i^k}{\partial t^k} > 0.$$
This holds irrespective of whether the pre-reform equilibrium is based on the Nash or the Stackelberg game, and whether in the latter case $i$ is the leader or the follower. QED

Given that a Pareto improvement is possible, it is natural to ask how much the government in country $i$ would be willing to pay country $k$ for a small increase in $t^k$. This clearly depends on the game. In the Nash case, $i$ will only consider the direct effect of $c^k$ on its own welfare, since country $k$ takes $i$'s consumption as given. Let $M^{ik}$ be $i$'s marginal willingness to pay for increasing the income tax in country $k$, i.e.,

$$M^{ik} = \frac{u_i' - u_i'}{u_i' + u_i'} \frac{\partial c^k}{\partial t^k} = -\beta^{ik} \frac{\partial c^k}{\partial t^k} > 0. \tag{20}$$

Equation (20) indicates that the (partial) degree of foreign positionality plays a key role for tax coordination, as it determines how much the government in country $i$ is willing to pay for a small decrease in the consumption in country $k$, ceteris paribus. The same algebraic expression holds in the Stackelberg game where $i$ is leader.

### 4.2. Efficient international negotiations on the tax rates

Here we consider the somewhat extreme case where countries can negotiate with each other about tax policy without transaction costs. Suppose for convenience that we have only the two countries $i$ and $k$, who can negotiate efficiently about the other country’s marginal income tax rate. Country $i$ would then be willing to buy a further marginal tax increase in country $k$ as long as the welfare cost to $i$ of paying $k$ is lower than the welfare gain to $i$ of the associated reduced consumption in $k$. Let us also assume that the countries succeed in finding an agreement such that no Pareto improvements are possible. This means that the marginal income tax rates will be (globally) Pareto efficient. An alternative interpretation of such a resource allocation is that it corresponds to the outcome of a global social planner aiming to obtain a globally Pareto-efficient allocation.

Consider the Lagrangean corresponding to the maximization of utility in country $i$ subject to a constraint that utility is held fixed in country $k$ and an overall resource constraint:
\[ L = u' + \mu [u^k - U^k] + \gamma [w' (\Omega - z') - c' + w^k (\Omega - z^k) - c^k]. \] (21)

and the corresponding first order conditions

\[ u'_c + u'_c + \mu u^k_c = \gamma, \] (22)
\[ u'_c + \mu u^k_c + \mu u^k_c = \gamma, \] (23)
\[ u'_z / w' = \gamma, \] (24)
\[ \mu u^k_c / w^k = \gamma. \] (25)

We have derived the following result:

**Proposition 4.** For country \( i \), which can negotiate with another country \( k \) without transaction costs, the optimal marginal income tax rate is given by

\[ t' = \alpha' + \frac{1 - \alpha' + \beta' \beta^k}{1 - \alpha^k + \beta^k} > 0. \]

**Proof:** Equations (24) and (25) imply

\[ \mu = \frac{u'_z / w'}{u'_z / w^k} = \frac{u'_z}{u'_z} \frac{w^k}{w'} \] (26)

Combine equations (22) and (23) and use equation (26) to substitute for \( \mu \)

\[ \frac{u'_k}{u'_c} \frac{w^k}{w'} \left[ 1 + \frac{u'_c}{u'_c} - \frac{u'_z}{u'_z} \right] = \frac{u'_z}{u'_c} \frac{w^k}{w'} \left[ 1 + \frac{u'_c}{u'_c} - \frac{u'_z}{u'_z} \right], \] (27)

while equations (22), (24), and (26) can be combined in a similar way to give

\[ \frac{u'_k}{u'_c} = \frac{u'_z / u'_c}{u'_z \frac{w^k}{w'} - u'_c \frac{w^k}{w'}}. \] (28)

Substituting equation (28) into equation (27) and using the individual budget constraints \( u'_z / w' = u'_c [1 - t'] \) imply
Finally, rewriting equation (29) in terms of positionality degrees such that $u'_e / u'_e = -\alpha^i$, $u'_e / u'_e = -\beta^i$, $u'_e / u'_e = -\alpha^k$, and $u'_e / u'_e = -\beta^k$ gives the formula in Proposition 4. QED

Thus, the optimal tax expression looks almost like a conventional Pigouvian tax based on the sum of all people’s (including people from other countries) marginal willingness to pay for reducing consumption by an individual in country $i$, which would be given by

$$i' = -\frac{u'_i}{u'_e} - \frac{u'_i}{u'_k} = \alpha^i + \beta^k.$$  

Yet, the second term in the tax formula in Proposition 4, related to the sum of the marginal willingness to pay by residents in the foreign country, $u'_e / u'_e$, has a modifying factor attached to it. We will return to this factor and the intuition behind Proposition 4. Let us first present the more straightforward results from the symmetric case where the positionality degrees are identical in both countries:

**Corollary 1.** If $\alpha^i = \alpha^k = \alpha$ and $\beta^i = \beta^k = \beta$, the optimal marginal income tax rate for country $i$, which can negotiate without transaction costs with another country $k$, is given by

$$i' = \alpha^i + \beta^k = \alpha + \beta = \rho > 0.$$  

Proof: Follows directly from Proposition 4.

Hence, the optimal tax in the symmetric case is a simple Pigouvian tax given by the aggregate global marginal willingness to pay for reduced consumption by an individual in country $i$. In turn, this sum equals the total degree of positionality, $\rho$. Basically, the tax reflects the part of consumption that is waste, due to zero-sum relative comparison effects, whereas leisure is purely non-positional (by assumption). As such, Corollary 1 provides a straightforward generalization of the efficient tax policy derived in the context of one-country economies in, e.g., Persson (1995), Ljungqvist and Uhlig (2000), and Dupor and Liu (2003).
Let us now turn to the modifying factor in the non-symmetric case, i.e.,

\[
\frac{1 - \alpha^i + \beta^i}{1 - \alpha^k + \beta^k}.
\]

Suppose first that the \( \beta \)-factors are small, such that the modifying factor can be approximated by \((1 - \alpha^i) / (1 - \alpha^k)\). Then, if \( \alpha^i > \alpha^k \), the modifying factor for the marginal income tax rate in country \( i \) becomes less than unity. The intuition is that \( \alpha^i > \alpha^k \) implies that the optimal marginal income tax in country \( i \) is larger than in country \( k \). In turn, this means that a larger fraction of an income increase in country \( i \) is taxed away, such that a smaller fraction of this income increase causes a negative consumption externality. In the more general case where the \( \beta \)-factors are not small, also these factors will affect how much of an income increase in relative terms will be taxed away. A large \( \beta \) in country \( i \) then implies that a larger fraction will be taxed away in country \( k \) (rather than in country \( i \)), and vice versa. As such, the relative weight given to domestic externality-correction is reduced in country \( i \), which also explains why the \( \beta \)-factors affect the modifying factor in the opposite direction compared with the \( \alpha \)-factors.

