Optimal Redistributive Income Taxation and Efficiency Wages*

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October 2017

*Aronsson would like to thank the Swedish Research Council (ref 2016-02371) for generous research grants.

Abstract

This paper integrates efficiency wage setting in the theory of optimal redistributive income taxation. In doing so, we use a model with two skill-types, where efficiency wage setting characterizes the labor market faced by the low-skilled, whereas the high-skilled face a conventional, competitive labor market. There are two types of jobs in this economy; a low-demanding job which can be carried out by everybody, and a high-demanding job which can only be carried out by the high-skilled, meaning that a potential mimicker may either adopt a conventional income-replication strategy or a job-replication strategy. In this framework, we show that the marginal income tax implemented for the high-skilled is negative under plausible assumptions. The marginal income tax facing the low-skilled can be either positive or negative in general, even if employment-related motives for policy intervention typically contribute to an increase in this marginal tax. An increase in the unemployment benefit contributes to relax the binding self-selection constraint (irrespective of the strategy adopted by a potential mimicker), which makes this instrument particularly useful from the perspective of redistribution.

JEL classification: H21, H42.

Keywords: Nonlinear income taxation, unemployment benefits, efficiency wages, redistribution.

1 Introduction

The modern theory of optimal redistributive taxation, as developed from the seminal contribution of Mirrlees (1971), is largely based on model economies where the labor market is competitive. Although analytically convenient, such a description of the labor market is clearly at odds with most real world developed economies, where unemployment has been an important social problem for a long time. Albeit small by comparison, a literature on optimal redistributive taxation under unemployment has gradually evolved during the latest decades, in which the incentives underlying an optimal tax policy partly differ from those following under perfect competition. The major mechanisms generating unemployment in these studies are trade-union wage formation (e.g., Aronsson and Sjögren, 2003, 2004; Aronsson *et al.*, 2009; Hummel and Jacobs, 2016), minimum wages (Marceau and Boadway, 1994), and search frictions (Lehmann *et al.*, 2015). Although fundamentally different, a common denominator is a policy incentive to reduce the level of unemployment, which is likely to result in higher marginal income tax rates than under perfect competition.

However, to our knowledge, there are no earlier studies on optimal redistributive taxation in economies where the labor market is characterized by efficiency wages. The overall purpose of the present paper is to fill this gap by integrating efficiency wages in the self-selection approach to optimal taxation. Such an extension is interesting for several reasons. First, efficiency wage theory has played an important role in labor economics for a long time by explaining involuntary unemployment as well as wage differentials across workers and sectors.[1] Second, it has been used in related areas of public economics as a framework for studying relationships between tax policy, wages, and unemployment in representative-agent models (e.g., Chang, 1995; Pisauro, 1995). Third, and compared to the related literature on trade-union behavior, efficiency wage theory does not rely on any (arbitrary) assumption of objective function for trade-

[1] See Katz (1986) for an overview of efficiency wage models. See also Shapiro and Stiglitz (1984).

unions,[2] since the wage rate is decided on unilaterally by firms realizing that increased wages leads to higher productivity.

Our study is based on a two-type model,[3] where efficiency wage setting characterizes the labor market faced by the low-skilled type, whereas the high-skilled type faces a conventional, competitive labor market. The rationale for this assumption is that the unemployment is typically higher and more persistent among the low-skilled than among the high-skilled, suggesting that mechanisms generating equilibrium unemployment are more important to examine in the context of agents with relatively low productivity. The government in our model uses a nonlinear income tax and an unemployment benefit to correct for the imperfection in the labor market and redistribute income from the high-skilled to the low-skilled. We assume that two types of jobs are available in the economy; a low-demanding job which can be carried out both by the low-skilled and high-skilled individuals, and a high-demanding job which can only be carried out by the high-skilled. We also assume that effort - which is thought of as the "effort exerted per hour spent at the workplace" - is a decision-variable in the low-demanding job, meaning that individuals employed in this type of job have the option to "shirk" with an exogenous probability of detection (through imperfect monitoring).[4] As a consequence, there are two ways for a high-skilled individual to mimic the income of the low-skilled type; either by choosing a low-demanding job, or by reducing the hours of work when employed in a high-demanding job. In turn, the government must recognize both these options when solving the optimal tax and expenditure problem.

[2] See Kaufmann (2001) for an overview of models with trade-unionized labor markets.

[3] The discrete two-type version of the Mirrleesian optimal income tax problem was developed in its original form by Stern (1982) and Stiglitz (1982).

[4] Notice that imperfect monitoring in the low-demanding job generates a second source of asymmetric information, between firms and workers employed in the low-demanding job, on top of the standard asymmetric information problem between the government and the workers (regarding their skill type). In this sense our paper also relates to a recent strand in the Mirrleesian literature (see Stantcheva, 2014, Bastani *et al.*, 2015 and Cremer and Roeder, 2017) analyzing optimal tax policy in settings with two sources of asymmetric information. The difference is that, whereas in the aforementioned papers the second source of asymmetric information manifested itself in an adverse selection problem, in our model it manifests itself in a moral hazard problem.

The outline of the study is as follows. In Section 2, we present the model by going through, in turn, the decision-problem and behavior of individual consumers and firms, respectively, and the optimal tax and expenditure problem facing the government. The optimal tax and expenditure policy is characterized and discussed in Section 3, while the conclusions are summarized in Section 4. Some of the derivations and proofs are given in the Appendix.

2 The Model

The economy is populated by two skill-types. High-skilled workers are paid the before-tax wage rate w^h, and low-skilled workers are paid the before-tax wage rate w^ℓ, where $w^h > w^\ell$. The total population is normalized to one and the fraction of individuals of type j is denoted by π^j (for $j = \ell, h$).

One single output good is produced by identical firms using labor as the only input, and the production process is characterized by constant returns to scale. Since the number of firms is treated as exogenous, it will be normalized to one for notational convenience. As we mentioned above, there are two different jobs available in the economy. One is low-demanding and can be carried out by all workers (low-skilled as well as high-skilled), while the other is high-demanding and exclusive to high-skilled workers. Whereas the second job pays a competitive wage, the first job is characterized by a monitoring technology such that the employers pay a wage above the market clearing level in order to boost the effort level of the employees. This generates involuntary unemployment among the low-skilled.

2.1 Individuals and Firms

Individual preferences are represented by the strictly quasi-concave utility function

$$U = v(c, L, e), \tag{1}$$

where c denotes consumption, L denotes the hours spent at the workplace, and $e \in [0, 1]$ denotes the effort the individual exerts at the workplace. Individuals derive utility from consumption and disutility from work hours and effort, respectively, i.e., $\partial v / \partial c > 0$, $\partial v / \partial L < 0$, and $\partial v / \partial e < 0$. To simplify the interpretation of the results, we also add the (quite realistic) assumption that $\partial^2 v / \partial e \partial L < 0$, i.e., the marginal utility of leisure $(-\partial v / \partial L)$ increases with the effort exerted when working.

For a worker employed in the high-demanding job, where there is no monitoring problem and competitive wages are paid, we assume that the effort exerted by workers is equal to one, which can be interpreted to mean that workers in the high-demanding job never shirk.[5] However, for workers employed in the low-demanding job, effort is a choice variable and affects the probability of being fired. The probability of being monitored in the low-demanding job is assumed to be exogenous, and a worker is fired if caught shirking.[6] Assuming that the firms monitor each worker in the low-demanding job with probability, p, and interpreting $1 - e$ as the fraction of time that an individual spends shirking while on the job, the probability of being fired is given by $(1 - e) p \equiv \varphi(e)$, where $\partial \varphi(e) / \partial e < 0$. If fired, a low-skilled worker faces two alternatives: he/she can either be hired by another firm or become unemployed.[7] The economy-wide unemployment rate is denoted by u.

In the tradition of the optimal income tax literature à la Mirrlees (1971), we assume that an individual's skill-type (as reflected in the before-tax wage rate) is private information, while the individual's income if employed, $I^j = w^j L^j$, is publicly observable. This rules out first-best type-specific lump-sum taxes but allows income to be taxed via

[5] This is clearly a simplification; albeit a consequence of the assumption of a perfectly competitive market for this type of job. A possible interpretation is that the job characteristics make shirking impossible or uninteresting for those employed in the high-demanding job.

[6] Assuming away turnover costs, this turns out to be the best strategy for a firm.

[7] With all firms in the economy being identical and under the assumption that agents are fully informed, the unitary wage of a low-skilled worker, as well as his/her choice of effort, will be the same in each firm.

a general, nonlinear tax schedule, $T\left(I^j\right)$. The other policy instrument at the disposal of the government is a transfer, b, paid to each unemployed individual. To characterize the set of (constrained) Pareto-efficient resource allocations, we will derive an optimal revelation mechanism. For our purpose, a mechanism consists of a set of type-specific before-tax incomes, I^j's, and disposable incomes, B^j's, for individuals in employment, an unemployment benefit, b, and an unemployment rate, u, which we will include as an artificial control variable in the social decision-problem. A complete solution to the optimal tax problem per se, such that I^j and B^j are determined via individual utility maximization, then requires the design of a general income tax function, $T(\cdot)$, such that $B^j = I^j - T\left(I^j\right)$. This will be described in greater detail below.

We begin by characterizing the decision-problems and behavior of individuals and firms, respectively, and then continue with the optimal tax and expenditure problem. An individual of any type j treats the hourly wage rates, the unemployment benefit, the unemployment rate, and the parameters of the tax function (including the structure of marginal taxation) as exogenous. Private consumption equals the disposable income B^j for an employed individual of skill-type $j = \ell, h$, and equals b for an unemployed individual.

Starting with an individual of the low-skilled type, note that this individual makes two decisions; (i) an optimal effort choice (which influences the likelihood of becoming unemployed) and (ii) a consumption-leisure choice conditional on being employed. For a given (I^ℓ, B^ℓ)-bundle, we can define the indirect expected utility function of a low-skilled individual as follows:

$$EV^\ell\left(B^\ell, I^\ell, u, b; w^\ell\right) = \max_{e^\ell}\left[1 - u\varphi\left(e^\ell\right)\right] v\left(B^\ell, \frac{I^\ell}{w^\ell}, e^\ell\right) + u\varphi\left(e^\ell\right) v\left(b, 0, 0\right),$$

where we have used $L^\ell = I^\ell / w^\ell$, and E denotes the expectations operator. The variable $u\varphi(e^\ell)$ is interpretable as the probability of being unemployed (the overall unemployment rate among the low-skilled times the probability of being fired when shirking).

The first order condition for this problem is given by

$$\left[1-u\varphi\left(e^{\ell}\right)\right]\frac{\partial v}{\partial e^{\ell}}=u\varphi'\left(e^{\ell}\right)\left[v\left(B^{\ell},\frac{I^{\ell}}{w^{\ell}},e^{\ell}\right)-v\left(b,0,0\right)\right]. \tag{2}$$

Therefore, when choosing effort level, this individual trades off the additional cost of effort against the gain in expected utility due to the decreased probability of becoming unemployed. Equation (2) implicitly defines the effort level, e^{ℓ}, as a function of B^{ℓ}, I^{ℓ}/w^{ℓ}, b, and u, i.e.,

$$e^{\ell}=e^{\ell}\left(B^{\ell},I^{\ell}/w^{\ell},b,u\right). \tag{3}$$

Based on equation (2), we can derive the following comparative statics properties of the effort function with respect to B^{ℓ}, I^{ℓ}, u, b, and w^{ℓ}:

$$\frac{\partial e^{\ell}}{\partial B^{\ell}} = -\frac{\left[1-u\varphi\left(e^{\ell}\right)\right]\frac{\partial^2 v}{\partial e^{\ell}\partial B^{\ell}}-u\varphi'\left(e^{\ell}\right)\frac{\partial v}{\partial B^{\ell}}}{\Phi}, \tag{4}$$

$$\frac{\partial e^{\ell}}{\partial I^{\ell}} = -\frac{\left[1-u\varphi\left(e^{\ell}\right)\right]\frac{\partial^2 v}{\partial e^{\ell}\partial L^{\ell}}-u\varphi'\left(e^{\ell}\right)\frac{\partial v}{\partial L^{\ell}}}{\Phi}\frac{1}{w^{\ell}}<0, \tag{5}$$

$$\frac{\partial e^{\ell}}{\partial u} = \frac{\varphi\left(e^{\ell}\right)\frac{\partial v}{\partial e^{\ell}}+\left[v\left(B^{\ell},\frac{I^{\ell}}{w^{\ell}},e^{\ell}\right)-v\left(b,0,0\right)\right]\varphi'\left(e^{\ell}\right)}{\Phi}>0, \tag{6}$$

$$\frac{\partial e^{\ell}}{\partial b} = -\frac{u\varphi'\left(e^{\ell}\right)\frac{\partial v(b,0,0)}{\partial b}}{\Phi}<0, \tag{7}$$

$$\frac{\partial e^{\ell}}{\partial w^{\ell}} = \frac{\left[1-u\varphi\left(e^{\ell}\right)\right]\frac{\partial^2 v}{\partial e^{\ell}\partial L^{\ell}}-u\varphi'\left(e^{\ell}\right)\frac{\partial v}{\partial L^{\ell}}}{\Phi}\frac{L^{\ell}}{w^{\ell}}>0, \tag{8}$$

where

$$\Phi\equiv\left[1-u\varphi\left(e^{\ell}\right)\right]\frac{\partial^2 v}{\partial e^{\ell}\partial e^{\ell}}-u\varphi'\left(e^{\ell}\right)\frac{\partial v}{\partial e^{\ell}}-\left[v\left(B^{\ell},\frac{I^{\ell}}{w^{\ell}},e^{\ell}\right)-v\left(b,0,0\right)\right]u\varphi''\left(e^{\ell}\right)<0.$$

These results are intuitive: an increase in the economy-wide unemployment rate increases the effort, in order to reduce the probability of ending up in unemployment, while an increase in the unemployment benefit leads to decreased effort as it reduces the income loss to the individual if becoming unemployed. The effect of an increase in the post-tax income in the employment state, B^ℓ, is ambiguous in general, since $sign\left(\partial^2 v/\partial e^\ell \partial B^\ell\right)$ is ambiguous. A sufficient (albeit not necessary) condition for $\partial e^\ell/\partial B^\ell > 0$ is that $\partial^2 v/\left(\partial e^\ell \partial B^\ell\right) \geq 0$, i.e., the marginal disutility of effort is a weakly decreasing function of the disposable income in the employment state. The latter condition is obviously satisfied in the special case where the utility function, as represented by equation (1), is separable between consumption and other goods (in which $\partial^2 v/\left(\partial e^\ell \partial B^\ell\right) = 0$).

