Optimal Redistributive Taxation and Provision of Public Input Goods in an Economy with Outsourcing and Unemployment***

Thomas Aronsson* and Erkki Koskela**

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Abstract
This paper concerns optimal redistributive income taxation and provision of a public input good in a two-type model with a minimum wage policy implemented for the low-ability type, where firms may use some of their resources for outsourcing by locating part of the production process abroad. Our results show that the incentive to relax the self-selection constraint and the incentive to increase employment among the low-skilled reinforce each other in terms of marginal income taxation. In addition, the appearance of equilibrium unemployment also provides an incentive for the government to directly tax outsourcing. Without a direct instrument for taxing outsourcing, the government may reduce the amount of resources spent on outsourcing by increased provision of the public input good, which is desirable in the sense that reduced outsourcing contributes to less wage inequality and increased employment.

Keywords: outsourcing, optimal nonlinear taxation, public goods, unemployment

JEL Classification: H21, H25, J31, J62.

* Department of Economics, Umeå University, SE – 901 87 Umeå, Sweden.
** Department of Economics, P.O. Box 17 (Arkadiankatu 17), 00014 University of Helsinki, Finland.
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1. Introduction

During the latest decades, outsourcing has become an increasingly important aspect of production. Outsourcing means that part of the production activity is located to another country. Large wage differences across countries are most likely important explanations for this behavior, as the production costs may be substantially reduced if part of the production is located to a country with lower wages.\(^1\) However, although earlier research addresses some of the implications of outsourcing for unemployment and welfare policy\(^2\), very little attention has so far been devoted to the implications of outsourcing for optimal redistributive taxation. An exception is the recent study by Aronsson and Koskela (2008), who incorporate outsourcing in an optimal income tax model with two ability-types and perfectly competitive markets. Their results show that if the government is able to control the amount of resources spent on outsourcing via a direct tax instrument (e.g. a payment made by firms per unit of resources transferred abroad), then outsourcing will not (by itself) contribute to the incentive structure underlying optimal income taxation, i.e. the marginal income tax structure is governed by the same incentives as in the absence of outsourcing. However, if the government lacks a direct instrument for taxing outsourcing, then outsourcing contributes to decrease the marginal income tax rate implemented for the low-ability type and increase the marginal income tax rate implemented for the high-ability type, which means that outsourcing contributes to a more progressive tax structure. The intuition behind this result is that increased labor tax progression leads to less outsourcing\(^3\), implying that increased tax progression also serves as an indirect instrument to reduce wage inequality.

The main purpose of the present paper is to analyze outsourcing and redistributive nonlinear taxation in an economy with involuntary unemployment. Such a study is important for at least two reasons. First, many countries have been characterized by relatively high unemployment rates for a long time, especially among low-skilled labor,

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\(^1\) For a wide range of industries, wage differences across countries constitute central explanations for outsourcing of production; see e.g. Amiti and Wei (2004), Rishi and Saxena (2004) and Sinn (2007).

\(^2\) See e.g. Keuschnigg and Ribi (2008).

\(^3\) This is a consequence of their assumption that outsourcing is complementary with skilled labor and substitutable for unskilled labor, meaning that a more progressive tax structure (in the sense described above) leads to less outsourcing and, therefore, contributes to a more equal wage distribution.
while at the same time production activities have been moved abroad to a greater extent than before (see e.g. Stefanova 2006 for the East-West dichotomy of outsourcing). Therefore, if outsourcing leads to reduced demand for low-skilled labor, this suggests that the policy implications of outsourcing and involuntary unemployment ought to be analyzed simultaneously. Second, although earlier literature shows that both outsourcing and involuntary unemployment imply restraint on redistributive policy, there have been no attempts so far to combine them in the study of redistributive nonlinear taxation. Another purpose is to extend the set of policy instruments by also considering provision of a public input good. The basic idea is that public input goods, which enhance the productivity of domestic labor, can be used to (partly) offset the increased inequality and unemployment that would otherwise be a consequence of outsourcing.

The present paper is also related to a relatively small – yet growing – literature dealing with optimal nonlinear taxation in economies with imperfect competition in the labor market. To our knowledge, Marceau and Boadway (1994) wrote the first paper in this area. Their study is based on an extension of the two-type model developed by Stern (1982) and Stiglitz (1982), and focuses on the welfare effects of minimum wages and unemployment insurance. Boadway and Cuff (2001) also examine the redistributive role of minimum wages; yet from a broader perspective as they consider the effects of minimum wage legislation in combination with either linear or nonlinear income taxation as well as distinguish between fixed and flexible labor supply among the employed. Aronsson and Sjögren (2003) analyze a mixed tax problem in combination with provision of a public consumption good in a two-type model with union wage formation. They show (among other things) how employment-related motives behind the public policy, which are due to the appearance of imperfect competition in the labor market, modify the use of income taxation, commodity taxation and public good provision by comparison with the use of these instruments in a competitive economy. Aronsson et al. (2008) extend the analysis to

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4 Earlier literature often emphasizes that outsourcing tends to increase the wage differentials between skilled and unskilled labor; see also section 2 below.