How would the analysis change if we were to include many countries? In principle, the problem of finding a Pareto-efficient allocation is both qualitatively and quantitatively equivalent in the many-country case. Yet, the coordination problem is much more complex, as the consumption in a single country will cause utility losses in many other countries. The optimal marginal income tax rate would, therefore, also take a more complex form than in Proposition 4; in particular, the second part of the tax formula would be expanded if additional countries were included. At the same time, the interpretation in terms of the outcome of a negotiation process without transaction costs is much less straightforward here, since there would be many potential coalitions and many Nash equilibria. Nevertheless, the interpretation in terms of the outcome of a global social planner would still be valid.

5. Distributional Concerns and Asymmetric Information

So far, we have assumed that people are identical within each country, and that the only reason for using income taxes is to correct for positional externalities. In reality, however, taxation has many purposes, a central one being to redistribute income. In this section, we
generalize the model to encompass heterogeneity and distributional concerns within each country. As a work horse, we utilize a modified version of the Stern-Stiglitz optimal nonlinear income taxation model with two ability types in each country.

Each country is characterized by asymmetric information between the government and the private sector, such that the government can observe (and hence tax) income but not leisure. Furthermore, we assume (as we did above) that the population in each country is fixed; this simplifies the analysis and allows us to abstract from the implications of labor mobility for redistributive policy at the national level.

There are two ability types in each country and \( n_j \) individuals of ability type \( j \) in country \( i \). Each such individual faces the following utility function:

\[
U_j^i = v_j^i \left( c_j^i, z_j^i, R_j^i, S_j^i \right) = v_j^i \left( c_j^i, z_j^i, r_j^i(c_j^i, \bar{c}^i), s_j^i(c_j^i, \bar{c}^i) \right) = u_j^i \left( c_j^i, z_j^i, \bar{c}^i, \bar{c}^i \right),
\]  

for \( j = 1, 2 \). Equation (30) allows for the same between-country differences in preferences as equation (1); yet, it also allows the two ability types in the same country to have different preferences and make different relative consumption comparisons. All notations are the same as in the previous two sections, with the exception that the variables are both ability-type specific and country-specific here (and not just country-specific as above).

The individual budget constraint is given by

\[
w_j^i l_j^i - T^i(w_j^i l_j^i) = c_j^i,
\]  

where \( l_j^i = \Omega - z_j^i \) denotes the hours of work by ability type \( j \) in country \( i \), and \( T^i(\cdot) \) is a nonlinear income tax decided by the government in country \( i \). The corresponding first order condition for the consumption-leisure tradeoff becomes

\[
u_{j,c}^i w_j^i [1 - T^i(w_j^i l_j^i)] - u_{j,c}^i = 0,
\]  

(32)
where \( u'_{j,c} = \partial u'_{j} / \partial c'_{j} \) and \( u''_{j,c} = \partial u'_{j} / \partial c'_{j} \), while \( T' (w'_{j,i}) \) denotes the marginal income tax rate facing ability type \( j \) in country \( i \).

5.1 Degrees of positionality

The positionality degrees are defined in the same general way as in the representative-agent framework set out above. Yet, since the utility functions may differ between types, the positionality degrees may differ too for that particular reason (and, of course, also because the two ability types face different constraints). By analogy to the positionality measures presented in subsection 2.1, the partial degrees of domestic and foreign positionality for an individual of type \( j \) in country \( i \) can be written as

\[
\alpha'_{j} = \frac{v'_{j,R}f'_{j,c}}{v'_{j,c} + v'_{j,R}f'_{j,c} + v'_{j,S}s'_{j,c}},
\]

(33)

\[
\beta'_{jk} = \frac{v'_{j,R}r'_{j,c}}{v'_{j,c} + v'_{j,R}f'_{j,c} + v'_{j,S}s'_{j,c}},
\]

(34a)

\[
\beta'_{j} = \sum_{k} \beta'_{jk} = \frac{v'_{j,S}s'_{j,c}}{v'_{j,c} + v'_{j,R}f'_{j,c} + v'_{j,S}s'_{j,c}},
\]

(34b)

where \( v'_{j,S} = \{v'_{j,S}^{1}, \ldots, v'_{j,S}^{i-1}, v'_{j,S}^{i+1}, \ldots, v'_{j,S}^{n}\} \) is a vector of the partial derivatives of the utility function for an individual of ability type \( j \) in country \( i \) with respect to each relative consumption vis-à-vis the average consumption in other countries. Correspondingly, \( s'_{j,c} = \{s'_{j,c}^{1}, \ldots, s'_{j,c}^{i-1}, s'_{j,c}^{i+1}, \ldots, s'_{j,c}^{n}\} \) is a (column) vector of partial derivatives of these cross-country relative consumption measures with respect to the individual’s own consumption, such that

\[
v'_{j,S} s'_{j,c} = \sum_{k} v'_{j,S} s'_{j,c}^{k}.
\]

The interpretations are the same as before, with the only exception that the degrees of positionality are ability-type specific here: \( \alpha'_{j} \) reflects the fraction of the overall utility increase from the last dollar consumed for an individual of ability type \( j \) in country \( i \) that is due to the increased relative consumption compared with others in the individual’s own country, while \( \beta'_{j} \) denotes the corresponding fraction of the utility increase that is due to the
increased relative consumption compared with people in other countries. The total degree of positionality for an individual of ability type $j$ in country $i$ is then defined as

$$\rho'_j = \alpha'_j + \sum_{k=1}^{2} \beta'_{jk} = \alpha'_j + \beta'_j. \quad (35)$$

For further use, we also calculate the corresponding average degrees of positionality in country $i$ as

$$\bar{\alpha}^i = \frac{n_i^1 \alpha_1^i + n_i^2 \alpha_2^i}{N^i}, \quad (36a)$$

$$\bar{\beta}_{ik} = \frac{n_i^1 \beta_{1k} + n_i^2 \beta_{2k}}{N^i}, \quad (36b)$$

$$\bar{\beta}^i = \frac{n_i^1 \beta_1^i + n_i^2 \beta_2^i}{N^i}, \quad (36c)$$

where $N^i = n_i^1 + n_i^2$ denotes the total population in country $i$.

### 5.2 The second-best problem of the government

Let type 1 be the low-ability type and type 2 the high-ability type, which means that $w_2^i > w_1^i$. The objective of the government in each county is again to obtain a Pareto-efficient resource allocation, which can be accomplished by maximizing the utility of the low-ability type subject to a minimum utility restriction for the high-ability type, as well as subject to a self-selection constraint and the budget constraint. We also follow the standard approach in assuming that the government wants to redistribute from the high-ability to the low-ability type. The self-selection constraint that must be imposed to prevent the high-ability type from mimicking the low-ability type can then be written as

$$U_2^i = u_2^i \left( c_2^i, z_2^i, c_2^i, \bar{c}^{-i} \right) \geq u_2^i \left( c_1^i, \Omega - \frac{w_2^i t_1^i}{w_2^i}, \bar{c}^i, \bar{c}^{-i} \right) = \hat{U}_2^i. \quad (37)$$
The expression on the left-hand side of the weak inequality is the utility of the high-ability type, while the right-hand side denotes the utility of the mimicker. A mimicking high-ability type faces the same before-tax income (and in this case also consumption) as the low-ability type; yet, since the mimicker is more productive, he/she can reach this income with less effort than the low-ability type. Throughout the paper, we use the hat symbol (\(^\hat{\cdot}\)) to denote mimicker variables.