Now, by noticing that an employed, low-skilled worker will behave as if he/she is maximizing

$$EV^\ell\left(I^\ell - T\left(I^\ell\right), I^\ell, u, b; w^\ell\right),$$

with respect to I^ℓ, we can implicitly characterize the marginal income tax rate faced by a low-skilled worker as follows based on the first order condition:

$$T'\left(I^\ell\right) = 1 + \frac{\partial EV^\ell/\partial I^\ell}{\partial EV^\ell/\partial B^\ell}. \tag{9}$$

Similarly, a high-skilled individual is maximizing $V^h\left(I^h - T\left(I^h\right), I^h; w^h\right) = v(I^h - T(I^h), I^h/w^h, e = 1)$ with respect to I^h, and we can implicitly characterize the marginal income tax rate facing a high-skilled individual as

$$T'\left(I^h\right) = 1 + \frac{\partial V^h/\partial I^h}{\partial V^h/\partial B^h} = 1 - \frac{1}{w^h}\frac{\partial v\left(B^h, I^h/w^h, e=1\right)/\partial L^h}{\partial v\left(B^h, I^h/w^h, e=1\right)/\partial B^h}. \tag{10}$$

Turning to firm behavior, let N^ℓ and N^h denote the number of workers employed in the low-demanding and high-demanding job, respectively. The (linearly homogeneous) production function is given by

$$F\left(e^\ell L^\ell N^\ell, L^h N^h\right). \tag{11}$$

In writing (11), we have used the assumption that $e^h = 1$. The production function is increasing in each argument, $F_1^{'} \equiv \partial F/\partial(e^\ell L^\ell N^\ell) > 0$ and $F_2^{'} \equiv \partial F/\partial(L^h N^h) > 0$, the marginal products are diminishing such that $F_{11}^{''} \equiv \partial^2 F/\partial(e^\ell L^\ell N^\ell)^2 < 0$ and $F_{22}^{''} \equiv \partial^2 F/\partial(L^h N^h)^2 < 0$, and the production factors are technical complements, i.e., $F_{12}^{''} = F_{21}^{''} \equiv \partial^2 F/\partial(e^\ell L^\ell N^\ell)\partial(L^h N^h) > 0$. Conditional on the choices made by the government, and by recognizing that the wage rate paid to workers in the low-demanding job affects their effort choice through equation (3), the representative firm chooses w^ℓ and N^ℓ. For a given bundle (I^ℓ,B^ℓ) intended by the government for low-skilled individuals, the first order conditions of a profit-maximizing firm imply

$$\frac{\partial e^\ell}{\partial L^\ell}\frac{L^\ell}{e^\ell} = -1, \tag{12}$$

$$F_1' = w^\ell/e^\ell. \tag{13}$$

Equation (12) implicitly characterizes the optimal wage rate paid to workers employed in the low-demanding job, and equation (13) implicitly characterizes the optimal number of agents employed in the low-demanding job.[8] With the disposable income in the employment state, B^ℓ, held constant, the wage rate enters the effort equation only through $I^\ell/w^\ell = L^\ell$. Therefore, equation (12) is just a variant of the standard condition for wage setting in an efficiency wage model, i.e., $(\partial e^\ell/\partial w^\ell)w^\ell/e^\ell = 1$. Finally, the equilibrium wage rate for workers in the high-demanding job, w^h, satisfies the condition

$$F_2' = w^h. \tag{14}$$

For later purposes, we need to evaluate how N^ℓ, w^ℓ and w^h vary in response to changes in I^ℓ, B^ℓ, I^h, B^h, u, and b. Denoting the elasticity of substitution between the two labor inputs in production by σ, we have derived the following comparative statics results in the Appendix A:

[8] Equations (12) and (13) are derived as the first order conditions with respect to w^ℓ and N^ℓ, respectively, of the following problem solved by a representative firm:

$$\max_{w^\ell, N^\ell} F\left(N^\ell \frac{I^\ell}{w^\ell} e^\ell\left(B^\ell, \frac{I^\ell}{w^\ell}, u, b\right), N^h \frac{I^h}{w^h}\right) - \sum_{i=\ell,h} N^i I^i.$$

$$\frac{dN^\ell}{dI^\ell} = \frac{1-\frac{1}{\sigma}}{\left(e^\ell L^\ell\right)^2 F''_{11}} - \frac{N^\ell}{I^\ell}, \quad \frac{dN^\ell}{dB^\ell} = \frac{-\frac{\partial e^\ell}{\partial B^\ell}\frac{w^\ell L^\ell}{e^\ell}\left[1-\frac{1}{\sigma}\right]}{\left(e^\ell L^\ell\right)^2 F''_{11}}, \quad \frac{dN^\ell}{dI^h} = \frac{N^\ell}{w^h L^h}, \quad \frac{dN^\ell}{dB^h} = 0, \tag{15}$$

$$\frac{dw^\ell}{dI^\ell} = \frac{1}{L^\ell}, \quad \frac{dw^\ell}{dB^\ell} = -\frac{\frac{\partial^2 e^\ell}{\partial L^\ell \partial B^\ell} + \frac{1}{L^\ell}\frac{\partial e^\ell}{\partial B^\ell}}{\frac{L^\ell}{w^\ell}\left[\frac{2e^\ell}{\left(L^\ell\right)^2} - \frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell}\right]}, \quad \frac{dw^\ell}{dI^h} = 0, \quad \frac{dw^\ell}{dB^h} = 0, \tag{16}$$

$$\frac{dw^h}{dI^\ell} = -\frac{N^\ell}{N^h L^h}, \quad \frac{dw^h}{dB^\ell} = \frac{N^\ell L^\ell \frac{\partial e^\ell}{\partial B^\ell}\frac{w^\ell}{e^\ell}}{N^h L^h}, \quad \frac{dw^h}{dI^h} = 0, \quad \frac{dw^h}{dB^h} = 0, \tag{17}$$

$$\frac{dN^\ell}{du} = \frac{\left[\frac{1}{\sigma}-1\right]\frac{w^\ell}{e^\ell}L^\ell\frac{\partial e^\ell}{\partial u}}{\left(e^\ell L^\ell\right)^2 F''_{11}}, \quad \frac{dw^\ell}{du} = -\frac{\frac{\partial^2 e^\ell}{\partial L^\ell \partial u} + \frac{1}{L^\ell}\frac{\partial e^\ell}{\partial u}}{\frac{L^\ell}{w^\ell}\left[\frac{2e^\ell}{\left(L^\ell\right)^2} - \frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell}\right]}, \quad \frac{dw^h}{du} = \frac{w^\ell}{e^\ell}\frac{N^\ell L^\ell}{N^h L^h}\frac{\partial e^\ell}{\partial u}, \tag{18}$$

$$\frac{dN^\ell}{db} = \frac{\left[\frac{1}{\sigma}-1\right]\frac{w^\ell}{e^\ell}L^\ell\frac{\partial e^\ell}{\partial b}}{\left(e^\ell L^\ell\right)^2 F''_{11}}, \quad \frac{dw^\ell}{db} = -\frac{\frac{\partial^2 e^\ell}{\partial L^\ell \partial b} + \frac{1}{L^\ell}\frac{\partial e^\ell}{\partial b}}{\frac{L^\ell}{w^\ell}\left[\frac{2e^\ell}{\left(L^\ell\right)^2} - \frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell}\right]}, \quad \frac{dw^h}{db} = \frac{w^\ell}{e^\ell}\frac{N^\ell L^\ell}{N^h L^h}\frac{\partial e^\ell}{\partial b}. \tag{19}$$

Whereas some of the comparative statics derivatives are signed, others are not without additional assumptions. In particular, note that the elasticity of substitution, σ, plays a key role for how N^ℓ responds to variations in I^ℓ, B^ℓ, u, and b. Some of these results will be discussed in greater detail below, where (15)-(19) are used to characterize the optimal marginal tax and expenditure policy.

2.2 Social Decision-Problem

We consider the general governmental objective of reaching a Pareto efficient resource allocation. This is accomplished by maximizing the (expected) utility of the low-skilled type subject to a minimum utility restriction for the high-skilled type, as well as subject to the appropriate self-selection and resource constraints. We also assume that the government (or social planner) wants to redistribute from the high-skilled to the low-skilled, which Stiglitz (1982) refers to as the "normal" case, meaning that the optimal resource allocation must be constrained to prevent high-skilled individuals from mimicking the low-skilled type. As such, the optimal marginal tax and expenditure policies characterized below will satisfy any social welfare function, which is increasing in the utility of both skill-types, if consistent with the assumed profile of the redistribution.

To ensure that each high-skilled individual prefers the allocation intended for his/her type (I^h, B^h) over the before-tax and disposable income combination intended for the employed low-skilled type (I^ℓ, B^ℓ), we impose self-selection constraints designed to make mimicking unattractive. In our setting, mimicking can occur in two alternative ways. One possibility would be for the high-skilled individual to reduce his/her labor supply in the high-demanding job to the extent required to earn I^ℓ instead of I^h. Since the high-skilled are more productive than the low-skilled, a high-skilled mimicker can reach the income level I^ℓ by supplying fewer hours of work than needed by a low-skilled individual. Another possibility for a high-skilled individual to act as a mimicker would be to take a low-demanding job, which also gives the before-tax income I^ℓ. With regards to the latter option, we assume that, for any given level of effort exerted in the workplace, the productivity of a high-skilled worker does not differ from the productivity of a low-skilled worker in the low-demanding job. A high-skilled individual working in the low-demanding job will thus be paid the wage rate w^ℓ (instead of the wage rate w^h). If choosing a low-demanding job, the effort exerted by a high-skilled individual will depend on the outside option if caught shirking and fired. Assuming that, if fired from a low-demanding job, a high-skilled individual can always find employment in a high-demanding job (so that he/she does not face any threat of becoming unemployed), the optimal effort choice of a mimicker in a low-demanding job would be zero. Note that the utility of a mimicker would, in this case, exceed the utility faced by the low-skilled type, since the effort provided by a high-skilled individual in the low-demanding job falls short of the effort chosen by individuals of the low-skilled type.

Even though both available mimicking strategies require a high-skilled mimicker to earn the same before-tax income as a low-skilled individual, we will hereafter use the term *income-replication* strategy to refer to the case when a high-skilled individual behaves as a mimicker and chooses the high-demanding job (working fewer hours than a low-skilled individual), whereas we will use the expression *job-replication* strategy to refer to the case when a high-skilled individual behaves as a mimicker and chooses the low-demanding job (exerting less effort than a low-skilled individual).

Now, since the government can implement any desired combination of work hours and disposable income for each skill-type subject to constraints, we follow convention in much earlier literature on optimal nonlinear taxation by writing the social decision-problem directly in terms of the before-tax and disposable incomes, instead of in terms of parameters of the tax function. The social-decision problem can then be written as follows:

$$\max_{I^\ell, B^\ell, I^h, B^h, b, u} \quad EV^\ell \left(B^\ell, I^\ell, u, b; w^\ell\right)$$

subject to

$$\begin{aligned}
V^h\left(B^h, I^h; w^h\right) &\geq \overline{V}^h & (\delta) \qquad (20)\\
V^h\left(B^h, I^h; w^h\right) &\geq V^h\left(B^\ell, I^\ell; w^h\right) & (\lambda) \qquad (21)\\
V^h\left(B^h, I^h; w^h\right) &\geq V^h\left(B^\ell, I^\ell, e=0; w^\ell\right) & (\zeta) \qquad (22)\\
F\left(e^\ell N^\ell \frac{I^\ell}{w^\ell}, N^h \frac{I^h}{w^h}\right) &\geq \pi^h B^h + N^\ell B^\ell + \left(\pi^\ell - N^\ell\right) b & (\mu) \qquad (23)\\
u &= 1 - \frac{N^\ell\left(B^\ell, I^\ell, u, b\right)}{\pi^\ell} & (\theta) \qquad (24)
\end{aligned}$$

where the Lagrange multipliers attached to the respective constraints are given in parentheses.

The δ-constraint is the minimum utility constraint, implying that the utility of each high-skilled individual must not fall short of $\overline{V}^h$. The λ-constraint and the ζ-constraint are two self-selection constraints that jointly ensure that a high-skilled individual has no incentive to act as a mimicker, i.e., has no incentive to earn the income level intended for the low-skilled type through the income-replication or job-replication strategy, respectively. Note also that, unless the two mimicking strategies available to the high-skilled are equally attractive in utility terms, at most one of these two self-selection constraints will be binding. The μ-constraint is the economy-wide resource constraint, while the θ-constraint is required to treat the unemployment rate as an artificial control variable for the government. We can interpret the resource constraint such that the aggregate output must not fall short of the aggregate consumption.

The first order condition of the social decision-problem are presented in Appendix B. Next, we turn to the implications of these first order conditions for the optimal tax and expenditure policy.

3 Optimal Tax and Expenditure Policy

To simplify the presentation, we introduce the following short notation for marginal rates of substitution between the before-tax income and disposable income for a low-skilled individual and a mimicker, respectively, where we have distinguished between the income-replication and job-replication strategies for the mimicker:

$$MRS_{I,B}^{\ell} = -\frac{\partial EV^{\ell}/\partial I^{\ell}}{\partial EV^{\ell}/\partial B^{\ell}} > 0, \qquad \widehat{MRS}_{I,B}^{h} = -\frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{h}}\right)/\partial I^{\ell}}{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{h}}\right)/\partial B^{\ell}} > 0,$$

$$\widehat{MRS}_{I,B}^{h,e=0} = -\frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{\ell}}, e=0\right)/\partial I^{\ell}}{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{\ell}}, e=0\right)/\partial B^{\ell}} > 0.$$

For notational convenience, we also define "utility compensated" wage and employment responses to an increase in the before-tax income, I^{ℓ}, such that

$$\frac{d\tilde{w}^{\ell}}{dI^{\ell}} = \frac{dw^{\ell}}{dI^{\ell}} + MRS^{\ell}_{I,B}\frac{dw^{\ell}}{dB^{\ell}}, \tag{25}$$

$$\frac{d\tilde{N}^{\ell}}{dI^{\ell}} = \frac{dN^{\ell}}{dI^{\ell}} + MRS^{\ell}_{I,B}\frac{dN^{\ell}}{dB^{\ell}}, \tag{26}$$

$$\frac{d\widetilde{w}^{h}}{dI^{\ell}} = \frac{dw^{h}}{dI^{\ell}} + MRS^{\ell}_{I,B}\frac{dw^{h}}{dB^{\ell}}. \tag{27}$$

Equations (25) and (26) measure how an increase in the before-tax income, I^{ℓ}, affects the wage rate, w^{ℓ}, and the number of employed persons, N^{ℓ}, respectively, of the low-skilled type, when the low-skilled are compensated via an increase in the disposable income, B^{ℓ}, to remain at the initial (expected) utility level. Similarly, equation (27) shows how a marginal increase in I^{ℓ}, compensated via a change in B^{ℓ} to leave the (expected) utility of low-skilled individuals unchanged, affects the wage rate paid to workers in the high-demanding job, w^{h}.

We are now ready to characterize the marginal income tax rates implemented for the two skill-types.