5 See also the related literature dealing with optimal linear taxation in economies with involuntary unemployment, e.g. Bovenberg and van der Ploeg (1996), Boeters and Schneider (1999) and Koskela and Schöb (2002).

6 See also the related literature dealing with labor income tax progression in trade-unionized economies; for instance, by considering the relationship between, on the one hand, the optimal degree of tax progression and, on the other, the structure of wage bargaining and/or the incentives characterizing the choice of work hours (Fuest and Huber 1997 and Aronsson and Sjögren 2004a, 2004b).
a dynamic economy and show that unemployment gives rise to intertemporal production inefficiency at the second best optimum which, in turn, constitutes an employment-related argument for using capital income taxation.

Following Marceau and Boadway (1994), our study is based on a two-type optimal income tax model with a minimum wage implemented for the low-ability type which, in turn, gives rise to unemployment at the equilibrium. A minimum wage approach to imperfect competition in the labor market is clearly relevant from a practical policy perspective, as many countries apply minimum wage policies at present. Only in Europe, there is a variety of minimum wage systems, where a formal distinction is made between systems where the minimum wage is decided upon by the government (i.e. via legislation), and systems where the minimum wage is the outcome of bargaining between the employers and representatives of the employees.7 A minimum wage model has also theoretical appeal, as it provides a simpler and more tractable alternative to models with trade-unionized labor markets often applied in earlier literature on optimal taxation under imperfect competition in the labor market.8

In a way similar to Aronsson and Koskela (2008), we distinguish between (i) a situation where the government can control the resources spent on outsourcing via a tax on outsourcing; and (ii) a situation where a direct tax instrument for controlling outsourcing is not available. This distinction is reasonable: whereas the former case is appealing from a welfare economic point of view (as we are dealing with normative aspects of taxation), the argument behind latter case is more practical because international agreements on factor mobility and trade may limit the availability of such instruments in practice. However, by contrast to Aronsson and Koskela, who analyze an economy with competitive markets, the distinction between Cases (i) and (ii) above is also relevant in the sense of highlighting an

7 See Dolado et al. (1996) for a comparative empirical study of the consequences for employment of using minimum wages. The empirical evidence is mixed, and no strong evidence of adverse effects on employment is found except possibly for young workers.

8 Real world labor markets may contain a variety of mechanisms - such as minimum wage legislation, wage bargaining between trade-unions and firms and wage-induced productivity (i.e. the idea underlying efficiency wages) – that give rise to involuntary unemployment. From our perspective, and except for the tractability-argument presented above, it does not matter so much which mechanism is chosen, since they all imply similar employment-related incentives for tax and expenditure policies.
employment-related motive for the government to exercise control over the resources spent on outsourcing.

The outline of the study is as follows. In section 2, we describe the decision-problems facing private agents, i.e. consumers and firms, and the outcome of private optimization. We also describe the labor market (with a minimum wage imposed on the low-ability type) and the outcome in terms of employment. Section 3 concerns the optimal tax and expenditure problem in Case (i), where the government has access to a tax on outsourcing. The results show that redistributive (i.e. relaxation of the self-selection constraint) and employment-related motives for taxation in general work in the same direction: the government implements a positive marginal income tax rate for the low-ability type, a negative marginal income tax rate for the high-ability type and a positive tax on outsourcing, respectively. For the public input good, however, the redistributive argument and the employment-related motive counteract each other. More specifically, the desire to relax the self-selection constraint provides an incentive to underprovide the public input good relative to the first best policy rule, whereas the desire to increase employment provides an incentive for overprovision. Section 4 concerns optimal income taxation and public provision in Case (ii), where the government lacks a direct tax instrument attached to outsourcing, meaning that the income tax and the public input good will (at least in part) serve as indirect instruments to reduce the level of outsourcing. Our results show that the incentive to reduce outsourcing does not (by itself) modify the formula for the marginal income tax rate of the low-ability type, while it may change the marginal income tax rate of the high-ability type in either direction. In addition, the incentive to reduce outsourcing contributes to overprovision the public input good relative to the first best policy rule. The results are summarized and discussed in section 5.

2. The Model

In this section, we present the decision-problems facing private agents, i.e. consumers and firms. We also describe the labor market and characterize the outcome of private optimization. The government’s decision-problem and the outcome in terms of optimal
taxation and provision of the public input good are addressed sections 3 (for Case (i)) and 4 (for Case (ii)).

2.1. Consumers

There are two types of consumers; a low-ability type (denoted by superindex 1) and a high-ability type (denoted by superindex 2). The distinction between ability-types refers to productivity, which is interpreted to mean that the high-ability type faces a higher before tax wage rate than the low-ability type. We denote the number of individuals of each ability-type by $\pi^1$ and $\pi^2$, respectively.