As we are considering a pure redistribution problem under positional externalities, it follows that the government’s overall resource constraint can be written as

\[
n_1^i(w_1^i l_1^i - c_1^i) + n_2^i(w_2^i l_2^i - c_2^i) = 0,
\]

i.e., overall production equals overall consumption.

Therefore, and by analogy with earlier literature based on the self-selection approach to optimal income taxation (see, e.g., Stiglitz, 1982; Boadway and Keen, 1993), the marginal income tax rates can be derived implicitly by choosing the number of work hours and private consumption for each ability type in each country based on the following Lagrangean for an arbitrary country \(i\):

\[
L' = u'_i + \delta'[u'_2 - \hat{u}'_2] + \lambda' [u'_2 - \hat{u}'_2] + \gamma'[n_1^i(w_1^i l_1^i - c_1^i) + n_2^i(w_2^i l_2^i - c_2^i)].
\] (38)

In equation (38), utility is written in terms of the function \(u'_j(\cdot)\) defined in equation (30), and \(\hat{u}'_2\) therefore denotes the utility of the mimicker. The variables \(\delta', \lambda', \) and \(\gamma'\) are Lagrange multipliers.

5.3 Optimal tax policy in the Nash-competition case

In the Nash equilibrium, the government in country \(i\) treats consumption (and hence average consumption) in all other countries as exogenous, and vice versa. The first order conditions for \(l_1^i, c_1^i, l_2^i, \) and \(c_2^i\) are then given as follows:
\[-u_{i,z}^i + \frac{w_i^i}{w_2^i} \lambda^i \hat{u}_{i,2z}^i + \gamma^i n_i^i w_i^i = 0, \tag{39}\]

\[u_{i,c}^i - \lambda^i \hat{u}_{i,2c}^i - \gamma^i n_i^i + \frac{\partial L_i^i}{\partial c_i^i} N_i^i = 0, \tag{40}\]

\[-(\delta^i + \lambda^i) u_{i,z}^i + \gamma^i n_i^i w_2^i = 0, \tag{41}\]

\[(\delta^i + \lambda^i) u_{i,c}^i - \gamma^i n_i^i + \frac{\partial L_i^i}{\partial c_i^i} N_i^i = 0, \tag{42}\]

where

\[\frac{\partial L_i^i}{\partial c_i^i} = u_{i,c}^i + (\delta^i + \lambda^i) u_{i,2c}^i - \lambda^i \hat{u}_{i,2c}^i. \tag{43}\]

Equation (43) measures the partial welfare effect for country \(i\) of an increase in \(\bar{c}_i\), which will be referred to as the \textit{within-country positionality effect} in what follows. We will return to this measure in more detail below.

Before presenting the results, let us introduce \(\sigma_j^i\) as a short notation for the optimal marginal income tax rate facing ability type \(j\) in country \(i\) in the absence of any relative consumption concerns, i.e., the expressions for optimal marginal income taxation in the standard two-type model, as

\[\sigma_{j,1}^i = \frac{\lambda^i \hat{u}_{i,c}^i}{\gamma^i n_i^i w_i^i} \left[ MRS_{1,2x}^i - \frac{w_i^i}{w_2^i} MRS_{2,2x}^i \right], \tag{44a}\]

\[\sigma_{j,2}^i = 0. \tag{44b}\]

In equation (44a), \(MRS_{1,2x}^i = \frac{[\partial u_i^i / \partial z_i^i]}{[\partial u_i^i / \partial c_i^i]}\) denotes the marginal rate of substitution between leisure and private consumption for the low-ability type, while \(MRS_{2,2x}^i\) denotes the corresponding marginal rate of substitution for the mimicker. Then, by using equations (39)-(42) and (44a)-(44b), we show in the Appendix that the optimal marginal income tax rate for individuals of ability type \(j\) in country \(i\) can be written in a general form as:

24
\[
T^i (w^i/L^i) = \sigma^i_j - \frac{MRS_j,\bar{c}}{\gamma^i w^i/N^i} \frac{\partial L}{\partial \bar{c}^i}. 
\]

In what follows, we will not focus on the interpretation of the variables \( \sigma^i_1 \) and \( \sigma^i_2 \), as these are well understood and explained elsewhere (e.g., Stiglitz, 1982). Instead, we will focus on how the marginal income tax rates are modified due to the relative consumption concerns. As can be observed from equation (45), the optimal marginal income tax rate of either ability type can be written as an additively separable expression, where in addition to \( \sigma^i_j \) there is a second term that is proportional to the within-country positionality effect, \( \partial L^i / \partial \bar{c}^i \). To be able to explore this positionality effect in greater detail, such that we can express the marginal income tax rates in terms of positionality degrees, we start by introducing a measure of the difference in the partial degree of domestic positionality between the mimicker and the low-ability type as follows:

\[
\alpha^{id} = \frac{\lambda^i \bar{u}^i_{2,c}}{\gamma^i N^i} \left[ \hat{\alpha}^i_2 - \hat{\alpha}^i_1 \right]. 
\]

We can then derive the following result with respect to the within-country positionality effect:

**Lemma 2.** For country \( i \), whose government is a Nash competitor to the governments in the other countries, the within-country positionality effect can be written as

\[
\frac{\partial L^i}{\partial \bar{c}^i} = -\gamma^i N^i \frac{\bar{\alpha}^i}{1 - \bar{\alpha}^i} + \lambda^i \bar{u}^i_{2,c} \frac{\bar{\alpha}^i_2 - \bar{\alpha}^i_1}{1 - \bar{\alpha}^i} = -\gamma^i N^i \frac{\bar{\alpha}^i - \alpha^{id}}{1 - \bar{\alpha}^i}. 
\]

Proof: See Appendix.

The within-country positionality effect for country \( i \) can be decomposed into two separate parts reflecting: (1) the average degree of domestic positionality, \( \bar{\alpha}^i \), and (2) the difference in the degree of domestic positionality between the mimicker and the low-ability type, \( \alpha^{id} \). Clearly, the average degree of domestic positionality contributes negatively to welfare in country \( i \), as it reflects the direct welfare cost of the externality that each individual imposes
on other domestic residents. On the other hand, the indicator of positionality differences leads to increased domestic welfare if the mimicker is more positional than the low-ability type (in which case $\alpha^{id} > 0$), and lower domestic welfare if the low-ability type is more positional than the mimicker (so $\alpha^{id} < 0$). The intuition is that if the mimicker is more (less) positional than the low-ability type, an increase in $c^i$ will contribute to relax (tighten) the self-selection constraint, ceteris paribus. Note also that since the government in country $i$ behaves as a Nash competitor vis-à-vis all other countries, it treats the average consumption in all other countries (as summarized by the elements of the vector $c^{-i}$ in equation (30)) as exogenous. As a consequence, the within-country positionality effect does not depend directly on any measure of foreign positionality.