Proposition 1. *The marginal income tax rate faced by high-skilled workers is negative (positive) if* $\left(I^{\ell} - B^{\ell} + b\right)\pi^{\ell} + \theta/\mu > 0$ (< 0) *and given as follows:*

$$T'\left(I^{h}\right) = -\frac{1-u}{\pi^{h}I^{h}}\left[\left(I^{\ell} - B^{\ell} + b\right)\pi^{\ell} + \frac{\theta}{\mu}\right]. \tag{28}$$

Let $\widetilde{\epsilon}_{N^{\ell},I^{\ell}}$ *and* $\widetilde{\epsilon}_{w^{h},I^{\ell}}$ *denote the compensated elasticity of* N^{ℓ} *and* w^{h}*, respectively, respect to* I^{ℓ}*, such that* $\widetilde{\epsilon}_{N^{\ell},I^{\ell}} = \frac{d\tilde{N}^{\ell}}{dI^{\ell}}\frac{I^{\ell}}{N^{\ell}}$ *and* $\widetilde{\epsilon}_{w^{h},I^{\ell}} = \frac{d\widetilde{w}^{h}}{dI^{\ell}}\frac{I^{\ell}}{w^{h}}$*. The marginal income tax rate faced by low-skilled workers can then be written as*

$$
\begin{aligned}
T'\left(I^{\ell}\right) &= \frac{\lambda \frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{h}}\right)}{\partial B^{\ell}}}{\mu\left(1-u\right)\pi^{\ell}}\left[MRS_{I,B}^{\ell}-\widehat{MRS}_{I,B}^{h}\left(1-\frac{I^{\ell}}{w^{h}}\frac{d\widetilde{w}^{h}}{dI^{\ell}}\right)\right] \\
&\quad +\frac{\zeta \frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{\ell}},e=0\right)}{\partial B^{\ell}}}{\mu\left(1-u\right)\pi^{\ell}}\left[MRS_{I,B}^{\ell}-\widehat{MRS}_{I,B}^{h,e=0}\right] \\
&\quad +\frac{1}{\mu\left(1-u\right)\pi^{\ell}}\left[\zeta\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{\ell}},e=0\right)}{\partial w^{\ell}}-\frac{\partial EV^{\ell}}{\partial w^{\ell}}\right]\frac{d\widetilde{w}^{\ell}}{dI^{\ell}} \\
&\quad -\frac{1}{I^{\ell}}\left[I^{\ell}-B^{\ell}+b+\frac{\theta}{\mu\pi^{\ell}}\right]\widetilde{\epsilon}_{N^{\ell},I^{\ell}}-\frac{N^{h}I^{h}}{N^{\ell}I^{\ell}}MRS_{I,B}^{h}\widetilde{\epsilon}_{w^{h},I^{\ell}}.
\end{aligned} \tag{29}
$$

Proof: see the Appendix C.

According to equation (28), high-skilled individuals should face a negative marginal income tax rate under realistic assumptions, implying that their labor supply is distorted upwards. This finding is reminiscent of a result derived by Stiglitz (1982) in a model with competitive labor markets, where a negative marginal income tax rate for high-skilled individuals works as a device to reduce the wage gap between the skill-types.[9] In our setting, a negative marginal income tax rate for the high-skilled is justified as a mechanism through which the government may stimulate the demand for low-skilled labor. This is, in turn, socially beneficial for two reasons: by (i) increasing the net revenue collected by the government (provided that the transfer paid to the unemployed is larger than the transfer paid to low-skilled workers), and (ii) reducing the unemployment rate, which is socially desirable whenever the social value of decreased unemployment measured in terms of public funds, θ/μ, is positive (see Proposition 2 below).

Turning to marginal income taxation of the low-skilled in equation (29), the first two terms on its right hand side, i.e., the first and second row, are induced by the self-selection constraints; these components would vanish in a first best environment where

[9] See also Pirttilä and Tuomala (2001), and Gahvari (2014).

individual skill is observable, in which case $\lambda = \zeta = 0$. In a second-best optimum where individual skill is private information, at most one of these two self-selection constraints will be binding (as mentioned before). If the self-selection constraint given in equation (21), i.e., the constraint associated with the income-replication strategy, is binding ($\lambda > 0$), the first term on the right hand side of equation (29) can either be positive or negative. This is interesting in itself: in a standard optimal income tax model with competitive labor markets, the corresponding term would be unambiguously positive due to the assumption that the marginal rates of substitution satisfy the condition $\partial MRS_{I,B}/\partial w < 0$.[10] In the standard model, normality of consumption is a sufficient condition to guarantee that the single-crossing condition holds. In our setting, however, this is not enough. When comparing a low-skilled individual with a high-skilled mimicker adopting the income-replication strategy, it is still true that a high-skilled individual needs fewer hours to earn the same before-tax income as a low-skilled individual (since $w^h > w^\ell$). At the same time, however, a high-skilled individual employed in the high-demanding job exerts more effort than a low-skilled individual in the low-demanding job. Therefore, if we were to make the reasonable assumption that $\partial MRS^j_{I,B}/\partial e^j > 0$, it is in principle possible that the first term on the right hand side of equation (29) takes a negative sign.[11] This is even more likely taking into account that $d\widetilde{w}^h/dI^\ell < 0$, as we show in the Appendix C (see equation (C13)).

Instead, if the self-selection constraint given in equation (22), i.e., the constraint associated with the job-replication strategy is binding ($\zeta > 0$), the second row of eq. (29) will be unambiguously positive under the plausible assumption that $\partial MRS^j_{I,B}/\partial e^j > 0$, thus contributing to a higher marginal income tax rate for the low-skilled. The reason is that a low-skilled individual exerts a positive level of effort, whereas a high-skilled

[10] This is typically referred to as the Agent-Monotonicity condition. See Seade (1982).

[11] The intuition behind the assumption $\partial MRS^j_{I,B}/\partial e^j > 0$ is that increased on-the-job effort makes leisure outside the workplace more valuable at the margin, *ceteris paribus* (i.e. for given values of I, B and w).

mimicker adopting the job-replication strategy exerts no effort at all, implying that a low-skilled individual attaches a higher marginal value to leisure outside the workplace compared to a mimicker (despite that they are paid the same wage rate, w^{ℓ}). An increase in the marginal income tax rate, which induces the low-skilled to reduce their hours of work, will thus hurt the mimicker more than it hurts the low-skilled, meaning that this policy opens up for more redistribution through a relaxation of the job-replication self-selection constraint.[12]

The third and fourth rows of equation (29) arise as a consequence of efficiency wage setting for the low-skilled type and would vanish if the labor market were competitive. Note that the choice of income-consumption bundle for the low-skilled, (I^{ℓ}, B^{ℓ}), affects the labor market outcome via two channels: (i) the hourly wage rates, w^{ℓ} and w^{h}, and (ii) the number of low-skilled workers that each firm finds it optimal to hire. Starting with the marginal tax response to an induced change in the hourly wage rate paid to the low-skilled, w^{ℓ}, note first that

$$\frac{\partial EV^{\ell}}{\partial w^{\ell}} = -\left[1 - u\varphi\left(e^{\ell}\right)\right] \frac{L^{\ell}}{w^{\ell}} \frac{\partial v\left(B^{\ell}, \frac{I^{\ell}}{w^{\ell}}, e^{\ell}\right)}{\partial L^{\ell}} > 0,$$

in which we have used that the indirect effects of w^{ℓ}, arising via the individuals' labor supply and effort choices, vanish as a consequence of optimality, i.e., by the Envelope Theorem. $\partial EV^{\ell}/\partial w^{\ell}$ is interpretable as a direct expected benefit faced by each individual of the low-skilled type following an increase in their wage rate. If the self-selection constraint associated with the job-replication strategy, i.e., equation (22), is binding, such a wage increase would also make mimicking more attractive, which explains the first term in square brackets in the third row.[13] We can then interpret the difference in square brackets in the third row of equation (29) as reflecting the net social cost of

[12]In the alternative - albeit less plausible - scenario where $\partial MRS^{j}_{I,B}/\partial e^{j} < 0$, the second row of equation (29) will, instead, be negative and contributes to a lower marginal tax rate for the low-skilled.

[13]We have $\zeta \frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{\ell}}, e=0\right)}{\partial w^{\ell}} = -\zeta \frac{I^{\ell}}{\left(w^{\ell}\right)^{2}} \frac{\partial v\left(B^{\ell}, \frac{I^{\ell}}{w^{\ell}}, e=0\right)}{\partial L^{\ell}} > 0.$

an increase in w^ℓ; if this difference is positive (negative), an increase in w^ℓ would lead to lower (higher) welfare, *ceteris paribus*. To see what this net social cost implies for the marginal income tax rate faced by the low-skilled, suppose first that $d\tilde{w}^\ell/dI^\ell > 0$, meaning that a combined increase in the before-tax and disposable income for the low-skilled would push up their hourly wage rate.[14] In this case, there will be an incentive for the government to increase (reduce) the labor supply of the low-skilled via a lower (higher) marginal tax rate if

$$\begin{aligned}&\zeta\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial w^\ell}-\frac{\partial EV^\ell}{\partial w^\ell} \\ =\ &\left\{\left[1-u\varphi\left(e^\ell\right)\right]\frac{\partial v\left(B^\ell,\frac{I^\ell}{w^\ell},e^\ell\right)}{\partial L^\ell}-\zeta\frac{\partial v\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial L^\ell}\right\}\frac{L^\ell}{w^\ell}<0\ \ (>0).\end{aligned} \tag{30}$$

The policy incentives are analogous in the alternative scenario where $d\tilde{w}^\ell/dI^\ell < 0$, in which case the government would implement a lower (higher) marginal tax rate for the low-skilled if the sign of (30) is positive (negative).[15]

A direct employment effect of marginal taxation is captured by the first term in the forth row of equation (29), showing how a compensated (for low-skilled agents) increase in I^ℓ affects N^ℓ. Notice first that the sign of $\widetilde{\epsilon}_{N^\ell,I^\ell}$ is in principle ambiguous. For interpretational purposes, we will focus on the case where $d\tilde{N}^\ell/dI^\ell < 0$ (as shown in the Appendix C, equation (C12), a sufficient condition for $d\tilde{N}^\ell/dI^\ell < 0$ is that $\sigma \geq 1$[16]). In this case, the government has an incentive to distort the labor supply of low-skilled workers downwards through higher marginal taxation for two reasons: first, when the

[14]Since we have established that $d\tilde{w}^h/dI^\ell < 0$, it also follows that a compensated (for the low-skilled) marginal increase in I^ℓ leads to an increase in w^ℓ/e^ℓ. Even though an increase in w^ℓ/e^ℓ does not necessarily imply that w^ℓ increases, for interpretational purposes we here focus on the case when $d\tilde{w}^\ell/dI^\ell > 0$.

[15]Recall that $\partial^2 v/\partial e\partial L < 0$ by assumption. Therefore, the sign of (30) is more likely to be positive the lower the effort level, e^ℓ, the higher the unemployment rate, u, and the higher the value of the Lagrange multiplier ζ.

[16]This condition is, for instance, satisfied under under a Cobb-Douglas production function where $\sigma = 1$. In this case, we can use equation (C12) to derive $d\tilde{N}^\ell/dI^\ell = -N^\ell/I^\ell$.

transfer paid to the unemployed is larger than the transfer paid to the employed low-skilled workers ($I^\ell - B^\ell + b > 0$), an increase in N^ℓ will increase the net tax revenue; second, an increase in N^ℓ leads to a lower unemployment rate, which is socially beneficial when the social value of decreased unemployment measured in terms of public funds, θ/μ, is positive.[17]

Finally, the second term on the fourth row of equation (29) reflects an interaction effect between the labor supply of the low-skilled and the tax revenue the government can collect from the high-skilled. We show in the Appendix C that $d\widetilde{w}^h/dI^\ell < 0$ (see equation (C13)). Therefore, this component also pushes in the direction of increasing the marginal income tax rate implemented for the low-skilled. The interpretation is that the wage rate paid to the high-skilled increases in response to a compensated (for low-skilled workers) marginal reduction in I^ℓ. In turn, this opens up the possibility to raise additional tax revenue from the high-skilled without violating the minimum utility restriction.

We will now turn to the factors determining the social value of decreased unemployment, θ/μ. For this purpose, let

$$MRS^\ell_{u,b} = -\frac{\partial EV^\ell/\partial u}{\partial EV^\ell/\partial b} = \frac{v\left(B^\ell, \frac{I^\ell}{w^\ell}, e^\ell\right) - v\left(b,0,0\right)}{u\partial v\left(b,0,0\right)/\partial b} > 0 \qquad (31)$$

denote the marginal rate of substitution between the unemployment rate and the unemployment benefit for a low-skilled individual,[18] and define the "utility compensated"

[17] A similar employment-related motive for tax policy intervention was found by Aronsson and Sjögren (2003, 2004), based on different models with trade-unionized labor markets.

[18] $v\left(B^\ell, \frac{I^\ell}{w^\ell}, e^\ell\right) - v\left(b,0,0\right) > 0$ is necessary to rule out the possibility that low-skilled individuals are voluntary unemployed.

wage and employment responses to an increase in the unemployment rate such that

$$\frac{d\tilde{w}^{\ell}}{du} = \frac{dw^{\ell}}{du} + MRS^{\ell}_{u,b}\frac{dw^{\ell}}{db}, \tag{32}$$

$$\frac{d\tilde{N}^{\ell}}{du} = \frac{dN^{\ell}}{du} + MRS^{\ell}_{u,b}\frac{dN^{\ell}}{db}, \tag{33}$$

$$\frac{d\tilde{w}^{h}}{du} = \frac{dw^{h}}{du} + MRS^{\ell}_{u,b}\frac{dw^{h}}{db}. \tag{34}$$

Proposition 2 gives an expression for the social value of decreased unemployment at a second-best optimum.

Proposition 2. *The social value of decreased unemployment can be expressed as follows:*

$$\frac{\theta}{\mu} = \frac{\pi^{\ell}}{\Gamma}\left\{uMRS^{\ell}_{u,b} + \left[I^{\ell} - B^{\ell} + b\right]\widetilde{\epsilon}_{N^{\ell},1-u} + \frac{N^{h}}{N^{\ell}}MRS^{h}_{I,B}I^{h}\widetilde{\epsilon}_{w^{h},1-u}\right\}$$
$$+\frac{\lambda}{\mu\Gamma}\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{h}}\right)}{\partial w^{h}}\frac{d\tilde{w}^{h}}{du} + \frac{1}{\mu\Gamma}\left[\zeta\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{\ell}},e=0\right)}{\partial w^{\ell}} - \frac{\partial EV^{\ell}}{\partial w^{\ell}}\right]\frac{d\tilde{w}^{\ell}}{du}, \tag{35}$$

where $\Gamma \equiv 1 + \frac{1}{\pi^{\ell}}\frac{d\tilde{N}^{\ell}}{du}$, $\widetilde{\epsilon}_{N^{\ell},1-u} \equiv -\frac{1-u}{N^{\ell}}\frac{d\tilde{N}^{\ell}}{du}$ *and* $\widetilde{\epsilon}_{w^{h},1-u} \equiv -\frac{1-u}{w^{h}}\frac{d\tilde{w}^{h}}{du}$.
Proof: see the Appendix D.

Note first that Γ in the denominator on the right hand side of equation (35) represents a feedback effect, which is reminiscent of the corresponding effect entering in expressions for the social shadow price of a consumption externality (e.g., Sandmo, 1980; Pirttilä and Tuomala, 1997). As shown by Sandmo (1980), stability requires that the feedback effect is positive. We will, therefore, base our discussion of the result in Proposition 2 on the assumption that $\Gamma > 0$. In our case, where we have used the social first order conditions for u and b to derive equation (35), it follows that the feedback effect depends on how the number of employed persons of the low-skilled type is influenced both by the unemployment rate and the unemployment benefit. The term $d\tilde{N}^{\ell}/du$ in the expression for Γ can be thought of as the employment response to a utility compensated increase in the unemployment rate, where the compensation is measured in terms of the unemployment benefit.