The utility function facing ability-type $i$ ($i=1,2$) is given by

$$u^i = u(c^i, z^i)$$

where $c$ is consumption and $z$ leisure. Leisure is, in turn, defined as a time endowment, $H$, less the time spent in market work, $l$. Let $w^i$ denote the hourly gross wage rate and $T(w^i l^i)$ the income tax payment by ability-type $i$. The individual budget constraint can then be written as

$$w^i l^i - T(w^i l^i) - c^i = 0 .$$

The first order condition for the hours of work becomes

$$u'_i w^i (1 - T'(w^i l^i)) - u'_z = 0$$

where $T'(w^i l^i) = \frac{\partial T(w^i l^i)}{\partial (w^i l^i)}$ is the marginal income tax rate.

As indicated above, some low-ability agents may become unemployed due to the minimum wage policy (see below). Each unemployed individual is assumed to receive an unemployment benefit, $b$, and consume the maximum amount of leisure. Therefore, the utility facing an unemployed individual becomes $u^u = u(b, H)$.
2.2. Production

Turning to the production side of the economy, we assume that the representative firm acts on a competitive product market and uses three variable inputs – domestic labor of each ability-type and the amount of resources outsourced to production abroad - together with a public input good (which appears as an externality production factor) to produce a homogenous good. The production function is characterized by decreasing returns to scale in the three production factors controlled by the firm. The production function is written as $F(L, L^2, M, G)$, where $L$ represents the total number of hours of work by ability-type $i$, measured as the hours of work per employee times the number of employed persons, whereas $M$ denotes the resources spent on outsourcing and $G$ represents the public input good.

Skilled and unskilled labor are assumed to be technical complements in production, i.e. $F_{LL}(L, L^2, M, G) > 0$. Following Koskela and Stenbacka (2007), we also assume that outsourcing is substitutable for unskilled labor and complementary with skilled labor; therefore, $F_{LM}(L, L^2, M, G) < 0$ and $F_{M}(L, L^2, M, G) > 0$, which means that outsourcing leads to wage inequality (as long as the before tax wage rates are determined by marginal productivity). In a study dealing with the effects of globalization on the skill premium, Ethier (2005) uses a similar type of production function to analyze the decision between international outsourcing and in-house production. We assume that while some activities are easy to outsource, other activities are more costly to outsource. Therefore, the marginal cost of outsourcing increases in the scope of activities to outsource, so that there is a cost of outsourcing, $\psi(M)$, which is increasing and convex, i.e. $\psi'(M) > 0, \psi''(M) \geq 0$. This captures the idea that outsourcing may necessitate costly investments into the establishment of network of suppliers in relevant host-countries. The public input good will be assumed to increase the marginal productivity of both skilled and unskilled labor,

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9 Therefore, we abstract from possible product market imperfections in what follows. Lommerud et al. (2006) have demonstrated how international mergers might curb the market power of unions giving socially excessive incentives for international mergers, unless products are close substitutes.

10 Empirical support for the idea that outsourcing leads to more inequality is provided by, e.g., Feenstra and Hanson (1999), Haskel and Slaughter (2001), Hijzen, Görg and Hine (2005), Hijzen (2007), Egger and Egger (2006), Munch and Skaksen (2005), Riley and Young (2007) and Geishecker and Görg (2008).
which means \( F_{L,G}(L^1, L^2, M, G) > 0 \) and \( F_{L,G}(L^1, L^2, M, G) > 0 \), whereas outsourcing and the public input good are weak substitutes in the sense that \( F_{M,G}(L^1, L^2, M, G) \leq 0 \).  

The objective function facing the firm can be written as

\[
\pi = F(L^1, L^2, M, G) - w^1 L^1 - w^2 L^2 - \psi(M) - t M
\]

where \( t \) is a tax per unit of the resources spent on outsourcing (which may, or may not, be operative). The first order conditions become

\[
F_{L}(L^1, L^2, M, G) - w^1 = 0 \quad (4)
\]
\[
F_{L}(L^1, L^2, M, G) - w^2 = 0 \quad (5)
\]
\[
F_{M}(L^1, L^2, M, G) - \psi_M(M) - t = 0. \quad (6)
\]

As mentioned in the introduction, we consider two possible cases with regards to the tax on outsourcing; Case (i) means that this tax is operative, i.e. part of the set of tax instruments facing the government (section 3), whereas Case (ii) means that it is not operative and, therefore, set equal to zero (section 4).

2.3. The Labor Market

As we indicated above, the labor market for high-skilled labor is assumed to be competitive, meaning that the equilibrium condition becomes \( L^2 = \bar{\pi}^2 \). Low-skilled workers, on the other hand, are subject to a minimum wage, i.e. \( w^1_{\text{min}} \), which is decided

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11 In real world economies, outsourcing often means that firms move part of their production structure and employ (primarily low-skilled) labor abroad. Therefore, if domestic public input goods contribute to increase the productivity of domestic production factors, increased public provision means, ceteris paribus, a stronger incentive for domestic production relative to outsourcing. Our assumption that outsourcing and the public input good are weak substitutes can be thought of as a ‘reduced form equivalent’ to this property. However, note also that all results derived below would still apply if outsourcing and the public input good are complements in the production function, provided that the degree of complementary is low relative to the degree of complementary between the public input good and domestic labor.
upon by the government. The minimum wage is assumed to imply a binding constraint in what follows, so \( w^l = w_{\text{min}} \).