By combining equation (45) and Lemma 2, we can now derive:

**Proposition 5.** In Nash equilibrium, the optimal marginal income tax rate facing individuals of ability type $j$ in country $i$ can be expressed in terms of positionality degrees such that (for $j=1,2$)

$$T^i(w_j^i) = \sigma_j^i + \bar{\alpha}^i (1 - \sigma_j^i) - (1 - \bar{\alpha}^i)(1 - \sigma_j^i) \frac{\alpha^{id}}{1 - \bar{\alpha}^{id}}.$$  

**Proof:** Substituting the within-country positionality effect from Lemma 2 into equation (45), we obtain

$$T^i(w_j^i) = \sigma_j^i + \frac{MRS^{i,j}}{w_j^i} \left[ \frac{\alpha^j}{1 - \alpha^j} - \frac{\lambda^i \hat{u}^i_{z^i \sigma}}{\gamma' \left[ n^1_i + n^2_i \right]} \frac{\hat{\alpha}_z^i - \alpha^i}{1 - \alpha^i} \right],$$  

(47)

Using $MRS^{i,j}/w_j^i = 1 - T^i(w_j^i)$ in equation (47) and then solving for $T^i(w_j^i)$ gives

$$T^i(w_j^i) = \frac{1 - \bar{\alpha}^i}{1 - \alpha^{id}} \sigma_j^i + \frac{\bar{\alpha}^i - \alpha^{id}}{1 - \alpha^{id}},$$

(48)

which can be re-arranged to give the tax formula in Proposition 5. QED
The first term on the right-hand side of the marginal income tax formula in Proposition 5 is again given by the expression for the optimal marginal income tax rate in the conventional case without any positional concerns. The second term reflects the incentive facing the domestic government to correct for positional externalities and depends on the average degree of within-country positionality. If $\sigma_i^j > 0$ (as in the standard optimal income tax model where the consumers share a common utility function), this corrective component is smaller for the low-ability type than for the high-ability type. The third term reflects an incentive for the government to relax the self-selection constraint by exploiting that the mimicker and the low-ability type may differ in terms of positional concerns. Consider first the case where $\hat{\alpha}_{id}^i > \alpha_i^i$, such that $\tilde{\alpha}_{id} > 0$. This means that an increase in average domestic consumption (with the average consumption in other countries held constant) will cause a larger utility loss, in monetary terms, for the mimicker than for the low-ability type. Hence, an increase in $\tilde{c}_i$ makes it less attractive to become a mimicker, such that the self-selection constraint is relaxed. This is clearly beneficial from a social point of view, and implies a corresponding reason for reducing the marginal income tax rate. The intuition for the opposite case where $\hat{\alpha}_{id}^i < 0$ is analogous. Note also that the result in Proposition 5 resembles the tax policy implications of positional concerns derived for a one-country economy by Aronsson and Johansson-Stenman (2008). This is so because all positional externalities caused to other countries are ignored by the Nash-competing national governments.

5.4 Optimal tax policy in the Stackelberg-competition case

Let us again assume, as we did in sub-section 3.2, that country $i$ is a Stackelberg leader in relation to country $k$, which is a Stackelberg follower, and that country $i$ plays the Nash game with all other countries. As in subsection 3.2 above, we focus on the policy incentives facing the Stackelberg leader; the policy incentives facing the follower are analogous to those facing the governments in the Nash game analyzed in subsection 5.3. Hence, the government in country $i$ will treat the average consumption in country $k$ as a function of the average consumption in country $i$. Let us rewrite the Lagrangean in (38) to make this explicit:
where the average consumption in countries other than \( i \) and \( k \) have been suppressed for notational convenience. The first order conditions with respect to the hours of work for both productivity types remain the same as in the Nash equilibrium case, i.e., as (39) and (41), while the first order conditions for \( c_1^i \) and \( c_2^i \) are given as

\[
\begin{align*}
& u_{1,c}^i - \lambda^i \hat{u}_{2,c}^i - \gamma^i n_i^i + \frac{dL_i^i}{dc^i} n_i^i = 0, \quad (50) \\
& (\delta^i + \lambda^i) u_{2,c}^i - \gamma^i n_{2,i}^i + \frac{dL_i^i}{dc^i} n_{2,i}^i = 0, \quad (51)
\end{align*}
\]

where

\[
\frac{dL_i^i}{dc^i} = u_{1,c}^i + u_{1,c}^i \frac{\partial c^k}{\partial c^i} + \left[ \delta^i + \lambda^i \right] \left[ u_{2,c}^i + u_{2,c}^i \frac{\partial c^k}{\partial c^i} \right] - \lambda^i \left[ \hat{u}_{2,c}^i + \hat{u}_{2,c}^i \frac{\partial c^k}{\partial c^i} \right]. \quad (52)
\]

In a way similar to equation (43), we can also interpret equation (52) as measuring the within-country positionality effect. The difference is that country \( i \) is here assumed to be first mover and will, therefore, also consider the indirect relationship between \( c^i \) and \( c^k \), which provides an additional channel through which the government may increase the domestic welfare.

Since the social first order conditions are analogous to those in the Nash equilibrium case, with the only exception being that \( \partial L_i^i / \partial c^i \) is replaced by \( dL_i^i / dc^i \) in (50) and (51) compared with (40) and (42), it follows that the optimal marginal income tax rates can be written as

\[
T^i \left( w_j^j \right) = \sigma^i_j - \frac{MRS^i_j \cdot \frac{dL}{dc^i}}{\gamma^i_j w_j^j N_i^i \frac{dL}{dc^i}}. \quad (53)
\]
Equation (53) takes the same form as equation (45) above, with the exception that the within-country positionality effect is now different. To be able to rewrite equation (53) in terms of degrees of positionality, let us first introduce the following measure of difference in the partial degree of foreign positionality between the mimicker and the low-ability type:

\[
\beta_{id}^{ikd} = \frac{\lambda_i \hat{d}_{i \cdot c}^k}{\gamma_i d_{i \cdot c}^k} \left[ \hat{\beta}_{i \cdot c}^k - \beta_i^{ik} \right].
\] (54)

As such, \( \beta_{id}^{ikd} > 0 \) \((< 0)\) if the mimicker in country \( i \) is more (less) positional than the low-ability type relative to the average consumption in country \( k \). The following result will be used to characterize the optimal tax policy of the Stackelberg leader:

**Lemma 2.** For country \( i \), where the government is a Stackelberg leader vis-à-vis country \( k \), the within-country positionality effect can be written as

\[
\frac{dL}{d\bar{c}^i} = -\gamma_i N_i \left( \bar{\alpha}^i + \bar{\beta}^{ikd} \frac{\partial \bar{c}^k}{\partial \bar{c}^i} - \left( \alpha^{id} + \beta^{ikd} \frac{\partial \bar{c}^k}{\partial \bar{c}^i} \right) \right) = -\gamma_i N_i \frac{\bar{\psi}^{ik} - \psi^{ikd}}{1 - \psi^{ikd}},
\]

where \( \bar{\psi}^{ik} = \bar{\alpha}^i + \bar{\beta}^{ikd} \frac{\partial \bar{c}^k}{\partial \bar{c}^i} \) and \( \psi^{ikd} = \alpha^{id} + \beta^{ikd} \frac{\partial \bar{c}^k}{\partial \bar{c}^i} \).