We can interpret $\pi^\ell u\ MRS_{u,b}^\ell$ in the first row of equation (35) as the sum of the marginal willingness to pay to avoid unemployment among the unemployed, where $\pi^\ell u$ represents the number of unemployed persons. This component thus resembles the sum of the marginal willingness to pay to avoid a public bad, with the modification that it is only measured over the relevant part of the population.

The remaining two terms in the first row of equation (35) are interpretable as public revenue effects. A direct public revenue effect of a variation in the unemployment rate is captured by the second term in curly brackets, which reflects that the net tax revenue increases by $I^\ell - B^\ell + b = T(I^\ell) + b$ when a low-skilled individual switches from unemployment to employment. The increased tax revenue is, in turn, multiplied by $\widetilde{\epsilon}_{N^\ell,1-u}$, measuring the extent to which each firm's demand for low-skilled workers is affected by a change in the economy-wide unemployment rate. By using equations (18)-(19) to derive

$$\frac{d\widetilde{N}^\ell}{du} = \frac{\left[\frac{1}{\sigma} - 1\right] \frac{w^\ell}{e^\ell} L^\ell}{\left(e^\ell L^\ell\right)^2 F_{11}''} \frac{\partial \widetilde{e}^\ell}{\partial u},$$

we can see that the sign of the compensated response of N^ℓ to a marginal increase in u depends both on the elasticity of substitution between the labor inputs, and on how the effort exerted by the low-skilled changes in response to a compensated increase in u. Therefore, this public revenue effect vanishes in the special case where $\sigma = 1$, in which $d\widetilde{N}^\ell/du = 0$. Instead, if $\sigma > 1$ and $\partial\widetilde{e}^\ell/\partial u < 0$ (or $\sigma < 1$ and $\partial\widetilde{e}^\ell/\partial u > 0$), we have $d\widetilde{N}^\ell/du < 0$, which implies that public revenue considerations lead to an increase in θ/μ.

The third term in curly brackets in the first row of equation (35) captures another public revenue effect, descending from the fact that a variation in the unemployment rate among the low-skilled affects the equilibrium wage rate paid in the high-demanding job. Using equations (18)-(19), we can see that the sign of this effect depends on the sign of

$$\frac{d\widetilde{w}^h}{du} = \frac{w^\ell}{e^\ell}\frac{N^\ell L^\ell}{N^h L^h}\frac{\partial \widetilde{e}^\ell}{\partial u}.$$

Therefore, if $\partial \widetilde{e}^\ell / \partial u > 0$, an increase in the unemployment rate will lead to an increase in the wage rate paid to the high-skilled. In turn, this opens up for the possibility of raising additional tax revenue from the high-skilled without violating the minimum utility restriction. As a consequence, the final term in curly brackets is negative and contributes to a downwards adjustment of θ/μ. Instead, if $\partial \widetilde{e}^\ell / \partial u < 0$, an increase in the unemployment rate leads to lower w^h, which would require a lower tax payment by the high-skilled to leave their utility unchanged. In the latter case, the final term in curly brackets is positive and contributes to an upward adjustment of θ/μ.

The self-selection constraints directly affect the social value of decreased unemployment through the second row of equation (35). The first term captures the effect of a variation in w^h, induced by a change in the unemployment rate, on the self-selection constraint associated with the income-replication strategy. If $d\widetilde{w}^h/du > (<)\ 0$, an increase in the unemployment rate makes this mimicking strategy more (less) attractive which, in turn, contributes to raising (lowering) the social value of decreased unemployment. Finally, the second term appears because an increase in the unemployment rate affects the wage rate paid in the low-demanding job, which leads to an additional social cost or benefit for reasons similar to those illustrated when discussing (29). In particular, $d\tilde{w}^\ell/du$ can be thought of as a utility compensated wage response to an increase in the unemployment rate, where the compensation appears in the form of an increase in the unemployment benefit, while

$$\zeta \frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^\ell}, e = 0\right)}{\partial w^\ell} - \frac{\partial EV^\ell}{\partial w^\ell} \tag{36}$$

is interpretable as the net social cost of an increase in the wage paid to the low-skilled type (as explained above in the context of marginal income tax policy). Therefore, if the product between $d\widetilde{w}^h/du$ and (36) is positive, the social value of decreased unemployment will be larger than implied by the remaining terms on the right hand side of equation (35). The intuition is, in this case, that reduced unemployment would also imply an indirect welfare benefit through an effect on the wage rate paid in the low-demanding job. The opposite policy incentive arises if the product between $d\tilde{w}^\ell/du$ and the bracketed term in the second row is negative.

To complete the characterization of the optimal second-best policy, we will now turn to the policy rule for the unemployment benefit. For this purpose, let

$$MRS^\ell_{b,B} = \frac{\partial EV^\ell/\partial b}{\partial EV^\ell/\partial B} = \frac{u\varphi\left(e^\ell\right)\partial v\left(b,0,0\right)/\partial b}{\left[1-u\varphi\left(e^\ell\right)\right]\partial v\left(B^\ell,\frac{I^\ell}{w^\ell},e^\ell\right)/\partial B^\ell} > 0$$

denote the marginal rate of substitution between the unemployment benefit and the disposable income in the employed state for a low-skilled individual, and define the "utility compensated" wage and employment responses to an increase in the unemployment benefit, b, such that

$$\frac{d\tilde{N}^\ell}{db} = \frac{dN^\ell}{db} - MRS^\ell_{b,B}\frac{dN^\ell}{dB^\ell}, \tag{37}$$

$$\frac{d\tilde{w}^\ell}{db} = \frac{dw^\ell}{db} - MRS^\ell_{b,B}\frac{dw^\ell}{dB^\ell}, \tag{38}$$

$$\frac{d\tilde{w}^h}{db} = \frac{dw^h}{db} - MRS^\ell_{b,B}\frac{dw^h}{dB^\ell}. \tag{39}$$

Proposition 3 characterizes the efficient level of the unemployment benefit at a second-best optimum.

Proposition 3. *The optimal unemployment benefit abides by the following policy rule:*

$$
\begin{aligned}
(1-u)\,\pi^{\ell} MRS^{\ell}_{b,B} &= u\pi^{\ell} - \left[I^{\ell} - B^{\ell} + b + \frac{\theta}{\mu\pi^{\ell}}\right]\frac{d\widetilde{N}^{\ell}}{db} - \pi^{h} L^{h} MRS^{h}_{I,B}\frac{d\widetilde{w}^{h}}{db} \\
&-\frac{MRS^{\ell}_{b,B}}{\mu}\left[\lambda\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{h}}\right)}{\partial B^{\ell}} + \zeta\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{\ell}},e=0\right)}{\partial B^{\ell}}\right] \\
&+\frac{\lambda}{\mu}\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{h}}\right)}{\partial w^{h}}\frac{d\widetilde{w}^{h}}{db} + \frac{1}{\mu}\left[\zeta\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{\ell}},e=0\right)}{\partial w^{\ell}} - \frac{\partial EV^{\ell}}{\partial w^{\ell}}\right]\frac{d\widetilde{w}^{\ell}}{db}.
\end{aligned}
\tag{40}
$$

Proof: see the Appendix E.

The left hand side of equation (40) is interpretable as the sum of the marginal willingness to pay for a higher unemployment benefit, measured among the employed individuals of the low-skilled type. In particular, $MRS^{\ell}_{b,B}$ reflects the amount of income that each low-skilled individual would be willing to forego, when employed, in order to marginally raise his/her consumption in the event of becoming unemployed. Thus, $(1-u)\,\pi^{\ell} MRS^{\ell}_{b,B}$ can be interpreted as measuring the aggregate insurance benefit for the low-skilled of a marginal increase in the consumption available if becoming unemployed.[19]

Turning to the right hand side, the direct public budget cost of a marginal increase in b (which is paid to the $u\pi^{\ell}$ workers being unemployed) is measured by the first term, while the second and third terms are employment and public revenue effects reminiscent of those described in the context of Proposition 1. More specifically, the second term in the first row captures the net social gain of the employment effect induced by a compensated marginal increase in b. By using equations (15) and (19), we can rewrite

[19] $(1-u)\,\pi^{\ell} MRS^{\ell}_{b,B}$ can also be interpreted as the additional income tax revenue that the government can collect if marginally raising the unemployment benefit in a compensated way, i.e., raising b while at the same time adjusting $T\left(I^{\ell}\right)$ upwards to leave the expected utility unchanged for the low-skilled individuals.

equation (37) to read

$$\frac{d\tilde{N}^{\ell}}{db}=\frac{\left[\frac{1}{\sigma}-1\right]\frac{w^{\ell}L^{\ell}}{e^{\ell}}}{\left(e^{\ell}L^{\ell}\right)^{2}F_{11}''}\left[\frac{\partial e^{\ell}}{\partial b}-MRS_{b,B}^{\ell}\frac{\partial e^{\ell}}{\partial B^{\ell}}\right]=\frac{\left[\frac{1}{\sigma}-1\right]\frac{w^{\ell}L^{\ell}}{e^{\ell}}}{\left(e^{\ell}L^{\ell}\right)^{2}F_{11}''}\frac{\partial\widetilde{e}^{\ell}}{\partial b}.$$

From our discussion in subsection 2.1, we know that $\partial e^{\ell}/\partial b<0$ and that $\partial^{2}v/\left(\partial e^{\ell}\partial B^{\ell}\right)\geq 0$ is a sufficient condition for $\partial e^{\ell}/\partial B^{\ell}>0$. As long as $\sigma>1$, we would thus expect $d\tilde{N}^{\ell}/db<0$. Therefore, if $I^{\ell}-B^{\ell}+b+\theta/\mu\pi^{\ell}>0$ (<0), the employment effects of a compensated increase in b calls for an upward (a downward) adjustment in the net resource cost of raising the unemployment benefit. This typically contributes to decrease (increase) the unemployment benefit, *ceteris paribus*. Similarly, the third term in the first row captures a public budget effect descending from the fact that a variation in the unemployment benefit affects the wage rate facing the workers in the high-demanding job. Using equations (17)-(19) and (39), we can derive

$$\frac{d\widetilde{w}^{h}}{db}=\frac{\frac{w^{\ell}}{e^{\ell}}N^{\ell}L^{\ell}}{N^{h}L^{h}}\frac{\partial\widetilde{e}^{\ell}}{\partial b},$$

implying that $sign\left(d\widetilde{w}^{h}/db\right)=sign\left(\partial\widetilde{e}^{\ell}/\partial b\right)$. Therefore, if $\partial\widetilde{e}^{\ell}/\partial b<0$ (which is plausible based on the arguments presented above), an increase in the unemployment benefit reduces the before-tax wage rate faced by the high-skilled. In turn, a lower wage necessitates a lower income tax payment by the high-skilled in order to leave their utility unchanged. This indirect public budget effect leads to an increase in the net marginal cost of raising the unemployment benefit.

The second row of equation (40), and the first term in the third row, reflect that a compensated marginal increase in b (an increase in b accompanied by a utility compensated upward adjustment in $T\left(I^{\ell}\right)$) contributes to relax the self-selection constraint. First, since low-skilled workers face the risk of becoming unemployed whereas a potential mimicker does not, a compensated (for low-skilled agents) increase in the unemployment benefit makes mimicking less attractive, irrespective of which strategy the mimicker chooses. This effect is summarized by the second row of equation (40), where the first term is negative under the income-replication strategy ($\lambda > 0$ and $\zeta = 0$), while the second term is negative under the job-replication strategy ($\lambda = 0$ and $\zeta > 0$). Intuitively, the transfer paid to the unemployed is an instrument better targeted to the low-skilled than a transfer to low-income earners in general. This effect works to reduce the net resource cost of making the unemployment benefit more generous. Second, the first term in the third row captures the effect of an induced change in w^h on the utility of a high-skilled mimicker adopting the income-replication strategy. As we have argued above, the most likely case is where $d\widetilde{w}^h/db < 0$, implying that a marginal increase in b has the further advantage of lowering the utility of a mimicker using the income-replication strategy. This component also contributes to reduce the net resource cost of making the unemployment benefit more generous.

The final term in the third row of equation (40) captures the net social cost of a change in w^ℓ induced by a compensated marginal increase in b. In general, this component can be either positive or negative. To go further, we use equations (16)-(19) and (38) to calculate an expression for $d\widetilde{w}^\ell/db$ as follows:

$$\frac{d\widetilde{w}^\ell}{db} = \frac{dw^\ell}{db} - MRS^\ell_{b,B}\frac{dw^\ell}{dB^\ell} = -\frac{\frac{\partial^2 e^\ell}{\partial L^\ell \partial b} - MRS^\ell_{b,B}\frac{\partial^2 e^\ell}{\partial L^\ell \partial B^\ell} + \frac{1}{L^\ell}\frac{\partial \widetilde{e}^\ell}{\partial b}}{\frac{L^\ell}{w^\ell}\left[\frac{2e^\ell}{\left(L^\ell\right)^2} - \frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell}\right]}.$$

With $\partial^2 e^\ell/\partial L^\ell \partial L^\ell < 0$, we have

$$sign\left(\frac{d\widetilde{w}^\ell}{db}\right) = sign\left(MRS^\ell_{b,B}\frac{\partial^2 e^\ell}{\partial L^\ell \partial B^\ell} - \frac{\partial^2 e^\ell}{\partial L^\ell \partial b} - \frac{1}{L^\ell}\frac{\partial \widetilde{e}^\ell}{\partial b}\right).$$

Since the most plausible case is the one where $\partial^2 e^\ell/\partial L^\ell \partial B^\ell > 0$, $\partial^2 e^\ell/\partial L^\ell \partial b < 0$, and $\partial \widetilde{e}^\ell/\partial b < 0$, one would expect that $d\widetilde{w}^\ell/db > 0$. Thus, if a compensated increase in b induces the firms to raise the wage rate paid to the low-skilled workers, a benefit (in terms of higher utility due to reduced work hours) would accrue both to employed low-skilled agents and to high-skilled mimickers choosing the job-replication strategy.

When the former (latter) effect dominates, such that

$$\zeta \frac{\partial V^h \left(B^\ell, \frac{I^\ell}{w^\ell}, e=0 \right)}{\partial w^\ell} - \frac{\partial EV^\ell}{\partial w^\ell} < 0 \ \ (> 0) \, ,$$

the wage response to a compensated marginal increase in b is socially beneficial (detrimental), thus lowering (raising) the net resource cost of making the unemployment benefit more generous.