To be able to derive expressions for the marginal income tax rates comparable to those derived in earlier studies on optimal redistributive income taxation under imperfect competition in the labor market, let us rewrite equation (4) such that

\[
F_L(n^l, \bar{\pi}^2, M, G) - w_{\text{min}} = 0
\]

(7)

where \( n^l \) is interpretable as the number of employed low-skilled individuals. As the binding minimum wage exceeds the market clearing wage rate, it follows that the low-skilled are partly unemployed, i.e. \( n^l < \bar{n}^l \). We can then use equation (7) to solve for the number of employed individuals of the low-ability type as a function of variables that the government controls via its tax and expenditure policies as follows

\[
n^l = n^l \left( \bar{l}^l, \bar{\pi}^2, M, G, w_{\text{max}} \right).
\]

(8)

In equation (8), the constant \( \pi^2 \) has been suppressed for notational convenience, and the sign above each argument indicates the comparative statics effect. With the assumptions made above, therefore, an increase in the hours of work per employed low-ability agent, increased resources spent on outsourcing and an increase in the minimum wage, respectively, tends to decrease the number of employed low-ability agents, whereas increases in the hours of work per high-ability agent and the public input good have the opposite effect.

3. **Optimal Taxation and Public Provision in Case (i)**

In this section, we analyze the optimal policy rules for marginal income taxation, taxation of outsourcing and provision of the public input good, respectively, that will
follow from the model set out above. The government is assumed to face the following general social welfare function\(^1\):

\[
W = W(n^1 u^1, \bar{n}^2 u^2, (\bar{n}^1 - n^1) u^\nu)
\]  

(9)

in which different welfare weights are attached to the utilities of different agent-types, whereas identical individuals are subject to equal treatment.

The informational assumptions are conventional: the government knows the income of each individual as well as the number of individuals of each ability-type, whereas ability is private information. This means that the government is not able to observe whether any given worker is a low-ability or high-ability type. By concentrating on the 'normal' case, where redistribution means income transfers from the high-ability to the low-ability type, one would like to prevent the high-ability type from mimicking the employed low-ability type in order to gain from redistribution. The self-selection constraint that may bind then becomes\(^1\):

\[
u^2 = u(c^2, z^2) \geq u(c^1, H - \phi l^1) = \hat{u}^2
\]

(10)

where \(\hat{u}^2\) denotes the utility of the mimicker and \(\phi = w^1 / w^2 < 1\) the wage ratio, i.e. the before-tax wage rate of the low-ability type relative to the before-tax wage rate of the high-ability type. Note also that the mimicker faces the same income and consumption point and, therefore, pays as much tax as the employed low-ability type. However, as the

\(^1\) Another approach (which is common in earlier literature on the self-selection approach to optimal income taxation) is to assume that the government aims at maximizing the utility of one agent-type subject to minimum utility restrictions for the others. If we were to use this alternative approach (instead of using the social welfare function), all qualitative results derived below would remain unchanged. The only (technical) difference is that, if we were to maximize the utility of one of the agent-types (say, the employed low-ability type) subject to minimum utility restrictions for the others (say, the unemployed and the high-ability type), the private utility gain of increased employment would not appear in the policy rules for optimal taxation and provision of the public input good (although the value of increased employment would still be positive as in our model).

\(^1\) This formulation, which only applies when the mimicker is employed, was also used by Marceau and Boadway (1994) in their study of minimum wage policy and unemployment insurance as means for redistribution. It is based on the assumption that the utility facing an employed low-ability type always exceeds the utility facing an unemployed individual. As a consequence, if the self-selection constraint in (10) is binding, it follows that the utility of the high-ability type always exceeds the utility facing an unemployed individual.
mimicker is more productive than the low-ability type, he/she spends more time on leisure. By using the first order conditions for the firm, one can see that \( \phi \) is a function of \( l^2, M, G \) and \( w_{\text{min}}^j \), i.e.

\[
\phi = \phi(l^2, M, G, w_{\text{min}}^j) = \frac{w_{\text{min}}^j}{F_{\phi}(n^1l^1, \bar{n}^2l^2, M, G)}
\]  

(11)

in which \( n^1 \) is determined by equation (8). With the assumptions made above, one can show that an increase in the minimum wage reduces the wage inequality, i.e. \( \partial \phi / \partial w_{\text{min}}^j > 0 \), while an increase in the public input good increases the wage inequality, so \( \partial \phi / \partial G < 0 \). The effects on the wage ratio of the other variables can be either positive or negative in general. However, by adding the assumption that the direct effect of \( l^2 \) and \( M \), respectively, on \( w^2 = F_{\phi}(n^1l^1, \bar{n}^2l^2, M, G) \) dominates the corresponding indirect effect that arises via \( n^1 \), then \( \partial \phi / \partial l^2 > 0 \) and \( \partial \phi / \partial M < 0 \), meaning that an increase in the hours of work by the high-ability type reduces the wage inequality, whereas an increase in the resources spent on outsourcing leads to more wage inequality.\(^{14}\) These properties appear to us to be reasonable and will be used in what follows. The effect of \( l^1 \) on the wage ratio is zero, because the hours of work per employee and the number of employed persons are perfect substitutes in terms of the production function.