Proof: See Appendix.

We can think of \( \bar{\psi}^{ik} \) and \( \psi^{ikd} \) as measuring the “average degree of effective positionality” and the “difference in the degree of effective positionality between the mimicker and the low-ability type,” respectively, from the point of view of the government in country \( i \), which is acting as a Stackelberg leader vis-à-vis country \( k \). In particular, note that these measures also reflect the partial degrees of foreign positionality (in addition to the partial degrees of domestic positionality), since the government in country \( i \) may explore the relationship
between $c^i$ and $c^k$ for purposes of externality correction and redistribution. The marginal income tax rates can be characterized as follows.\(^9\)

**Proposition 6.** (i) The optimal second-best marginal income tax rate for individuals of ability type $j$ in country $i$, where the government is a Stackelberg leader vis-à-vis country $k$, is given by (for $j=1,2$)

$$T^i(w^i, l^i) = \sigma_j' + \left[1 - \sigma_j'\right] \psi_j^i - \frac{\psi_j^id}{1 - \psi_j^i} \left[1 - \sigma_j'\right].$$

(ii) Given the levels of $\sigma_j'$, $\alpha^i$, $\alpha^{id}$, $\beta^i$, and $\beta^{id}$, and if $\beta^i > \beta^{id}$, the optimal marginal income tax rates chosen by the Stackelberg leader are larger (smaller) than the Nash equilibrium rates if the consumption in country $k$ is characterized by a cross-country keeping-up-with-the-Joneses (staying-away-from-the-Joneses) property with respect to the consumption in country $i$ such that

$$\frac{\partial c^k}{\partial c^i} > 0 \text{ (}<0).$$

**Proof:** The first part follows by analogy to the proof of Proposition 5 by replacing $\alpha^i$ and $\alpha^{id}$ with $\psi_j^i$ and $\psi_j^{id}$, respectively. To prove the second part, multiply and divide the second term on the right-hand side of the tax formula in the proposition by $1 - \psi_j^{id}$ and rearrange to derive

$$T^i(w^i, l^i) = \sigma_j' + \left(1 - \sigma_j'\right) (\psi_j^i - \psi_j^{id}) = \sigma_j' + \left(1 - \sigma_j'\right) (\alpha^i - \alpha^{id}) + \frac{(\beta^i - \beta^{id}) \partial c^k}{\partial c^i}.$$ (55)

With $\beta^i > \beta^{id}$, and if $\partial c^k / \partial c^i > 0 \text{ (}<0)$, the right-hand side of equation (55) is larger (smaller) than in the Nash case where all $\beta$-terms are absent. QED

---

\(^9\) Lemma 1 is not necessarily applicable here, since the two-type model with asymmetric information has a much more complicated structure than the representative-agent model in Section 3.
The marginal income tax formula implemented by the Stackelberg leader takes the same general form as that implemented by a follower. The difference is that the Stackelberg leader behaves as if $\psi$ is the appropriate measure of positionality, whereas the follower behaves as in the Nash game, where $\alpha$ is the appropriate measure of positionality. Therefore, if the consumption of the Stackelberg follower is characterized by the keeping-up-with-the-Joneses property discussed above, the policy decided by country $i$, i.e., the Stackelberg leader, may be closer to a globally optimal policy than that implemented by the follower. We will return to this comparison below. Note also that the interpretation of Proposition 6 is close – yet not equivalent – to the interpretation of Proposition 5, due to that the magnitudes (and possibly also the signs) of $\bar{\psi}^{ik}$ and $\psi^{id}$ depend on the reaction function $c^k(c^i)$. If $\partial c^k / \partial c^i > 0$, which appears to be a plausible assumption, the incentives to correct for positional externalities are stronger for a government acting as a Stackelberg leader than for a Nash competitor, whereas these incentives instead are weaker if $\partial c^k / \partial c^i < 0$ (in fact, if $\beta^{ik}$ is sufficiently large, we cannot rule out the possibility that $\bar{\psi}^{ik} < 0$, although this outcome appears very unlikely). Similarly, the interpretation of the variable $\psi^{id}$ is more complex than the interpretation of $\alpha^{il}$, as $\psi^{id}$ reflects differences in the degree of positionality between the mimicker and the low-ability type in two dimensions. The practical importance of Propositions 5 and 6 is, nevertheless, clear: the two propositions show exactly what information the national policy maker (who acts as a Nash competitor and Stackelberg leader, respectively) needs in order to implement the desired resource allocation through tax policy in a decentralized setting.

5.5 The scope for a Pareto-improving tax reform

We showed in subsection 4.1 that each government in the Nash equilibrium and Stackelberg equilibrium, respectively, is willing to pay a positive amount to other countries for them to increase their income taxes. Hence, there exists a Pareto-improving tax reform. The situation in the more general second-best case is similar. That there are two types of individuals in each country does not matter per se, since it would be beneficial for other countries if the marginal income tax rates were increased for both types. Yet, what is crucial is that welfare in one
country is affected negatively by increased consumption in other countries, i.e., the mimicker must not be so much more positional than the low-ability type that the welfare loss due to the direct positional externality is fully offset by a welfare benefit of increased reference consumption through the self-selection constraint. To be more specific, we have the following result:

**Proposition 7.** Based on either the Nash equilibrium or the Stackelberg game equilibrium, there is scope for Pareto-improving tax reforms through small increases in the marginal income tax rates, provided that the direct positional externality dominates the self-selection effect in the sense that \( \frac{\partial L_i}{\partial c^k} < 0 \) for \( i = 1, 2 \) and \( k \neq i \).

In Proposition 7, \( L_i \) is given by equation (38) in the Nash-equilibrium case and equation (49) in the Stackelberg-equilibrium case. As such, for a coordinated tax increase to be welfare improving, the positionality effect referred to in the proposition means that the average degree of foreign positionality must dominate the effect of the difference in this degree of positionality between the mimicker and the low-ability type. This is analogous to the results derived above.

### 5.6 Efficient international negotiations on the tax rates

Consider, as in subsection 4.2, the case where the countries can negotiate with each other about the tax policy without transaction costs. For convenience, suppose also that we only have two countries, \( i \) and \( k \). Country \( i \) would then be willing to buy a further tax increase in country \( k \) as long as the welfare cost for country \( i \) of paying country \( k \) is smaller than the welfare gain of the reduced consumption in country \( k \) due to the tax increase. This means that country \( k \) will take into account the welfare effects caused to country \( i \), and vice versa.