4 Concluding Remarks

This paper has integrated efficiency wage setting in the theory of optimal redistributive income taxation. In doing so, we used a model with two skill-types, where efficiency wage setting characterizes the labor market faced by the low-skilled, while the high-skilled face a conventional, competitive labor market. Furthermore, there are two types of jobs in this economy; a low-demanding job which can be carried out by all individuals, and a high-demanding job which can only be carried out by the high-skilled. The high-demanding job requires maximum effort per hour spent at the workplace, whereas effort per work hour is a decision-variable for individuals employed in the low-demanding job, such that individuals employed in this type of job have the option to "shirk" with an exogenous probability of detection. The government uses a nonlinear income tax and an unemployment benefit to redistribute income from the high-skilled to the low-skilled and to correct for imperfect competition in the labor market. As such, the government must also recognize that a high-skilled individual has two different options of mimicking the income of the low-skilled type; either by reducing the hours of work when employed in a high-demanding job (referred to as the income-replication strategy) or by choosing a low-demanding job (referred to as the job-replication strategy).

We would like to emphasize five results. First, the marginal income tax rate implemented for the high-skilled is likely to be negative. Albeit reminiscent of a result derived by Stiglitz (1982), the underlying mechanism is fundamentally different here: the negative marginal income tax rate implemented for the high-skilled provides a mechanism for increasing the demand for low-skilled labor. As such, it contributes to increase the net tax revenue and reduce the unemployment rate; both of which are socially desirable under plausible assumptions.

Second, the marginal income tax rate implemented for the low-skilled is not necessarily positive (as it would be in a standard model with competitive labor markets and no extensive margin of labor supply).[20] Although employment-related motives behind the tax policy (i.e., the incentive to increase the employment among the low-skilled) are likely to push up this marginal income tax rate, its sign may also depend on which of the two self-selection constraints that is binding. Whereas the self-selection constraint designed to prevent the job-replication strategy typically works to increase the marginal income tax rate of the low-skilled, the qualitative effect of the self-selection constraint designed to prevent the income-replication strategy is ambiguous. The intuition is that, although the income-replication strategy allows the high-skilled individual to spend more time on leisure than the low-skilled, a high-skilled mimicker employed in the high-demanding job still exerts more effort per work hour than the mimicked, low-skilled individual under the income-replication strategy.

Third, the social value of decreased unemployment takes a form reminiscent of shadow prices of public bads in the sense of depending on (i) the sum of the marginal willingness to pay to avoid unemployment among the unemployed, (ii) effects induced by the self-selection constraint, and (iii) tax revenue effects created by varying the unemployment rate. Note also that the social value of decreased unemployment directly affects the marginal income tax rates facing both skill-types at the second-best optimum,

[20]The result that the optimal marginal tax rate faced by low-skilled workers is not necessarily positive when firms pay efficiency wages in the low-demanding job is reminiscent of a similar finding obtained in a recent contribution by da Costa and Maestri (2017). By modifying the canonical Mirrleesian model to accommodate the assumption that firms have market power in the labor market, they show that almost all workers face negative marginal tax rates.

despite that unemployment may only arise among the low-skilled in our model.

Fourth, an increase in the unemployment benefit typically leads to a relaxation of the relevant self-selection constraint, irrespective of whether potential mimickers adopt an income-replication or job-replication strategy. As such, an increase in the unemployment benefit serves as a device to relax this constraint, which contributes to reduce the social resource cost of the unemployment benefit.

Fifth, the sign of the tax revenue effects influencing the social value of decreased unemployment and the optimal unemployment benefit, respectively, crucially depend on the elasticity of substitution between different types of labor inputs (in particular, whether the elasticity of substitution is larger or smaller than one). It also depends on the direction of the effort response by low-skilled individuals to a compensated variation in the unemployment rate and the unemployment benefit, respectively.

There are several interesting directions for future research. One would be to add a life-cycle dimension, where temporary unemployment spells have long term implications for income formation (since lost work experience may influence future wage profiles). In turn, this is relevant from the perspective of redistribution of life-time incomes and thus also for pension design. Another extension would be to introduce a spatial dimension and labor mobility, where unemployment may induce individuals to move in order to find employment. The latter is likely to affect the scope for redistribution as well as the employment-related motives behind the marginal tax policy, and may also call for a multi-level government approach to optimal taxation and public expenditure. Both these extensions are highly relevant and clearly comprehensive enough to motivate their own papers.

Appendix A

Derivations of the comparative statics results (15)-(19): Totally differentiating the systems of equations (12)-(14) gives, in matrix terms

$$\begin{bmatrix} 0 & -\frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell}\frac{I^\ell}{\left(w^\ell\right)^2}+2\frac{e^\ell}{L^\ell}\frac{1}{L^\ell}\frac{I^\ell}{\left(w^\ell\right)^2} & 0 \\ \frac{I^\ell}{w^\ell}e^\ell F_{11}'' & -\frac{1}{e^\ell}-\frac{w^\ell}{\left(e^\ell\right)^2}\frac{\partial e^\ell}{\partial L^\ell}\frac{I^\ell}{\left(w^\ell\right)^2} & -\frac{I^h}{\left(w^h\right)^2}N^h F_{12}'' \\ e^\ell\frac{I^\ell}{w^\ell}F_{12}'' & 0 & -N^h\frac{I^h}{\left(w^h\right)^2}F_{22}''-1 \end{bmatrix}\begin{bmatrix} dN^\ell \\ dw^\ell \\ dw^h \end{bmatrix}$$

$$= \begin{bmatrix} \left\{ \begin{array}{c} -\frac{\partial^2 e^\ell}{\partial L^\ell \partial B^\ell}dB^\ell-\frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell}\frac{dI^\ell}{w^\ell}-\frac{\partial^2 e^\ell}{\partial L^\ell \partial b}db-\frac{\partial^2 e^\ell}{\partial L^\ell \partial u}du \\ -\left[\frac{1}{L^\ell}\left(\frac{\partial e^\ell}{\partial B^\ell}dB^\ell+\frac{\partial e^\ell}{\partial L^\ell}\frac{dI^\ell}{w^\ell}+\frac{\partial e^\ell}{\partial b}db+\frac{\partial e^\ell}{\partial u}du\right)-\frac{e^\ell}{\left(L^\ell\right)^2}\frac{dI^\ell}{w^\ell}\right] \end{array} \right\} \\ \left\{ \begin{array}{c} -\left(\frac{\partial e^\ell}{\partial B^\ell}dB^\ell+\frac{\partial e^\ell}{\partial b}db+\frac{\partial e^\ell}{\partial u}du+\frac{\partial e^\ell}{\partial L^\ell}\frac{dI^\ell}{w^\ell}\right)\frac{I^\ell}{w^\ell}N^\ell F_{11}''-\frac{dI^\ell}{w^\ell}N^\ell e^\ell F_{11}'' \\ -\frac{dI^h}{w^h}N^h F_{12}''-\frac{w^\ell}{\left(e^\ell\right)^2}\left(\frac{\partial e^\ell}{\partial B^\ell}dB^\ell+\frac{\partial e^\ell}{\partial b}db+\frac{\partial e^\ell}{\partial u}du+\frac{\partial e^\ell}{\partial L^\ell}\frac{dI^\ell}{w^\ell}\right) \end{array} \right\} \\ \left\{-\left[\frac{\partial e^\ell}{\partial B^\ell}dB^\ell+\frac{\partial e^\ell}{\partial b}db+\frac{\partial e^\ell}{\partial u}du+\frac{\partial e^\ell}{\partial L^\ell}\frac{dI^\ell}{w^\ell}\right]\frac{I^\ell}{w^\ell}N^\ell F_{12}''-e^\ell\frac{dI^\ell}{w^\ell}N^\ell F_{12}''-N^h\frac{dI^h}{w^h}F_{22}''\right\} \end{bmatrix}. \tag{A1}$$

Starting with the comparative statics with respect to I^ℓ, we have

$$\begin{bmatrix} 0 & -\frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell}\frac{I^\ell}{\left(w^\ell\right)^2}+2\frac{e^\ell}{L^\ell}\frac{1}{L^\ell}\frac{I^\ell}{\left(w^\ell\right)^2} & 0 \\ \frac{I^\ell}{w^\ell}e^\ell F_{11}'' & -\frac{1}{e^\ell}-\frac{w^\ell}{\left(e^\ell\right)^2}\frac{\partial e^\ell}{\partial L^\ell}\frac{I^\ell}{\left(w^\ell\right)^2} & -\frac{I^h}{\left(w^h\right)^2}N^h F_{12}'' \\ e^\ell\frac{I^\ell}{w^\ell}F_{12}'' & 0 & -N^h\frac{I^h}{\left(w^h\right)^2}F_{22}''-1 \end{bmatrix}\begin{bmatrix} \frac{dN^\ell}{dI^\ell} \\ \frac{dw^\ell}{dI^\ell} \\ \frac{dw^h}{dI^\ell} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell}\frac{1}{w^\ell}+\frac{e^\ell}{w^\ell}\frac{2}{\left(L^\ell\right)^2} \\ \frac{1}{e^\ell L^\ell} \\ 0 \end{bmatrix}.$$

By exploiting the assumption that the F-function is linearly homogeneous, the determinant of the 3x3 matrix on the left hand side is equal to $\frac{\left(L^\ell\right)^2}{F_1'}\left[2\frac{e^\ell}{L^\ell}\frac{1}{L^\ell}-\frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell}\right]F_{11}''$. We can then derive

$$\frac{dN^\ell}{dI^\ell}=\frac{\frac{1}{e^\ell}\frac{1}{w^\ell}\left[2\frac{e^\ell}{L^\ell}\frac{1}{L^\ell}-\frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell}\right]\left[N^h\frac{I^h}{\left(w^h\right)^2}F_{22}''+1\right]}{\frac{\left(L^\ell\right)^2}{F_1'}\left[2\frac{e^\ell}{L^\ell}\frac{1}{L^\ell}-\frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell}\right]F_{11}''}=\frac{\left[N^h\frac{L^h}{w^h}F_{22}''+1\right]}{\left(e^\ell L^\ell\right)^2 F_{11}''}, \tag{A2}$$

$$\frac{dw^\ell}{dI^\ell} = \frac{\left[\frac{e^\ell}{w^\ell}\frac{2}{\left(L^\ell\right)^2} - \frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell}\frac{1}{w^\ell}\right] L^\ell e^\ell F''_{11}}{\frac{\left(L^\ell\right)^2}{F'_1}\left[2\frac{e^\ell}{L^\ell}\frac{1}{L^\ell} - \frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell}\right] F''_{11}} = \frac{1}{L^\ell}, \tag{A3}$$

$$\frac{dw^h}{dI^\ell} = \frac{\left[-\frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell} + 2\frac{e^\ell}{L^\ell}\frac{1}{L^\ell}\right] F''_{12}\frac{L^\ell}{w^\ell}}{\frac{\left(L^\ell\right)^2}{F'_1}\left[2\frac{e^\ell}{L^\ell}\frac{1}{L^\ell} - \frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell}\right] F''_{11}} = \frac{F''_{12}}{e^\ell L^\ell F''_{11}} = -\frac{N^\ell}{N^h L^h}. \tag{A4}$$

In the derivation of equation (A4), we have exploited the fact that $F''_{11} = -\left(N^h L^h F''_{12}\right) / \left(e^\ell N^\ell L^\ell\right)$ due to the assumption that the production function is linearly homogeneous.

Proceeding in a similar way, the comparative statics results with respect to B^ℓ are obtained by solving the system

$$\begin{bmatrix} 0 & -\frac{\partial^2 e^\ell}{\partial L^\ell \partial L^\ell}\frac{I^\ell}{\left(w^\ell\right)^2} + 2\frac{e^\ell}{L^\ell}\frac{1}{L^\ell}\frac{I^\ell}{\left(w^\ell\right)^2} & 0 \\ \frac{I^\ell}{w^\ell}e^\ell F''_{11} & -\frac{1}{e^\ell} - \frac{w^\ell}{\left(e^\ell\right)^2}\frac{\partial e^\ell}{\partial L^\ell}\frac{I^\ell}{\left(w^\ell\right)^2} & -\frac{I^h}{\left(w^h\right)^2}N^h F''_{12} \\ e^\ell \frac{I^\ell}{w^\ell}F''_{12} & 0 & -N^h\frac{I^h}{\left(w^h\right)^2}F''_{22} - 1 \end{bmatrix} \begin{bmatrix} \frac{dN^\ell}{dB^\ell} \\ \frac{dw^\ell}{dB^\ell} \\ \frac{dw^h}{dB^\ell} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\partial^2 e^\ell}{\partial L^\ell \partial B^\ell} - \frac{1}{L^\ell}\frac{\partial e^\ell}{\partial B^\ell} \\ -\frac{\partial e^\ell}{\partial B^\ell}\left(L^\ell N^\ell F''_{11} + \frac{w^\ell}{\left(e^\ell\right)^2}\right) \\ -\frac{\partial e^\ell}{\partial B^\ell}L^\ell N^\ell F''_{12} \end{bmatrix}.$$

The comparative statics results with respect to the remaining control variables in the social decision-problem, i.e., I^h, B^h, u, and b are obtained starting from the system (A1) and following a similar procedure.

Appendix B

First order conditions of the social decision-problem: The first order conditions with respect to I^h, B^h, I^ℓ, B^ℓ, u, and b can be written as

$$(\delta + \lambda + \zeta)\frac{\partial V^h\left(B^h, \frac{I^h}{w^h}\right)}{\partial I^h} = -\mu\left[e^\ell \frac{I^\ell}{w^\ell}\frac{dN^\ell}{dI^h}F'_1 + \frac{N^h}{w^h}F'_2 - \frac{dN^\ell}{dI^h}\left(B^\ell - b\right)\right] - \frac{\theta}{\pi^\ell}\frac{dN^\ell}{dI^h}, \tag{B1}$$

$$(\delta+\lambda+\zeta)\,\frac{\partial V^h\left(B^h,\frac{I^h}{w^h}\right)}{\partial B^h}=\mu\pi^h, \tag{B2}$$

$$\begin{aligned}
\frac{\partial EV^\ell}{\partial I^\ell} &= -\frac{\partial EV^\ell}{\partial w^\ell}\frac{dw^\ell}{dI^\ell}-(\delta+\lambda+\zeta)\,\frac{\partial V^h\left(B^h,\frac{I^h}{w^h}\right)}{\partial w^h}\frac{dw^h}{dI^\ell}+\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial I^\ell}\\
&+\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial w^h}\frac{dw^h}{dI^\ell}+\zeta\left[\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial I^\ell}+\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial w^\ell}\frac{dw^\ell}{dI^\ell}\right]\\
&-\mu\left\{L^\ell\left[N^\ell\frac{\partial e^\ell}{\partial L^\ell}\left(\frac{1}{w^\ell}-\frac{L^\ell}{w^\ell}\frac{dw^\ell}{dI^\ell}\right)+e^\ell\frac{dN^\ell}{dI^\ell}\right]+e^\ell N^\ell\left(\frac{1}{w^\ell}-\frac{L^\ell}{w^\ell}\frac{dw^\ell}{dI^\ell}\right)\right\}F_1'\\
&+\mu N^h\frac{L^h}{w^h}\frac{dw^h}{dI^\ell}F_2'+\mu\frac{dN^\ell}{dI^\ell}\left[B^\ell-b\right]-\frac{\theta}{\pi^\ell}\frac{dN^\ell}{dI^\ell},
\end{aligned} \tag{B3}$$