By using the short notation \( T^i = T(w^i l^i) \), the budget constraint of the government is given by

\[
\pi + n^1T^1 + \bar{n}^2T^2 + tM - (n^1 - n^1)b - G = 0
\]

where the production price of the public good (i.e. the marginal rate of transformation between the public good and the private consumption good) has been normalized to one for notational convenience. The term \( \pi \) represents possible pure profits, which we assume accrue to the government, as the government is the owner of the factor treated as fixed by

\(^{14}\) These properties are in conformity with the empirical evidence mentioned earlier.
the firm. The component \((n_1 - n^1)b\) represents the public expenditures on unemployment benefits.

Note that \(T(\cdot)\) is a general income tax in the sense that it may be used to implement any desired combination of \(l^1, c^1, l^2,\) and \(c^2\). It is, therefore, convenient to follow earlier comparable literature by using \(l^1, c^1, l^2,\) and \(c^2\), instead of the parameters of \(T(\cdot)\), as direct decision-variables for the government. Similarly, since the government can use \(t\) to exercise perfect control over \(M\) (given that it also exercises control over \(l^1, t^2, G\) and \(w^1_{\text{min}}\)), we may also use \(M\) as a direct decision-variable in what follows. By using the private budget constraint and the objective function of the firm, we may rewrite the budget constraint of the government to read

\[
F(n^1l^1, \bar{n}^2l^2, M, G) - n^1c^1 - \bar{n}^2c^2 - (\bar{n}^1 - n^1)c^u - \psi(M) - G = 0. \quad (12)
\]

The government’s decision-problem will be to choose tax and expenditure policies in order to maximize the social welfare function, presented in equation (9), subject to the self-selection constraint and budget constraint given by equations (10) and (12), respectively, as well as subject to equations (8) and (11), which determine the number of employed persons of the low-ability type and the wage ratio, respectively. The Lagrangean corresponding to the optimal tax and expenditure problem can now be written as

\[
L = W + \lambda[u^2 - \bar{u}^2] + \gamma[F - n^1c^1 - \bar{n}^2c^2 - (\bar{n}^1 - n^1)c^u - \psi(M) - G]
\]

in which \(F\) denotes the production function as specified in equation (12). The first order conditions for the hours of work, private consumption and outsourcing, which are the conditions governing the optimal tax structure, are presented in the Appendix 1.

3.1. Marginal income tax rates and the tax on outsourcing

We are now in the position to analyze how the simultaneous appearances of equilibrium unemployment and outsourcing affect the optimal tax structure. The marginal income tax rate of the low-ability type might be derived by combining equations (3), (A1) and (A2), whereas the marginal income tax rate of the high-ability type is derived by
combining equations (3), (A3) and (A4). The tax rate on outsourcing can be derived by combining equations (6) and (A5). Now, let

\[ MRS_{z,c}^i = \frac{u_i'}{u_c'} \quad \text{and} \quad \hat{MRS}_{z,c}^2 = \frac{\hat{u}_i^2}{\hat{u}_c^2} \]

denote the marginal rate of substitution between leisure and private consumption for ability-type \( i \) and the mimicker, respectively. In addition, to shorten the notation, define the value that the government attaches to the private utility gain of going from unemployment to employment measured in terms of public funds

\[ \Delta = \frac{1}{\gamma} \left[ \frac{\partial W}{\partial (n'u')} u' - \frac{\partial W}{\partial ((n^1 - n^1)u^v)} u^v \right]. \]

Then, by using \( \lambda^* = \frac{\hat{\lambda}^2}{\gamma} \), the marginal income tax rates and the tax on outsourcing can be written as

\[ T'(w^1l^1) = \frac{\lambda^*}{w'n}[MRS_{z,c}^1 - \phi \hat{MRS}_{z,c}^2] - \frac{1}{w'n}[T^1 + b + \Delta] \frac{\partial n^1}{\partial l^1} \]  

(13)

\[ T'(w^2l^2) = -\frac{\lambda}{\gamma w^2n^2} \hat{u}_i^2 l^1 \frac{\partial \phi}{\partial l^2} - \frac{1}{w^2n^2}[T^1 + b + \Delta] \frac{\partial n^1}{\partial l^2} \]  

(14)

\[ t = \frac{\lambda}{\gamma} \hat{u}_i^2 l^1 \frac{\partial \phi}{\partial M} - [T^1 + b + \Delta] \frac{\partial n^1}{\partial M}. \]  

(15)