Consider the Lagrangean corresponding to the maximization of the utility facing the low-ability type in country \( i \), while holding constant the utility of the high-ability type in country \( i \) and the utility facing both ability types in country \( k \) subject to a self-selection constraint in each country and an overall resource constraint:
\begin{equation}
L' = u_i' + \delta[u_i' - \tilde{u}_i'] + \lambda[u_i' - \tilde{u}_i'] + \omega[u_i' - \tilde{u}_i'] + \mu[u_i' - \tilde{u}_i'] + \nu[u_i' - \tilde{u}_i'] + \gamma(u_i'(w_i'l_i' - c_i') + n_i'(w_i'\bar{l}_i' - c_i')) + n_i'(w_i'\bar{l}_i' - c_i') + n_i'(w_i'\bar{l}_i' - c_i') \right),
\end{equation}

where \( \tilde{u}_i' \) for \( j=1,2 \) denote the minimum utility levels for residents in country \( k \). The first order conditions with respect to leisure and consumption for the individuals in country \( i \) take the same general form as equations (39)-(42), implying that equation (45) holds here as well. Yet, the relevant positionality effect is now different and given by

\begin{equation}
\frac{\partial L'}{\partial c_i} = u_i' + (\delta + \lambda)u_{2,i,c} - \lambda \tilde{u}_{2,i,c} + (\omega + \nu)u_{2,i,c} - \omega \tilde{u}_{2,i,c} + \mu u_{1,i,c},
\end{equation}

since country \( i \) will, in this case, recognize the welfare effects it causes on country \( k \). The corresponding social first order conditions with respect to leisure and consumption for country \( k \) are given by

\begin{equation}
-\mu u_{1,c} + \omega \tilde{u}_{1,c} \phi + \gamma n_i^k w_i^k = 0,
\end{equation}

\begin{equation}
\mu u_{1,c} - \omega \tilde{u}_{2,c} - \gamma n_i^k + \frac{\partial L'}{\partial c_k} n_i^k = 0,
\end{equation}

\begin{equation}
-(\nu + \omega)u_{2,c} + \gamma n_i^k w_i^k = 0,
\end{equation}

\begin{equation}
(\nu + \omega)u_{2,c} - \gamma n_i^k + \frac{\partial L'}{\partial c_k} n_i^k = 0,
\end{equation}

where \( N_i^k = n_i^k + n_i^k \) and

\begin{equation}
\frac{\partial L'}{\partial c_k} = u_{1,c} + (\delta' + \lambda')u_{1,c} - \lambda' \tilde{u}_{1,c} + (\omega + \nu)u_{1,c} - \omega \tilde{u}_{1,c} + \mu u_{1,c}.
\end{equation}

Therefore, since the positionality effect for each country is now different compared with those associated with the non-cooperative regimes analyzed above, the optimal marginal income tax rates as expressed in terms of relative consumption comparisons will of course also be different.
Without loss of generality, we simplify the analysis by focusing on a symmetric equilibrium where the two countries are identical in the sense of the following assumption:

**A1.** \( \bar{\alpha}^i = \bar{\alpha}^k = \bar{\alpha}, \ \bar{\beta}^i = \bar{\beta}^k = \bar{\beta}, \ \alpha^{id} = \alpha^{kd} = \alpha^d, \ \beta^{id} = \beta^{kd} = \beta^d, \) and \( N^i = N^k. \)

Note that these assumptions do not mean that the two countries are identical also in other respects; they may still differ in terms of wage and population distributions and preferences for leisure and private consumption. We can then derive the following result:

**Lemma 4.** For a country \( i \) that can negotiate without transaction costs with another country \( k, \) and under assumption A1, the positionality effect is given by

\[
\frac{\partial L^i}{\partial \bar{c}^i} = -\gamma N \frac{\bar{\beta} - \rho^d}{1 - \bar{\beta}},
\]

where \( \bar{\rho} = \bar{\alpha} + \bar{\beta} \) and \( \rho^d = \alpha^d + \beta^d. \)

Proof: See Appendix.

Note that the positionality effect in Lemma 4 reflects the welfare effects of an increase in \( \bar{c}^i \) facing both countries, i.e., when country \( i \) can negotiate over tax policy with country \( k, \) it will also recognize the welfare cost of its policy for country \( k. \) This is also the reason why the positionality effect is governed by the average degree of total positionality, \( \bar{\rho}, \) and the difference in the degree of total positionality between the mimicker and the low-ability type, \( \rho^d, \) instead of the corresponding measures (\( \bar{\alpha} \) and \( \alpha^d \)) as in the non-cooperative Nash equilibrium (see Lemma 2). With this (quite substantial) modification, the intuition behind the formula in Lemma 4 is the same as that behind the corresponding expression in Lemma 2. We can now characterize the optimal marginal income tax rates.

**Proposition 8.** For a country \( i \) that can negotiate without transaction costs with another country \( k, \) and under assumption A1, the optimal marginal income tax rate implemented for ability type \( j \) in country \( i \) can be written as

\[
\frac{\partial L^i}{\partial \bar{c}^i} = -\gamma N \frac{\bar{\beta} - \rho^d}{1 - \bar{\beta}},
\]

This allows us to avoid unnecessarily complex expressions due to differences in population sizes or degrees of positionality across countries. It does not affect the basic intuition behind the results.
\[ T^i_i'(w_i^i l_i^i) = \sigma^i_j + \bar{\rho}(1 - \sigma^i_j) - (1 - \bar{\rho})(1 - \sigma^i_j) \frac{\rho^d}{1 - \rho^d}. \]

**Proof:** By using the first order conditions, we can derive

\[ T^i_i'(w_i^i l_i^i) = \sigma^i_j - \frac{\text{MRS}^i_{j,c}}{\gamma^j w_j^i N^j} \frac{\partial L_j}{\partial c^i}, \]

which takes the same general form as equation (45). By using Lemma 4, the proof of Proposition 8 then follows by analogy to the proofs of Propositions 5 and 6. QED

Proposition 8 generalizes the Pareto-efficient optimal tax structure derived by Aronsson and Johansson-Stenman (2008) for a one-country economy to a multi-country economy with international positional externalities. As such, an increase in the average degree of total positionality (which is here given by the average degree of domestic positionality in country \( i \) plus the average degree of foreign positionality in country \( k \)) contributes to increase the marginal income tax rates implemented for the residents of country \( i \). Furthermore, if the mimicker is more (less) positional than the low-ability type, here measured in terms of the total degree of positionality, there is an incentive to relax the self-selection constraint through a lower (higher) marginal income tax rate for each ability type.