$$\begin{aligned}
\frac{\partial EV^\ell}{\partial B^\ell} &= -\frac{\partial EV^\ell}{\partial w^\ell}\frac{dw^\ell}{dB^\ell}-(\delta+\lambda+\zeta)\,\frac{\partial V^h\left(B^h,\frac{I^h}{w^h}\right)}{\partial w^h}\frac{dw^h}{dB^\ell}+\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial B^\ell}\\
&+\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial w^h}\frac{dw^h}{dB^\ell}+\zeta\left[\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial B^\ell}+\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial w^\ell}\frac{dw^\ell}{dB^\ell}\right]\\
&-\mu\left\{L^\ell\left[N^\ell\left(\frac{\partial e^\ell}{\partial B^\ell}-\frac{\partial e^\ell}{\partial L^\ell}\frac{L^\ell}{w^\ell}\frac{dw^\ell}{dB^\ell}\right)+e^\ell\frac{dN^\ell}{dB^\ell}\right]-e^\ell N^\ell\frac{L^\ell}{w^\ell}\frac{dw^\ell}{dB^\ell}\right\}F_1'\\
&+\mu N^h\frac{L^h}{w^h}\frac{dw^h}{dB^\ell}F_2'+\mu\frac{dN^\ell}{dB^\ell}\left[B^\ell-b\right]+\mu N^\ell-\frac{\theta}{\pi^\ell}\frac{dN^\ell}{dB^\ell},
\end{aligned} \tag{B4}$$

$$\begin{aligned}
&\frac{\partial EV^\ell}{\partial u}+\frac{\partial EV^\ell}{\partial w^\ell}\frac{dw^\ell}{du}+(\delta+\lambda+\zeta)\,\frac{\partial V^h}{\partial w^h}\frac{dw^h}{du}-\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial w^h}\frac{dw^h}{du}\\
&-\zeta\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial w^\ell}\frac{dw^\ell}{du}\\
&+\mu\left\{L^\ell\left[N^\ell\left(\frac{\partial e^\ell}{\partial u}-\frac{\partial e^\ell}{\partial L^\ell}\frac{L^\ell}{w^\ell}\frac{dw^\ell}{du}\right)+e^\ell\frac{dN^\ell}{du}\right]-e^\ell N^\ell\frac{L^\ell}{w^\ell}\frac{dw^\ell}{du}\right\}F_1'\\
&-\mu N^h\frac{L^h}{w^h}\frac{dw^h}{du}F_2'-\mu\frac{dN^\ell}{du}\left[B^\ell-b\right]+\theta\left[1+\frac{1}{\pi^\ell}\frac{dN^\ell}{du}\right]\\
=\;\;&0,
\end{aligned} \tag{B5}$$

$$
\begin{aligned}
& \frac{\partial EV^{\ell}}{\partial b} + \frac{\partial EV^{\ell}}{\partial w^{\ell}}\frac{dw^{\ell}}{db} + (\delta+\lambda+\zeta)\frac{\partial V^{h}}{\partial w^{h}}\frac{dw^{h}}{db} - \lambda\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{h}}\right)}{\partial w^{h}}\frac{dw^{h}}{db} \\
& -\zeta\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{\ell}},e=0\right)}{\partial w^{\ell}}\frac{dw^{\ell}}{db} \\
& +\mu\left\{L^{\ell}\left[N^{\ell}\left(\frac{\partial e^{\ell}}{\partial b} - \frac{\partial e^{\ell}}{\partial L^{\ell}}\frac{L^{\ell}}{w^{\ell}}\frac{dw^{\ell}}{db}\right) + e^{\ell}\frac{dN^{\ell}}{db}\right] - e^{\ell}N^{\ell}\frac{L^{\ell}}{w^{\ell}}\frac{dw^{\ell}}{db}\right\}F_1' \\
& -\mu N^{h}\frac{L^{h}}{w^{h}}\frac{dw^{h}}{db}F_2' - \mu\frac{dN^{\ell}}{db}\left[B^{\ell}-b\right] - \mu\left(\pi^{\ell}-N^{\ell}\right) + \frac{\theta}{\pi^{\ell}}\frac{dN^{\ell}}{db} \\
= \; & 0. \qquad \text{(B6)}
\end{aligned}
$$

Appendix C

Proof of Proposition 1: Let us start by deriving equation (28). Combining equations (B1) and (B2) gives

$$
\frac{\frac{\partial V^{h}\left(B^{h},\frac{I^{h}}{w^{h}}\right)}{\partial I^{h}}}{\frac{\partial V^{h}\left(B^{h},\frac{I^{h}}{w^{h}}\right)}{\partial B^{h}}}\mu\pi^{h} = -\mu\left[e^{\ell}\frac{I^{\ell}}{w^{\ell}}\frac{dN^{\ell}}{dI^{h}}F_1' + \frac{N^{h}}{w^{h}}F_2' - \frac{dN^{\ell}}{dI^{h}}\left(B^{\ell}-b\right)\right] - \frac{\theta}{\pi^{\ell}}\frac{dN^{\ell}}{dI^{h}}. \qquad \text{(C1)}
$$

Taking into account that $F_1' = w^{\ell}/e^{\ell}$, $F_2' = w^{h}$, and $N^{h} = \pi^{h}$, we can rewrite equation (C1) such that

$$
\left[1 + \frac{\frac{\partial V^{h}\left(B^{h},\frac{I^{h}}{w^{h}}\right)}{\partial I^{h}}}{\frac{\partial V^{h}\left(B^{h},\frac{I^{h}}{w^{h}}\right)}{\partial B^{h}}}\right]\mu\pi^{h} = -\left[\left(I^{\ell}-B^{\ell}+b\right)\mu + \theta\frac{1}{\pi^{\ell}}\right]\frac{dN^{\ell}}{dI^{h}}. \qquad \text{(C2)}
$$

The result stated in the first part of Proposition 1 can then be obtained by dividing both sides of equation (C2)by $\mu\pi^{h}$, using (10), and taking into account that $\frac{dN^{\ell}}{dI^{h}} = \frac{N^{\ell}}{w^{h}L^{h}}$ (see (15).

Let us now turn to equation (29). Using equations (12)-(14), and the fact that

$\frac{dw^\ell}{dI^\ell} = \frac{1}{L^\ell}$, we can simplify (B3)-(B4) and rewrite them to read

$$\begin{aligned}
\frac{\partial EV^\ell}{\partial I^\ell} &= -\frac{\partial EV^\ell}{\partial w^\ell}\frac{dw^\ell}{dI^\ell} - (\delta+\lambda+\zeta)\frac{\partial V^h\left(B^h, \frac{I^h}{w^h}\right)}{\partial w^h}\frac{dw^h}{dI^\ell} \\
&+\lambda\frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^h}\right)}{\partial I^\ell} + \lambda\frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^h}\right)}{\partial w^h}\frac{dw^h}{dI^\ell} \\
&+\zeta\left[\frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^\ell}, e=0\right)}{\partial I^\ell} + \frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^\ell}, e=0\right)}{\partial w^\ell}\frac{dw^\ell}{dI^\ell}\right] \\
&+\mu N^h L^h \frac{dw^h}{dI^\ell} + \mu\frac{dN^\ell}{dI^\ell}\left[-I^\ell + B^\ell - b\right] - \frac{\theta}{\pi^\ell}\frac{dN^\ell}{dI^\ell},
\end{aligned} \tag{C3}$$

$$\begin{aligned}
\frac{\partial EV^\ell}{\partial B^\ell} &= -\frac{\partial EV^\ell}{\partial w^\ell}\frac{dw^\ell}{dB^\ell} - (\delta+\lambda+\zeta)\frac{\partial V^h\left(B^h, \frac{I^h}{w^h}\right)}{\partial w^h}\frac{dw^h}{dB^\ell} \\
&+\lambda\frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^h}\right)}{\partial B^\ell} + \lambda\frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^h}\right)}{\partial w^h}\frac{dw^h}{dB^\ell} \\
&+\zeta\left[\frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^\ell}, e=0\right)}{\partial B^\ell} + \frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^\ell}, e=0\right)}{\partial w^\ell}\frac{dw^\ell}{dB^\ell}\right] \\
&-\mu L^\ell N^\ell \frac{\partial e^\ell}{\partial B^\ell}\frac{w^\ell}{e^\ell} + \mu N^h L^h \frac{dw^h}{dB^\ell} + \mu\frac{dN^\ell}{dB^\ell}\left[-I^\ell + B^\ell - b\right] + \mu N^\ell - \frac{\theta}{\pi^\ell}\frac{dN^\ell}{dB^\ell}.
\end{aligned} \tag{C4}$$

By noting that $-\frac{\partial V^h\left(B^h, \frac{I^h}{w^h}\right)}{\partial w^h} = \frac{\partial V^h\left(B^h, \frac{I^h}{w^h}\right)}{\partial I^h}L^h$, we can rewrite (B2) as

$$(\delta+\lambda+\zeta)\frac{\partial V^h\left(B^h, \frac{I^h}{w^h}\right)}{\partial B^h}\frac{\partial V^h\left(B^h, \frac{I^h}{w^h}\right)}{\partial I^h}L^h / \frac{\partial V^h\left(B^h, \frac{I^h}{w^h}\right)}{\partial B^h} = -\mu\pi^h MRS^h_{I,B}L^h,$$

and substitute $-\mu\pi^h MRS^h_{I,B}L^h$ for $-(\delta+\lambda+\zeta)\frac{\partial V^h\left(B^h, \frac{I^h}{w^h}\right)}{\partial w^h}$ in (C3)-(C4). We can then rewrite equations (C3) and (C4) as follows:

$$
\begin{aligned}
&\frac{\partial EV^{\ell}}{\partial I^{\ell}} = -\frac{\partial EV^{\ell}}{\partial w^{\ell}}\frac{dw^{\ell}}{dI^{\ell}} - \mu\pi^{h} MRS^{h}_{I,B} L^{h}\frac{dw^{h}}{dI^{\ell}} \\
&+\lambda\frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{h}}\right)}{\partial I^{\ell}} + \lambda\frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{h}}\right)}{\partial w^{h}}\frac{dw^{h}}{dI^{\ell}} \\
&+\zeta\left[\frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{\ell}}, e=0\right)}{\partial I^{\ell}} + \frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{\ell}}, e=0\right)}{\partial w^{\ell}}\frac{dw^{\ell}}{dI^{\ell}}\right] \\
&+\mu N^{h} L^{h}\frac{dw^{h}}{dI^{\ell}} + \mu\frac{dN^{\ell}}{dI^{\ell}}\left[-I^{\ell} + B^{\ell} - b\right] - \frac{\theta}{\pi^{\ell}}\frac{dN^{\ell}}{dI^{\ell}},
\end{aligned}
\tag{C5}
$$

$$
\begin{aligned}
&\frac{\partial EV^{\ell}}{\partial B^{\ell}} = -\frac{\partial EV^{\ell}}{\partial w^{\ell}}\frac{dw^{\ell}}{dB^{\ell}} - \mu\pi^{h} MRS^{h}_{I,B} L^{h}\frac{dw^{h}}{dB^{\ell}} \\
&+\lambda\frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{h}}\right)}{\partial B^{\ell}} + \lambda\frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{h}}\right)}{\partial w^{h}}\frac{dw^{h}}{dB^{\ell}} \\
&+\zeta\left[\frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{\ell}}, e=0\right)}{\partial B^{\ell}} + \frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{\ell}}, e=0\right)}{\partial w^{\ell}}\frac{dw^{\ell}}{dB^{\ell}}\right] \\
&+\mu N^{\ell} - \mu L^{\ell} N^{\ell}\frac{\partial e^{\ell}}{\partial B^{\ell}}\frac{w^{\ell}}{e^{\ell}} + \mu N^{h} L^{h}\frac{dw^{h}}{dB^{\ell}} + \mu\frac{dN^{\ell}}{dB^{\ell}}\left[-I^{\ell} + B^{\ell} - b\right] - \frac{\theta}{\pi^{\ell}}\frac{dN^{\ell}}{dB^{\ell}}.
\end{aligned}
\tag{C6}
$$

Dividing (C5) by (C6) and multiplying by the right hand side of (C6) gives the following expression:

$$
\begin{aligned}
&\frac{\frac{\partial EV^\ell}{\partial I^\ell}}{\frac{\partial EV^\ell}{\partial B^\ell}}\left\{-\frac{\partial EV^\ell}{\partial w^\ell}\frac{dw^\ell}{dB^\ell}+\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial B^\ell}+\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial w^h}\frac{dw^h}{dB^\ell}\right\}\\
&+\frac{\frac{\partial EV^\ell}{\partial I^\ell}}{\frac{\partial EV^\ell}{\partial B^\ell}}\zeta\left[\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial B^\ell}+\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial w^\ell}\frac{dw^\ell}{dB^\ell}\right]-\frac{\frac{\partial EV^\ell}{\partial I^\ell}}{\frac{\partial EV^\ell}{\partial B^\ell}}\mu\pi^h MRS^h_{I,B}L^h\frac{dw^h}{dB^\ell}\\
&+\frac{\frac{\partial EV^\ell}{\partial I^\ell}}{\frac{\partial EV^\ell}{\partial B^\ell}}\left\{\mu N^\ell-\mu L^\ell N^\ell\frac{\partial e^\ell}{\partial B^\ell}\frac{w^\ell}{e^\ell}+\mu N^h L^h\frac{dw^h}{dB^\ell}+\mu\frac{dN^\ell}{dB^\ell}\left[-I^\ell+B^\ell-b\right]-\frac{\theta}{\pi^\ell}\frac{dN^\ell}{dB^\ell}\right\}\\
=\;&-\frac{\partial EV^\ell}{\partial w^\ell}\frac{dw^\ell}{dI^\ell}+\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial I^\ell}+\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial w^h}\frac{dw^h}{dI^\ell}\\
&+\zeta\left[\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial I^\ell}+\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial w^\ell}\frac{dw^\ell}{dI^\ell}\right]\\
&-\mu\pi^h MRS^h_{I,B}L^h\frac{dw^h}{dI^\ell}+\mu N^h L^h\frac{dw^h}{dI^\ell}+\mu\frac{dN^\ell}{dI^\ell}\left[-I^\ell+B^\ell-b\right]-\frac{\theta}{\pi^\ell}\frac{dN^\ell}{dI^\ell}. \qquad \text{(C7)}
\end{aligned}
$$