The main difference between, on the one hand, equations (13)-(15) and, on the other, the corresponding results derived by Aronsson and Koskela (2008) for an economy with a competitive labor market refers to the final term in each equation above, which appears because each tax instrument can be used to influence the number of employed persons. To sign this effect, we use the first order condition for the minimum wage

\[ \frac{\hat{\lambda}^2}{w^1 \text{min}} \frac{\partial \phi}{\partial l^1} l^1 + \gamma[T^1 + b + \Delta] \frac{\partial n^1}{\partial w^1 \text{min}} = 0. \]  

(16)
The first term on the left hand side of equation (16) is clearly positive, as an increase in the minimum wage leads to an increase in the wage ratio (i.e. reduced wage inequality). Therefore, since an increase in the minimum wage also contributes to reduce the number of employed persons, i.e. \( \frac{\partial n^l}{\partial w^l_{\text{min}}} < 0 \), we have \( T^l + b + \Delta > 0 \).

We have derived the following result;

**Proposition 1.** Suppose that the government can control outsourcing via a direct tax instrument. With the assumptions made above, it follows that

(i) the government implements a positive marginal income tax rate for the low-ability type, a negative marginal income tax rate for the high-ability type and a positive tax on outsourcing, and

(ii) the employment effect (captured by the final term in each tax formula) provides an incentive for the government to implement a higher marginal income tax rate for the low-ability type, a lower marginal income tax rate for the high-ability type and a higher tax on outsourcing, ceteris paribus.

Proposition 1 follows by observing that \( MRS^1_{z,c} > \phi MRS^2_{z,c} \) due to single crossing; that \( \frac{\partial \phi}{\partial l^2} > 0, \frac{\partial \phi}{\partial M} < 0, \frac{\partial n^l}{\partial l^1} < 0, \frac{\partial n^l}{\partial l^2} > 0 \) and \( \frac{\partial n^l}{\partial M} < 0 \) by our earlier assumptions; and that \( T^l + b + \Delta > 0 \) according to equation (16). As a consequence, the redistributive component (via the self-selection constraint) and corrective component (via the employment effect) work in the same direction in each tax formula.

To be able to provide a more thorough interpretation of the second part of the proposition, note that \( T^l + b + \Delta > 0 \) measures the social value of increased employment among the low-skilled. As a consequence, there is an incentive for the government to use tax policy to increase the number of employed persons captured by the final term on the right hand side of each tax formula. The final term on the right hand side is positive in equation (13), negative in equation (14) and positive in equation (15), respectively, whereas each of these terms would have been equal to zero under full employment (where \( n^l = \pi^l \)). The intuition is that a higher marginal income tax rate for the employed low-ability type contributes to reduce the hours of work supplied by each employed low-ability individual; a lower marginal income tax rate implemented for the high-ability type leads to
increased hours of work by the high-ability type; and a higher tax on outsourcing leads to 
less outsourcing. Each such change implies increased employment among low-ability 
agents.\footnote{It is important to observe that the increased marginal income tax rate of the low-ability type caused by the 
employment effect in equation (13) would also follow under the weaker assumption that the hours of 
work by the high-ability type; and a higher tax on outsourcing leads to 
less outsourcing. Each such change implies increased employment among low-ability 
agents. However, note that the second part of the proposition does not necessarily mean 
that the government implements a higher marginal income tax rate for the low-ability type, 
a lower marginal income tax rate for the high-ability type and a higher tax on outsourcing 
than it would have done in a competitive economy, since the effects of public policy on the 
appropriate tools of social science analysis.}

It is interesting to compare the expressions for the marginal income tax rates derived 
above with those in earlier studies on optimal redistributive income taxation under 
imperfect competition in the labor market. Equations (13) and (14) are similar to the policy 
rules for marginal income taxation derived by Aronsson and Sjögren (2003) in an economy 
without outsourcing, with the exception that union wage setting is the mechanism behind 
the unemployment in their study. The main difference by comparison with their study is 
that we are in this case able to sign the qualitative contribution to the marginal income tax 
rates of the incentive to increase employment, i.e. we can sign the employment effects in 
the expressions for the marginal income tax rates. In addition, and by comparison with 
Aronsson and Koskela (2008) who consider outsourcing and optimal taxation in a 
competitive economy, we are also able to address (and sign the qualitative contribution of) 
an employment-related motive for taxing outsourcing.

3.2. Provision of the public input good

The first-order condition for the public input good can be written as

\[ F_G(\cdot) - 1 = -\frac{\lambda}{\gamma} \lambda u^2 l \frac{\partial \phi}{\partial G} - [T^1 + b + \Delta] \frac{\partial n^1}{\partial G}. \]  

Equation (17) means that the optimal provision of the public good deviates from the first 
best policy rule, i.e. \( F_G(\cdot) - 1 = 0 \), because a change in the public input good directly affects
(i) the wage distribution and (ii) the number of employed persons. To be more specific, increased provision of the public input good reduces the wage ratio (i.e. leads to more wage inequality), $\frac{\partial \phi}{\partial G} < 0$, and increases the number of employed persons of the low-ability type, $\frac{\partial n^l}{\partial G} > 0$. One can think of the first term on the right hand side of equation (17) as capturing the redistributive motive for public provision, whereas the second term captures the employment-related motive.