Note also that in the symmetric case analyzed here we can obtain a straightforward relationship between the socially efficient marginal income tax rates and the rates implemented in the non-cooperative regimes (addressed in Propositions 5 and 6). Let \( T^i_n'(w_i^i l_i^i) \), \( T^i_s'(w_i^i l_i^i) \), and \( T^i_c'(w_i^i l_i^i) \) denote the marginal income tax rate implemented for ability type \( j \) when the government in country \( i \) acts as a Nash competitor, Stackelberg leader, and in accordance with the cooperative game set out here, respectively. The following result is an immediate consequence of Propositions 5, 6, and 8:

**Corollary 2.** Under assumption A1, and given the levels of \( \sigma^i_j, \bar{\alpha}, \bar{\beta}, \alpha^d, \) and \( \beta^d \), we have

\[ T^i_n'(w_i^i l_i^i) < T^i_s'(w_i^i l_i^i) < T^i_c'(w_i^i l_i^i) \text{ for } j = 1, 2, \]

if (i) \( \bar{\beta} > \beta^d \) and (ii) \( \bar{c}c^k / \bar{c}c^i \in (0,1) \).
Clearly, $\bar{\beta} > \beta^d$ means that the marginal income tax rate implemented in the non-cooperative Nash equilibrium falls short of the socially efficient rate, i.e., $T_n^i'(w_i^j) < T_c^i'(w_i^j)$; the second condition then means that the marginal income tax rate decided by the Stackelberg leader falls between these two extremes.

6. Discussion

It is intuitively reasonable and consistent with recent empirical evidence that the increased globalization in recent decades has influenced the social comparisons inherent in individual well-being, such that consumption comparisons with people in other countries have increased in importance. In the present paper, we take this evidence seriously and analyze optimal income taxation under relative consumption concerns in a multi-country framework, where each individual in each country compares his/her own consumption both with that of other domestic residents and with that of people in other countries. Furthermore, our framework allows for differences in relative consumption concerns, depending on whether it refers to within-country or between-country comparisons, as well as for differences in preferences for relative consumption between individuals and across countries. We distinguish between the tax policy implicit in non-cooperative regimes where the national governments act as Nash competitors to one another or engage in Stackelberg competition and the tax policy implicit in a cooperative regime where the countries can negotiate over tax policy.

We start by examining a simple model where each country is modeled as a representative-agent economy, which means that we abstract from distributional concerns within each country. The results show that if the national governments behave as Nash competitors to one another, the resulting tax policy only internalizes the externalities that are due to within-country comparisons, whereas the tax policy chosen by the leader country in a Stackelberg game reflects between-country comparisons as well. Furthermore, if the residents of the Stackelberg follower country are characterized by cross-country keeping-up-with-the-Joneses preferences, then the marginal income tax implemented by the leader country in the Stackelberg game will exceed that implemented by the follower, as well as exceed the marginal income tax rates implicit in the Nash equilibrium. We also derive the globally
efficient tax structure in the cooperative regime and show that cooperation leads to higher marginal income tax rates than implicit in the non-cooperative Nash equilibrium, and under certain conditions also higher marginal income tax rates than implemented by the leader country in the Stackelberg game.

In the second part of the paper, we extend the analysis by allowing the consumers within each country to differ in ability (productivity) and assume that such ability is private information. This is well motivated because earlier research based on second-best analysis of one-country model economies shows that the policy implications of positional concerns may differ substantially from those that follow from representative-agent models. Once again, we compare the three regimes mentioned above. In general, these comparisons give ambiguous results, as externality correction may either tighten or relax the self-selection constraint. However, under a relatively mild additional assumption, namely that the difference in the degree of foreign positionality between the mimicker and the low-ability type is not too large, the qualitative results for the representative-agent framework referred to above will continue to hold also in the second-best setting.

A possible extension of our analysis would be to introduce a time dimension so that the importance of between-country comparisons may vary over time. This would allow us to capture how the policy responses are modified due to changes in preferences, as well as analyze the role of optimal capital income taxation in this context (and not just labor income taxation as we did above). Whether such extensions will be undertaken or not, we believe that the insights from the present paper, as well as from further theoretical and empirical research on related issues, will grow more important over time. This is because we anticipate that the globalization process will continue and that cross-country social comparisons will correspondingly increase further in importance.

**Appendix**

*Proof of Lemma 1*

The social first order condition in country \( k \) can be written as
By total differentiation, while recognizing that $c^k = \overline{c}^k$, we have

$$
\left[ u^k + u^k_{i'} \right] w^k - u^k_{i'} = 0
$$

(A1)

Use the resource constraint in country $k$,

$$
w^k (\Omega - z^k) - c^k = 0,
$$

to derive

$$
dz^{k^i} = - \frac{1}{w^k}.
$$

(A2)

Substituting equation (A2) into equation (A1) and solving for $dc^k / dc^i$ gives

$$
dc^k
\frac{dc^k}{dc^i} = \frac{(u^k_{i'} + u^k_{i'} c^k)}{\Phi / w^k}
$$

(A3)

where

$$
\Phi = - (u^k_{i'} + u^k_{i'} c^k) (w^k)^2 - (u^k_{i'} + u^k_{i'} c^k) (w^k)^2 + 2 (u^k_{i'} + u^k_{i'} c^k) w^k - u^k_{i'} > 0
$$

from the second order conditions. Therefore, the right-hand side of equation (A3) is positive if

$$
(u^k_{i'} + u^k_{i'} c^k) w^k - u^k_{i'} > 0
$$

and vice versa.

Now, differentiate $SMRS^k_{i'} = (u^k_{i'} + u^k_{i'} c^k) / u^k_{i'}$ with respect to $\overline{c}^i$,
\[
\frac{\partial \text{SMRS}^k_{zt}}{\partial \tilde{c}^i} = \left( \frac{u_{c', k}^i + u_{c'v, k}^i}{u_{c'}^k + u_{c'v, k}^i} \right) w^k - u_{c'v, k}^i,
\]

where we have used \( w^k = u_{c'}^i / (u_{c'}^k + u_{c'v, k}^i) \). We can then rewrite equation (A3) such that

\[
\frac{dc^k}{d\tilde{c}^i} = \frac{u_{c'}^k}{\Phi} \frac{\partial \text{SMRS}^k_{zt}}{\partial \tilde{c}^i}.
\] (A4)

Therefore, \( dc^k / d\tilde{c}^i > 0 \) \((-0\) if \( \partial \text{SMRS}^k_{zt} / \partial \tilde{c}^i > 0 \) \((-0\).

**Derivation of equation (45)**

Combine equations (39) and (40), which gives

\[
\text{MRS}_{1,\infty}^{i} \left[ \lambda_i^1 \dot{u}_{2,c}^i - \frac{\partial L_i^1}{\partial c^i} \frac{n_1^i}{n_1^i + n_2^i} \right] = \frac{w_1^i}{w_2^i} \lambda_i^1 \dot{u}_{2,c}^i + \gamma' n_1^i \left[ w_1^i - \text{MRS}_{1,\infty}^{i} \right].
\] (A5)

From equation (32) we have \( w_1^i - \text{MRS}_{1,\infty}^{i} = T_i^1 (w_1^i) \). Substituting into equation (A5) and solving for \( T_i^1 (w_1^i) \) gives equation (45) for the low-ability type. The result for the high-ability type is obtained equivalently by instead combining equations (32), (41), and (42).