Rearranging and collecting terms give

$$
\begin{aligned}
&-\mu N^h L^h\frac{dw^h}{dI^\ell}+\frac{\frac{\partial EV^\ell}{\partial I^\ell}}{\frac{\partial EV^\ell}{\partial B^\ell}}\mu N^\ell\\
=\;&-\frac{\partial EV^\ell}{\partial w^\ell}\frac{dw^\ell}{dI^\ell}+\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial I^\ell}+\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial w^h}\frac{dw^h}{dI^\ell}\\
&+\zeta\left[\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial I^\ell}+\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial w^\ell}\frac{dw^\ell}{dI^\ell}\right]\\
&-\mu\pi^h MRS^h_{I,B}L^h\left[\frac{dw^h}{dI^\ell}+MRS^\ell_{I,B}\frac{dw^h}{dB^\ell}\right]+\mu\frac{dN^\ell}{dI^\ell}\left[-I^\ell+B^\ell-b\right]-\frac{\theta}{\pi^\ell}\frac{dN^\ell}{dI^\ell}\\
&+MRS^\ell_{I,B}\left\{-\frac{\partial EV^\ell}{\partial w^\ell}\frac{dw^\ell}{dB^\ell}+\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial B^\ell}+\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial w^h}\frac{dw^h}{dB^\ell}\right\}\\
&+MRS^\ell_{I,B}\zeta\left[\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial B^\ell}+\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial w^\ell}\frac{dw^\ell}{dB^\ell}\right] \qquad \text{(C8)}\\
&+MRS^\ell_{I,B}\left\{-\mu L^\ell N^\ell\frac{\partial e^\ell}{\partial B^\ell}\frac{w^\ell}{e^\ell}+\mu N^h L^h\frac{dw^h}{dB^\ell}+\mu\frac{dN^\ell}{dB^\ell}\left[-I^\ell+B^\ell-b\right]-\frac{\theta}{\pi^\ell}\frac{dN^\ell}{dB^\ell}\right\}.
\end{aligned}
$$

Since $\frac{dw^h}{dI^\ell} = -\frac{N^\ell}{N^h L^h}$, the left hand side of equation (C8) can be written as $\mu N^\ell \left(1 + \frac{\frac{\partial EV^\ell}{\partial I^\ell}}{\frac{\partial EV^\ell}{\partial B^\ell}}\right)$. Thus, using (9) and (25)-(26), we have

$$
\begin{aligned}
&\mu N^\ell T'\left(I^\ell\right) \\
= &\left[\zeta \frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^\ell}, e=0\right)}{\partial w^\ell} - \frac{\partial EV^\ell}{\partial w^\ell}\right] \frac{d\widetilde{w}^\ell}{dI^\ell} + \zeta \frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^\ell}, e=0\right)}{\partial B^\ell} \left[MRS_{I,B}^\ell - \widehat{MRS}_{I,B}^{h,e=0}\right] \\
&+\lambda \frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^h}\right)}{\partial B^\ell} \left[MRS_{I,B}^\ell - \widehat{MRS}_{I,B}^{h}\right] + \lambda \frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^h}\right)}{\partial w^h} \frac{d\widetilde{w}^h}{dI^\ell} - \frac{\theta}{\pi^\ell} \frac{d\widetilde{N}^\ell}{dI^\ell} \\
&-\mu \pi^h MRS_{I,B}^h L^h \frac{d\widetilde{w}^h}{dI^\ell} - \mu \frac{d\widetilde{N}^\ell}{dI^\ell} \left[I^\ell - B^\ell + b\right] + \mu MRS_{I,B}^\ell \left\{N^h L^h \frac{dw^h}{dB^\ell} - L^\ell N^\ell \frac{\partial e^\ell}{\partial B^\ell} \frac{w^\ell}{e^\ell}\right\}.
\end{aligned}
\tag{C9}
$$

By recognizing that $\frac{dw^h}{dB^\ell} = \frac{N^\ell L^\ell \frac{\partial e^\ell}{\partial B^\ell} \frac{w^\ell}{e^\ell}}{N^h L^h}$ and $\frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^h}\right)}{\partial w^h} = -\frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^h}\right)}{\partial I^\ell} \frac{I^\ell}{w^h}$, we can derive the following expression for the marginal income tax rate:

$$
\begin{aligned}
T'\left(I^\ell\right) = &\frac{1}{\mu(1-u)\pi^\ell} \left[\zeta \frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^\ell}, e=0\right)}{\partial w^\ell} - \frac{\partial EV^\ell}{\partial w^\ell}\right] \frac{d\widetilde{w}^\ell}{dI^\ell} \\
&+ \frac{\zeta \frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^\ell}, e=0\right)}{\partial B^\ell}}{\mu(1-u)\pi^\ell} \left[MRS_{I,B}^\ell - \widehat{MRS}_{I,B}^{h,e=0}\right] \\
&+ \frac{\lambda \frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^h}\right)}{\partial B^\ell}}{\mu(1-u)\pi^\ell} \left[MRS_{I,B}^\ell - \widehat{MRS}_{I,B}^{h} \left(1 - \frac{I^\ell}{w^h} \frac{d\widetilde{w}^h}{dI^\ell}\right)\right] - \frac{\theta}{\mu\pi^\ell} \frac{1}{N^\ell} \frac{d\widetilde{N}^\ell}{dI^\ell} \\
&- \frac{N^h}{N^\ell} MRS_{I,B}^h L^h \frac{d\widetilde{w}^h}{dI^\ell} - \frac{1}{N^\ell} \frac{d\widetilde{N}^\ell}{dI^\ell} \left[I^\ell - B^\ell + b\right],
\end{aligned}
\tag{C10}
$$

or, equivalently, in elasticity form:

$$T'\left(I^{\ell}\right)=\frac{1}{\mu\left(1-u\right)\pi^{\ell}}\left[\zeta\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{\ell}},e=0\right)}{\partial w^{\ell}}-\frac{\partial EV^{\ell}}{\partial w^{\ell}}\right]\frac{d\widetilde{w}^{\ell}}{dI^{\ell}}$$
$$+\frac{\zeta\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{\ell}},e=0\right)}{\partial B^{\ell}}}{\mu\left(1-u\right)\pi^{\ell}}\left[MRS_{I,B}^{\ell}-\widehat{MRS}_{I,B}^{h,e=0}\right]$$
$$+\frac{\lambda\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{h}}\right)}{\partial B^{\ell}}}{\mu\left(1-u\right)\pi^{\ell}}\left[MRS_{I,B}^{\ell}-\widehat{MRS}_{I,B}^{h}\left(1-\frac{I^{\ell}}{w^{h}}\frac{d\widetilde{w}^{h}}{dI^{\ell}}\right)\right]$$
$$-\frac{\widetilde{\epsilon}_{N^{\ell},I^{\ell}}}{I^{\ell}}\left[\left(I^{\ell}-B^{\ell}+b\right)+\frac{\theta}{\mu\pi^{\ell}}\right]-\frac{N^{h}I^{h}}{N^{\ell}I^{\ell}}MRS_{I,B}^{h}\widetilde{\epsilon}_{w^{h},I^{\ell}}. \qquad \text{(C11)}$$

Finally, by using the comparative statics results for $\frac{dN^{\ell}}{dI^{\ell}}$, $\frac{dN^{\ell}}{dB^{\ell}}$, $\frac{dw^{h}}{dI^{\ell}}$, and $\frac{dw^{h}}{dB^{\ell}}$ together with equations (4)-(5) and (12), we can derive the following comparative statics results for $\widetilde{N}^{\ell}$ and $\widetilde{w}^{h}$:

$$\begin{aligned}
\frac{d\widetilde{N}^{\ell}}{dI^{\ell}} &= \frac{dN^{\ell}}{dI^{\ell}}+\frac{dN^{\ell}}{dB^{\ell}}MRS_{I,B}^{\ell}=\frac{1-\frac{1}{\sigma}}{\left(e^{\ell}L^{\ell}\right)^{2}F_{11}''}-\frac{N^{\ell}}{I^{\ell}}\\
&+MRS_{IB}^{\ell}\frac{\frac{1}{\sigma}-1}{\left(e^{\ell}L^{\ell}\right)^{2}F_{11}''}\left[MRS_{IB}^{\ell}+\frac{\frac{\partial^{2}v^{\ell}}{\partial L\partial e}\left[1-u\varphi\left(e^{\ell}\right)\right]}{w^{\ell}\frac{\partial v^{\ell}}{\partial B}u\varphi'\left(e^{\ell}\right)}\right]^{-1}\\
&= \frac{1-\frac{1}{\sigma}}{\left(e^{\ell}L^{\ell}\right)^{2}F_{11}''}\left[1-\frac{MRS_{IB}^{\ell}w^{\ell}\frac{\partial v^{\ell}}{\partial B}u\varphi'\left(e^{\ell}\right)}{MRS_{IB}^{\ell}w^{\ell}\frac{\partial v^{\ell}}{\partial B}u\varphi'\left(e^{\ell}\right)+\frac{\partial^{2}v^{\ell}}{\partial L\partial e}\left[1-u\varphi\left(e^{\ell}\right)\right]}\right]-\frac{N^{\ell}}{I^{\ell}}\\
&= \frac{1-\frac{1}{\sigma}}{\left(e^{\ell}L^{\ell}\right)^{2}F_{11}''}\left[\frac{\frac{\partial^{2}v^{\ell}}{\partial L\partial e}\left[1-u\varphi\left(e^{\ell}\right)\right]}{MRS_{IB}^{\ell}w^{\ell}\frac{\partial v^{\ell}}{\partial B}u\varphi'\left(e^{\ell}\right)+\frac{\partial^{2}v^{\ell}}{\partial L\partial e}\left[1-u\varphi\left(e^{\ell}\right)\right]}\right]-\frac{N^{\ell}}{I^{\ell}} \qquad \text{(C12)}
\end{aligned}$$

$$
\begin{aligned}
\frac{d\widetilde{w}^h}{dI^\ell} &= \frac{dw^h}{dI^\ell} + \frac{dw^h}{dB^\ell} MRS^\ell_{I,B} = -\frac{N^\ell}{N^h L^h} + \frac{N^\ell}{N^h L^h} \frac{w^\ell \frac{\partial v^\ell}{\partial B} u\varphi'\left(e^\ell\right)}{w^\ell \frac{\partial v^\ell}{\partial B} u\varphi'\left(e^\ell\right) + \frac{\frac{\partial^2 v^\ell}{\partial L \partial e}}{MRS^\ell_{IB}} \left[1 - u\varphi\left(e^\ell\right)\right]} \\
&= \frac{N^\ell}{N^h L^h} \frac{w^\ell \frac{\partial v^\ell}{\partial B} u\varphi'\left(e^\ell\right) - w^\ell \frac{\partial v^\ell}{\partial B} u\varphi'\left(e^\ell\right) - \frac{\frac{\partial^2 v^\ell}{\partial L \partial e}}{MRS^\ell_{IB}} \left[1 - u\varphi\left(e^\ell\right)\right]}{w^\ell \frac{\partial v^\ell}{\partial B} u\varphi'\left(e^\ell\right) + \frac{\frac{\partial^2 v^\ell}{\partial L \partial e}}{MRS^\ell_{IB}} \left[1 - u\varphi\left(e^\ell\right)\right]} \\
&= -\frac{N^\ell}{N^h L^h} \frac{\frac{\frac{\partial^2 v^\ell}{\partial L \partial e}}{MRS^\ell_{IB}} \left[1 - u\varphi\left(e^\ell\right)\right]}{w^\ell \frac{\partial v^\ell}{\partial B} u\varphi'\left(e^\ell\right) + \frac{\frac{\partial^2 v^\ell}{\partial L \partial e}}{MRS^\ell_{IB}} \left[1 - u\varphi\left(e^\ell\right)\right]} < 0. \qquad \text{(C13)}
\end{aligned}
$$

Appendix D

Proof of Proposition 2: Multiply equation (B6) by $MRS^\ell_{ub} = -\left(\partial EV^\ell / \partial u\right) / \left(\partial EV^\ell / \partial b\right)$ and then add the resulting equation to (B5). We obtain

$$
\begin{aligned}
& \frac{\partial EV^\ell}{\partial b} MRS^\ell_{u,b} + MRS^\ell_{u,b} \frac{\partial EV^\ell}{\partial w^\ell} \frac{dw^\ell}{db} + MRS^\ell_{u,b} \left(\delta + \lambda + \zeta\right) \frac{\partial V^h}{\partial w^h} \frac{dw^h}{db} \\
& -\lambda \frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^h}\right)}{\partial w^h} \frac{dw^h}{db} MRS^\ell_{u,b} - \zeta \frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^\ell}, e = 0\right)}{\partial w^\ell} \frac{dw^\ell}{db} MRS^\ell_{u,b} \\
& +\mu \left\{ L^\ell \left[N^\ell \left(\frac{\partial e^\ell}{\partial b} - \frac{\partial e^\ell}{\partial L^\ell} \frac{L^\ell}{w^\ell} \frac{dw^\ell}{db} \right) + e^\ell \frac{dN^\ell}{db} \right] - e^\ell N^\ell \frac{L^\ell}{w^\ell} \frac{dw^\ell}{db} \right\} F_1' MRS^\ell_{u,b} \\
& -\mu N^h \frac{L^h}{w^h} \frac{dw^h}{db} F_2' MRS^\ell_{u,b} - \mu \frac{dN^\ell}{db} \left[B^\ell - b\right] MRS^\ell_{u,b} - \mu \left(\pi^\ell - N^\ell\right) MRS^\ell_{u,b} \\
& +\frac{\theta}{\pi^\ell} \frac{dN^\ell}{db} MRS^\ell_{u,b} + \frac{\partial EV^\ell}{\partial u} + \frac{\partial EV^\ell}{\partial w^\ell} \frac{dw^\ell}{du} + \left(\delta + \lambda + \zeta\right) \frac{\partial V^h}{\partial w^h} \frac{dw^h}{du} \\
& -\lambda \frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^h}\right)}{\partial w^h} \frac{dw^h}{du} - \zeta \frac{\partial V^h\left(B^\ell, \frac{I^\ell}{w^\ell}, e = 0\right)}{\partial w^\ell} \frac{dw^\ell}{du} \\
& +\mu \left\{ L^\ell \left[N^\ell \left(\frac{\partial e^\ell}{\partial u} - \frac{\partial e^\ell}{\partial L^\ell} \frac{L^\ell}{w^\ell} \frac{dw^\ell}{du} \right) + e^\ell \frac{dN^\ell}{du} \right] - e^\ell N^\ell \frac{L^\ell}{w^\ell} \frac{dw^\ell}{du} \right\} F_1' \\
& -\mu N^h \frac{L^h}{w^h} \frac{dw^h}{du} F_2' - \mu \frac{dN^\ell}{du} \left[B^\ell - b\right] + \theta \left[1 + \frac{1}{\pi^\ell} \frac{dN^\ell}{du}\right] \\
= \; & 0. \qquad \text{(D1)}
\end{aligned}
$$