We summarize the main qualitative insight from equation (17) as follows;

**Proposition 2.** If the government can control outsourcing via a direct tax instrument, it will underprovide (overprovide) the public input good relative to the first best policy rule if the redistributive motive for public provision dominates (is dominated by) the employment-related motive. The employment-related motive contributes, unambiguously, to increase the provision of the public input good.

The mechanisms behind Proposition 2 are that increased public provision has two counteracting effects: it makes the income distribution more unequal (by increasing the wage rate facing the high-ability type given the minimum wage rate facing the low-ability type) and increases the employment. The relative strength of these two effects then determines whether the optimal policy rule means overprovision or underprovision relative to the first best policy rule.

4. **Optimal Taxation and Public Provision in Case (ii)**

A possible objection to the analysis set out above is that international agreements regarding factor mobility and trade may limit the possibility for national governments to implement direct taxes on outsourcing. In this section, therefore, we analyze optimal income taxation and provision of the public input good in Case (ii), where the government lacks a direct instrument by which to tax outsourcing.

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work per employee and the number of employed persons are imperfect substitutes in terms of the production function.
Note that the firm’s first order conditions for low-skilled labor and outsourcing, respectively, can be written as (with $t = 0$)

\begin{align}
F_l(n^1l^1, \pi^2l^2, M, G) - w_{min}^1 &= 0 \\
F_M(n^1l^1, \pi^2l^2, M, G) - \psi_M(M) &= 0.
\end{align}

By solving equation system (18) and (19) for $n^1$ and $M$, we have

\begin{align}
n^1 &= n^1(l^1, l^2, G, w_{min}^1) \\
M &= M(l^2, G, w_{min}^1)
\end{align}

in which the constant $\pi^2$ has been suppressed. With the assumptions made in Section 2, one can show that that an increase in the minimum wage leads to increased outsourcing, while an increase in the public input good reduces outsourcing\(^{16}\), i.e. $\partial M / \partial w_{min}^1 > 0$ and $\partial M / \partial G < 0$. An increase in the hours of work supplied by the high-ability type, on the other hand, may either increase or decrease the amount of resources spent on outsourcing, meaning that $\partial M / \partial l^2$ can be either positive or negative. Note also that $\partial M / \partial l^1 = 0$ by the assumptions made earlier, because the two effects via which the hours of work supplied by the low-ability type affect outsourcing - a direct effect and an indirect effect via the number of employed persons - cancel out. We will return to the properties of equation (21) below.

The optimal tax and expenditure problem can be written as if the government chooses $l^1, c^1, l^2, c^2, G$ and $w_{min}^1$ to maximize the Lagrangean

$$L = W + \lambda [u^2 - \bar{u}^2] + \rho [F - n^1c^1 - \pi^2c^2 - (\pi^1 - n^1)c^\alpha - \psi(M) - G]$$

\(^{16}\) The relationship between public infrastructure and outsourcing has also analyzed by Egger and Falkinger (2006); yet in a context different from ours.
subject to equations (8), (11) and (21). The marginal income tax rates are derived by using equations (A6)-(A9) in the Appendix 2. To analyze the optimal income tax structure, it is useful to begin by discussing the first-order condition for the minimum wage, which can be written as

$$\lambda \dot{w}_{\min}^2 \frac{d \phi}{dw_{\min}^i} l^i + \gamma [T^i + b + \Delta] \frac{dn^i}{dw_{\min}^i} = 0.$$  \hspace{1cm} (22)$$

The derivatives of the wage ratio and the number of employed persons of the low-ability type, respectively, with respect to the minimum wage in equation (22) can be decomposed into two parts; a direct effect (with $M$ held constant) and an indirect effect via equation (21). We have

$$\frac{d \phi}{dw_{\min}^i} = \frac{\partial \phi}{\partial w_{\min}^i} \bigg|_{M=0} + \frac{\partial \phi}{\partial M} \frac{\partial M}{\partial w_{\min}^i} > 0$$  \hspace{1cm} (23)$$

$$\frac{dn^i}{dw_{\min}^i} = \frac{\partial n^i}{\partial w_{\min}^i} \bigg|_{M=0} + \frac{\partial n^i}{\partial M} \frac{\partial M}{\partial w_{\min}^i} < 0$$  \hspace{1cm} (24)$$

where each direct effect is conditioned on the second best optimal level of outsourcing, $M^0$. Therefore, by the assumptions made above, the total effect on the number of employed persons of an increase in the minimum wage is unambiguously negative, whereas the total effect on the wage ratio wage can be either positive or negative. For purposes of interpretation, let us add the assumption that the positive direct effect of the minimum wage on the wage ratio dominates the negative indirect effect via the change in outsourcing, so that $d \phi / dw_{\min}^i > 0$. In this case, and by analogy to the analysis carried out in the previous section, we can use equation (22) to show that $T^i + b + \Delta > 0$, which will be useful below.