**Proof of Lemma 2**

We start by re-expressing the terms of equation (43). From the utility function (30) follows that

\[
u_{j,c}^i = v_{j,r,c}^i = v_{j,c}^i r_{j,c}^i = -\alpha_i^j u_{j,c}^i,
\] (A6)

where we have used (33) and \( r_{j,c}^i / r_{j,c}^i = -1 \). Substituting equation (A6) into equation (43) gives
\[ \frac{\partial L^i}{\partial c^i} = -\alpha_i u^i_{1,c} - (\delta^i + \lambda^i) \alpha^i_{2,c} u^i_{2,c} + \lambda^i \hat{\alpha}^i_{2,c} u^i_{2,c}. \] (A7)

Now, equations (40) and (42) can be rewritten as

\[ u^i_{1,c} = \lambda^i \hat{u}^i_{2,c} + \gamma^i n^i_1 - \frac{\partial L^i}{\partial c^i n^i_1 + n^i_2}, \]

\[ (\delta^i + \lambda^i) u^i_{2,c} = \gamma^i n^i_2 - \frac{\partial L^i}{\partial c^i n^i_1 + n^i_2}, \]

which if substituted into equation (A7) imply (after collecting terms and using equation (36a))

\[ \frac{\partial L^i}{\partial c^i} = -\gamma^i \left[ n^i_1 + n^i_2 \right] \left[ \frac{\alpha^i}{1 - \alpha^i} + \lambda^i \hat{u}^i_{2,c} \right] \frac{\alpha^i_{2,c} - \alpha^i}{1 - \alpha^i}. \] (A8)

QED

**Proof of Lemma 3**

By using equation (30), we can derive

\[ u^i_{j,c^k} = v^i_{j,s} s^i_{j,c^k} = v^i_{j,s} s^i_{j,c^k} \frac{s^i_{j,c^k}}{s^i_{j,c}} = -\beta^i_{j,k} u^i_{j,c}, \] (A9)

where we have used (36b) and \[ s^i_{j,c^k} \frac{s^i_{j,c}}{s^i_{j,c}} = -1. \] Substituting equation (A9) into equation (54) gives

\[ \frac{dL^i}{dc^i} = -u^i_{j,c} \left[ \alpha^i_{j,k} + \beta^i_{j,k} \frac{\partial c^k}{\partial c^i} \right] - u^i_{2,c} \left[ \delta^i + \lambda^i \right] \left[ \alpha^i_{2,k} + \beta^i_{2,k} \frac{\partial c^k}{\partial c^i} \right] \]

\[ + \lambda^i \hat{\alpha}_{2,c} \left[ \alpha^i_{2,k} + \beta^i_{2,k} \frac{\partial c^k}{\partial c^i} \right]. \] (A10)

Rewriting equations (52) and (53) such that
\[ u'_{i,c} = \lambda' \hat{u}'_{2,c} + \gamma' n'_{i} - \frac{dL'}{dc'_{i}} n'_{i} \]

\[ (\delta' + \lambda')u'_{2,c} = \gamma' n'_{2} - \frac{dL}{dc'_{i}} n'_{2} \]

and then substituting into equation (A10) gives (after collecting terms)

\[ \frac{dL'}{dc'_{i}} = -\gamma' \left[ n'_{i} + n'_{2} \right] \frac{\alpha' + \beta_{2} \frac{\partial}{\partial c'_{i}} \hat{u}^{k} - \alpha' + \beta_{1} \frac{\partial}{\partial c'_{i}} \hat{u}^{k}}{1 - \alpha' - \beta_{1} \frac{\partial}{\partial c'_{i}}} + \lambda' \hat{u}'_{2,c} \]

\[ \frac{dL}{dc'_{i}} = -\gamma' \left[ n'_{2} \right] \frac{\alpha' + \beta_{2} \frac{\partial}{\partial c'_{i}} \hat{u}^{k} - \alpha' + \beta_{1} \frac{\partial}{\partial c'_{i}} \hat{u}^{k}}{1 - \alpha' - \beta_{1} \frac{\partial}{\partial c'_{i}}} + \lambda' \hat{u}'_{2,c} \]

which can be rearranged to give the result in Lemma 3. QED

**Proof of Lemma 4**

Substituting the first three terms in equation (57) as was done in the derivation of equation (A9), we get

\[ \frac{\partial L'}{\partial c'} = -\alpha' \left[ \lambda' \hat{u}'_{2,c} + \gamma n'_{i} - \frac{\partial L'}{\partial c'} n'_{i} \right] - \alpha' \left[ \gamma n'_{2} - \frac{\partial L'}{\partial c'} n'_{2} \right] + \lambda' \hat{u}'_{2,c} \]

\[ - \beta_{k} \left[ \omega \hat{u}'_{2,c} + \gamma n'_{i} - \frac{\partial L'}{\partial c'} n'_{i} \right] - \beta_{k} \left[ \gamma n'_{2} - \frac{\partial L'}{\partial c'} n'_{2} \right] + \omega \hat{u}'_{2,c} \]

Using the positionality measures defined in equations (36a)-(36c) and rearranging gives

\[ \frac{\partial L'}{\partial c'} (1 - \alpha') = -\gamma N' \alpha' + \lambda' \hat{u}'_{2,c} (\hat{u}'_{2} - \alpha'_{i}) - \gamma N' \beta_{k} + \omega \hat{u}'_{2,c} (\hat{u}'_{2} - \beta_{k}) \]

\[ + \beta_{k} \frac{\partial L'}{\partial c'} \]

We can derive the analogous equation for country k:
\begin{equation}
\frac{\partial L'}{\partial c^i} (1 - \tilde{\alpha}^k) = -\gamma N' \tilde{\beta}^i + \lambda u^i_{2,i} (\tilde{\beta}^i - \beta_i) - \gamma N'^k \tilde{\alpha}^k + \omega u^k_{2,i} (\tilde{\alpha}^k - \alpha_i) + \beta^i \frac{\partial L'}{\partial c^i}.
\end{equation}

We then define \( \alpha^{id} \) and \( \beta^{id} \) according to equation (46) and (54), respectively, and apply analogous definitions for country \( k \), i.e., to measure \( \alpha^{kd} \) and \( \beta^{kd} \). By substituting equation (A14) into equation (A13) and then using \( \tilde{\alpha}^i = \tilde{\alpha}^k = \tilde{\alpha} \), \( \tilde{\beta}^i = \tilde{\beta}^k = \tilde{\beta} \), \( \alpha^{id} = \alpha^{kd} = \alpha^d \), \( \beta^{id} = \beta^{kd} = \beta^d \), and \( N'^i = N'^k \), we obtain the positionality effect in Lemma 4. QED

References