By simplifying, rearranging terms, and defining

$$\begin{aligned}\partial\widetilde{e}^{\ell}/\partial u &= \partial e^{\ell}/\partial u + MRS^{\ell}_{ub}\partial e^{\ell}/\partial b, \quad d\widetilde{N}^{\ell}/du = dN^{\ell}/du + MRS^{\ell}_{ub}dN^{\ell}/db,\\ d\widetilde{w}^{\ell}/du &= dw^{\ell}/du + MRS^{\ell}_{ub}dw^{\ell}/db, \quad d\widetilde{w}^{h}/du = dw^{h}/du + MRS^{\ell}_{ub}dw^{h}/db,\end{aligned}$$

we can rewrite (D1) to read

$$\begin{aligned}&\theta\left[1+\frac{1}{\pi^{\ell}}\frac{d\widetilde{N}^{\ell}}{du}\right]+(\delta+\lambda+\zeta)\frac{\partial V^{h}}{\partial w^{h}}\frac{d\widetilde{w}^{h}}{du}\\ =\;& \mu u\pi^{\ell}MRS^{\ell}_{u,b}+\mu N^{h}L^{h}\frac{d\widetilde{w}^{h}}{du}+\mu\frac{d\widetilde{N}^{\ell}}{du}\left[B^{\ell}-b\right]\\ &-\mu\left\{L^{\ell}\left[N^{\ell}\left(\frac{\partial\widetilde{e}^{\ell}}{\partial u}-\frac{\partial e^{\ell}}{\partial L^{\ell}}\frac{L^{\ell}}{w^{\ell}}\frac{d\widetilde{w}^{\ell}}{du}\right)+e^{\ell}\frac{d\widetilde{N}^{\ell}}{du}\right]-e^{\ell}N^{\ell}\frac{L^{\ell}}{w^{\ell}}\frac{d\widetilde{w}^{\ell}}{du}\right\}F_{1}'\\ &-\left[\frac{\partial EV^{\ell}}{\partial w^{\ell}}\frac{d\widetilde{w}^{\ell}}{du}-\lambda\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{h}}\right)}{\partial w^{h}}\frac{d\widetilde{w}^{h}}{du}-\zeta\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{\ell}},e=0\right)}{\partial w^{\ell}}\frac{d\widetilde{w}^{\ell}}{du}\right]. \quad \text{(D2)}\end{aligned}$$

Using the social first order condition with respect to B^h, i.e., equation (B2), to substitute $-\mu\pi^{h}MRS^{h}_{I,B}L^{h}$ for $-(\delta+\lambda+\zeta)\frac{\partial V^{h}\left(B^{h},\frac{I^{h}}{w^{h}}\right)}{\partial w^{h}}$, equation (D2) can be rewritten as

$$\begin{aligned}&\theta\left[1+\frac{1}{\pi^{\ell}}\frac{d\widetilde{N}^{\ell}}{du}\right]\\ =\;& -\mu\pi^{h}MRS^{h}_{I,B}L^{h}\frac{d\widetilde{w}^{h}}{du}+\mu u\pi^{\ell}MRS^{\ell}_{u,b}+\mu N^{h}L^{h}\frac{d\widetilde{w}^{h}}{du}+\mu\frac{d\widetilde{N}^{\ell}}{du}\left[B^{\ell}-b\right]\\ &-\mu\left\{L^{\ell}\left[N^{\ell}\left(\frac{\partial\widetilde{e}^{\ell}}{\partial u}-\frac{\partial e^{\ell}}{\partial L^{\ell}}\frac{L^{\ell}}{w^{\ell}}\frac{d\widetilde{w}^{\ell}}{du}\right)+e^{\ell}\frac{d\widetilde{N}^{\ell}}{du}\right]-e^{\ell}N^{\ell}\frac{L^{\ell}}{w^{\ell}}\frac{d\widetilde{w}^{\ell}}{du}\right\}F_{1}'\\ &-\left[\frac{\partial EV^{\ell}}{\partial w^{\ell}}\frac{d\widetilde{w}^{\ell}}{du}-\lambda\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{h}}\right)}{\partial w^{h}}\frac{d\widetilde{w}^{h}}{du}-\zeta\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{\ell}},e=0\right)}{\partial w^{\ell}}\frac{d\widetilde{w}^{\ell}}{du}\right]. \quad \text{(D3)}\end{aligned}$$

Next, using equations (12)-(13), and noticing that $\frac{d\widetilde{w}^{h}}{du}=\frac{w^{\ell}}{e^{\ell}}\frac{N^{\ell}L^{\ell}}{N^{h}L^{h}}\frac{\partial\widetilde{e}^{\ell}}{\partial u}$, we can simplify equation (D3) such that

$$
\begin{aligned}
&\theta\left[1+\frac{1}{\pi^{\ell}}\frac{d\widetilde{N}^{\ell}}{du}\right] \\
= \;& \mu u\pi^{\ell}MRS^{\ell}_{u,b}-\mu\pi^{h}MRS^{h}_{I,B}L^{h}\frac{d\widetilde{w}^{h}}{du}-\mu\frac{d\widetilde{N}^{\ell}}{du}\left[I^{\ell}-B^{\ell}+b\right] \\
&+\lambda\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{h}}\right)}{\partial w^{h}}\frac{d\widetilde{w}^{h}}{du}+\left[\zeta\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{\ell}},e=0\right)}{\partial w^{\ell}}-\frac{\partial EV^{\ell}}{\partial w^{\ell}}\right]\frac{d\widetilde{w}^{\ell}}{du}. \qquad \text{(D4)}
\end{aligned}
$$

Finally, defining $\Gamma \equiv 1+\frac{1}{\pi^{\ell}}\left(\frac{dN^{\ell}}{du}-\frac{dN^{\ell}}{db}\frac{\frac{\partial EV^{\ell}}{\partial u}}{\frac{\partial EV^{\ell}}{\partial b}}\right)=1+\frac{1}{\pi^{\ell}}\frac{\partial\tilde{N}^{\ell}}{\partial u}$, equation (D4) can be rearranged to read

$$
\begin{aligned}
\frac{\theta}{\mu}=\;&\frac{1}{\Gamma}\left\{u\pi^{\ell}MRS^{\ell}_{u,b}-\pi^{h}MRS^{h}_{I,B}L^{h}\frac{d\widetilde{w}^{h}}{du}-\frac{d\widetilde{N}^{\ell}}{du}\left[I^{\ell}-B^{\ell}+b\right]\right\} \\
&+\frac{\lambda}{\mu\Gamma}\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{h}}\right)}{\partial w^{h}}\frac{d\widetilde{w}^{h}}{du}-\frac{1}{\mu\Gamma}\left[\frac{\partial EV^{\ell}}{\partial w^{\ell}}-\zeta\frac{\partial V^{h}\left(B^{\ell},\frac{I^{\ell}}{w^{\ell}},e=0\right)}{\partial w^{\ell}}\right]\frac{d\widetilde{w}^{\ell}}{du}, \qquad \text{(D5)}
\end{aligned}
$$

where $\frac{d\widetilde{N}^{\ell}}{du}=\frac{\left[\frac{1}{\sigma}-1\right]\frac{w^{\ell}}{e^{\ell}}L^{\ell}}{\left(e^{\ell}L^{\ell}\right)^{2}F''_{11}}\frac{\partial\widetilde{e}^{\ell}}{\partial u}$. Equation (35) in Proposition 2 is obtained by expressing the second and third terms in the first row of equation (D5) in elasticity form.

Appendix E

Proof of Proposition 3: Multiply equation (B4) by $-MRS^{\ell}_{bB}=-\left(\partial EV^{\ell}/\partial b\right)/\left(\partial EV^{\ell}/\partial B^{\ell}\right)$

and then add the resulting expression to equation (B6). This gives

$$
\begin{aligned}
& -\frac{\partial EV^{\ell}}{\partial B^{\ell}} MRS^{\ell}_{b,B} - \frac{\partial EV^{\ell}}{\partial w^{\ell}} \frac{dw^{\ell}}{dB^{\ell}} MRS^{\ell}_{b,B} - (\delta+\lambda+\zeta) \frac{\partial V^{h}\left(B^{h}, \frac{I^{h}}{w^{h}}\right)}{\partial w^{h}} \frac{dw^{h}}{dB^{\ell}} MRS^{\ell}_{b,B} \\
& +\lambda \frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{h}}\right)}{\partial B^{\ell}} MRS^{\ell}_{b,B} + \lambda \frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{h}}\right)}{\partial w^{h}} \frac{dw^{h}}{dB^{\ell}} MRS^{\ell}_{b,B} \\
& +\zeta \left[\frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{\ell}}, e=0\right)}{\partial B^{\ell}} + \frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{\ell}}, e=0\right)}{\partial w^{\ell}} \frac{dw^{\ell}}{dB^{\ell}} \right] MRS^{\ell}_{b,B} \\
& -\mu I^{\ell} \frac{dN^{\ell}}{dB^{\ell}} MRS^{\ell}_{b,B} - \mu N^{\ell} \frac{I^{\ell}}{e^{\ell}} \frac{\partial e^{\ell}}{\partial B^{\ell}} MRS^{\ell}_{b,B} \\
& +\mu N^{h} L^{h} \frac{dw^{h}}{dB^{\ell}} MRS^{\ell}_{b,B} + \mu \frac{dN^{\ell}}{dB^{\ell}} \left[B^{\ell} - b\right] MRS^{\ell}_{b,B} + \mu N^{\ell} MRS^{\ell}_{b,B} - \frac{\theta}{\pi^{\ell}} \frac{dN^{\ell}}{dB^{\ell}} MRS^{\ell}_{b,B} \\
& +\frac{\partial EV^{\ell}}{\partial b} + \frac{\partial EV^{\ell}}{\partial w^{\ell}} \frac{dw^{\ell}}{db} + (\delta+\lambda+\zeta) \frac{\partial V^{h}}{\partial w^{h}} \frac{dw^{h}}{db} - \lambda \frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{h}}\right)}{\partial w^{h}} \frac{dw^{h}}{db} \\
& -\zeta \frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{\ell}}, e=0\right)}{\partial w^{\ell}} \frac{dw^{\ell}}{db} \\
& +\mu \left\{ L^{\ell} \left[N^{\ell} \left(\frac{\partial e^{\ell}}{\partial b} - \frac{\partial e^{\ell}}{\partial L^{\ell}} \frac{L^{\ell}}{w^{\ell}} \frac{dw^{\ell}}{db} \right) + e^{\ell} \frac{dN^{\ell}}{db} \right] - e^{\ell} N^{\ell} \frac{L^{\ell}}{w^{\ell}} \frac{dw^{\ell}}{db} \right\} F_1' \\
& -\mu N^{h} \frac{L^{h}}{w^{h}} \frac{dw^{h}}{db} F_2' - \mu \frac{dN^{\ell}}{db} \left[B^{\ell} - b\right] - \mu \left(\pi^{\ell} - N^{\ell}\right) + \frac{\theta}{\pi^{\ell}} \frac{dN^{\ell}}{db} \\
= \; & 0. \qquad \text{(E1)}
\end{aligned}
$$

Simplifying and rearranging terms give

$$
\begin{aligned}
& \mu(1-u)\pi^{\ell} MRS^{\ell}_{b,B} = \mu \pi^{\ell} u + \left[\zeta \frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{\ell}}, e=0\right)}{\partial w^{\ell}} - \frac{\partial EV^{\ell}}{\partial w^{\ell}} \right] \frac{d\widetilde{w}^{\ell}}{db} - \frac{\theta}{\pi^{\ell}} \frac{d\widetilde{N}^{\ell}}{db} \\
& -\left[\lambda \frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{h}}\right)}{\partial B^{\ell}} + \zeta \frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{\ell}}, e=0\right)}{\partial B^{\ell}} \right] MRS^{\ell}_{b,B} \\
& +\lambda \frac{\partial V^{h}\left(B^{\ell}, \frac{I^{\ell}}{w^{h}}\right)}{\partial w^{h}} \frac{d\widetilde{w}^{h}}{db} - (\delta+\lambda+\zeta) \frac{\partial V^{h}\left(B^{h}, \frac{I^{h}}{w^{h}}\right)}{\partial w^{h}} \frac{d\widetilde{w}^{h}}{db} \\
& -\mu I^{\ell} N^{\ell} \frac{\partial \widetilde{e}^{\ell}}{\partial b} \frac{1}{e^{\ell}} + \mu N^{h} L^{h} \frac{d\widetilde{w}^{h}}{db} - \mu \frac{d\widetilde{N}^{\ell}}{db} \left[I^{\ell} - B^{\ell} + b\right]. \qquad \text{(E2)}
\end{aligned}
$$

By using the social first order condition for B^h in equation (B2) to substitute $-\mu\pi^h MRS^h_{I,B}L^h$ for $-(\delta+\lambda+\zeta)\frac{\partial V^h\left(B^h,\frac{I^h}{w^h}\right)}{\partial w^h}$, equation (E2) can be rewritten to read

$$\begin{aligned}
&\mu(1-u)\pi^\ell MRS^\ell_{b,B} = \mu\pi^\ell u + \left[\zeta\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial w^\ell} - \frac{\partial EV^\ell}{\partial w^\ell}\right]\frac{d\widetilde{w}^\ell}{db} - \frac{\theta}{\pi^\ell}\frac{d\widetilde{N}^\ell}{db} \\
&-\left[\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial B^\ell} + \zeta\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial B^\ell}\right]MRS^\ell_{b,B} \\
&+\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial w^h}\frac{d\widetilde{w}^h}{db} - \mu\pi^h L^h MRS^h_{I,B}\frac{d\widetilde{w}^h}{db} \\
&-\mu I^\ell N^\ell\frac{\partial\widetilde{e}^\ell}{\partial b}\frac{1}{e^\ell} + \mu N^h L^h\frac{d\widetilde{w}^h}{db} - \mu\frac{d\widetilde{N}^\ell}{db}\left[I^\ell - B^\ell + b\right].
\end{aligned} \tag{E3}$$

Therefore, since $T'\left(I^h\right) = 1 - MRS^h_{I,B}$,

$$\begin{aligned}
&(1-u)\pi^\ell MRS^\ell_{b,B} = \pi^\ell u + \frac{1}{\mu}\left[\zeta\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial w^\ell} - \frac{\partial EV^\ell}{\partial w^\ell}\right]\frac{d\widetilde{w}^\ell}{db} \\
&-\frac{MRS^\ell_{b,B}}{\mu}\left[\lambda\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial B^\ell} + \zeta\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^\ell},e=0\right)}{\partial B^\ell}\right] + \frac{\lambda}{\mu}\frac{\partial V^h\left(B^\ell,\frac{I^\ell}{w^h}\right)}{\partial w^h}\frac{d\widetilde{w}^h}{db} \\
&+\pi^h L^h T'\left(I^h\right)\frac{d\widetilde{w}^h}{db} - I^\ell N^\ell\frac{\partial\widetilde{e}^\ell}{\partial b}\frac{1}{e^\ell} - \left[I^\ell - B^\ell + b + \frac{\theta}{\mu\pi^\ell}\right]\frac{d\widetilde{N}^\ell}{db}.
\end{aligned} \tag{E4}$$

Since $\frac{d\widetilde{w}^h}{db} = \frac{dw^h}{db} - MRS^\ell_{b,B}\frac{dw^h}{dB^\ell} = \frac{\frac{w^\ell}{e^\ell}N^\ell L^\ell}{N^h L^h}\frac{\partial\widetilde{e}^\ell}{\partial b}$, the first two terms in the third row of equation (E4) can be written as

$$\pi^h L^h T'\left(I^h\right)\frac{d\widetilde{w}^h}{db} - I^\ell N^\ell\frac{\partial\widetilde{e}^\ell}{\partial b}\frac{1}{e^\ell} = -N^h L^h MRS^h_{I,B}\frac{d\widetilde{w}^h}{db} = -\pi^h L^h MRS^h_{I,B}\frac{d\widetilde{w}^h}{db}. \tag{E5}$$

Using equation (E5) to substitute in equation (E4) gives equation (40) in Proposition 3.

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