4.1. Marginal income tax rates

As the social welfare function is equal to the Lagrangean at the second best optimum, we can use $\Lambda = [\partial L / \partial M] / \gamma$ to measure the welfare effect of a marginal increase in the resources spent on outsourcing. The marginal income tax rates can then be written as
\[
T'(w^1l^1) = \lambda^* \left[ \frac{MRS_{z^1}}{w_n} - \phi M\hat{R}S^2_{z^1} \right] - \frac{1}{w_n} \left[ T^1 + b + \Delta \right] \frac{\partial n^1}{\partial l^1} \bigg|_{M=M^0} (25)
\]
\[
T'(w^2l^2) = -\frac{\lambda}{\gamma w^2n^2} \frac{\partial^2 \phi}{\partial l^2} \bigg|_{M=M^0} - \frac{1}{w^2n^2} \left[ T^1 + b + \Delta \right] \frac{\partial n^1}{\partial l^2} \bigg|_{M=M^0} - \frac{\Lambda}{\gamma w^2n^2} \frac{\partial M}{\partial l^2} . (26)
\]

Equation (25) means that the incentives underlying the marginal labor income tax rate implemented for the low-ability type remain as in Case (i), were the government had access to a direct instrument to tax outsourcing, whereas equation (26) contains an additional incentive due the relationship between \( l^2 \) and \( M \), which can be either positive or negative. The intuition as to why a corresponding relationship between \( l^1 \) and \( M \) is absent in equation (25) was discussed in connection to equation (21) above: a change in the hours of work per employed low-ability agent will not affect the total number of hours worked by the low-ability type, as the hours of work per employee and the number of employed persons are perfect substitutes in terms of the production function. To interpret the final term on the right hand of equation (26), we take the derivative of the Lagrangean with respect to \( M \) and use \( F_\gamma(\cdot) - \psi_{\nu_\gamma}(\cdot) = 0 \) from equation (19) to derive

\[
\Lambda = \frac{1}{\gamma} \frac{\partial L}{\partial M} = \frac{\lambda}{\gamma} \frac{\partial^2 \phi}{\partial M} l^1 + \left[ T^1 + b + \Delta \right] \frac{\partial n^1}{\partial M} < 0 . (27)
\]

We can then interpret equations (25) and (26) as follows;

**Proposition 3.** Suppose that the government does not have access to a direct instrument to tax outsourcing. The incentives underlying the marginal income tax rate implemented for the low-ability type remain as they were in section 3 (i.e. where a direct tax on outsourcing was available). The relationship between the hours of work supplied by the high-ability type and the level of outsourcing provides an incentive for the government to increase (decrease) the marginal income tax rate implemented for the high-ability type – relative to the policy outcome that would be chosen with \( M = M^0 \) - if \( \partial M / \partial l^2 > 0 \) (\(< 0\)).
It is interesting to compare equations (25) and (26) with the corresponding marginal income tax rates derived by Aronsson and Koskela (2008) in an economy with a competitive labor market. They show that the lack of a direct tax instrument for outsourcing provides an incentive for the government to implement a lower marginal income tax rate for the low-ability type and a higher marginal income tax rate for the high-ability type than it would otherwise have done (i.e. if such an instrument were available).

With a binding minimum wage for the low-ability type, on the other hand, there is no direct additional effect of outsourcing on the marginal labor income tax rate implemented for the low-ability type, whereas the direct effect of outsourcing on the marginal income tax rate implemented for the high-ability type can be either positive or negative, as the effect of an increase in the hours of work by the high-ability type may either increase or decrease the amount of resources spent on outsourcing. The intuition behind this ambiguity is that an increase in the hours of work supplied by the high-ability type will both have a direct positive effect on outsourcing due to complementarity between high-skilled labor and outsourcing, and a negative effect due to complementarity between high-skilled labor and the number of employed low-skilled persons. The latter relationship would, of course, vanish under perfect competition, where \( n^1 = \bar{n}^1 \).

4.2 Provision of the public input good

Finally, turning to the provision of the public input good, we have

\[
F_G(\cdot) - 1 = -\frac{\lambda}{\gamma} \lambda \hat{u}^2 \left[ \frac{\partial \phi}{\partial G} \right]_{M=M^0} - [T^1 + b + \Delta] \frac{\partial n^1}{\partial G} \bigg|_{M=M^0} - \Lambda \frac{\partial M}{\partial G}.
\]

The following result can be derived from equation (28);

**Proposition 4.** Without a direct instrument to tax outsourcing, there is an incentive for the government to use the public input good to reduce the level of outsourcing. This incentive effect – summarized by the third term on the right hand side of equation (28) - works to increase the provision of the public input good, ceteris paribus.
Note that the first two terms on the right hand side of equation (28) are analogous to the formula for public provision that applies when the government can tax outsourcing directly, i.e. equation (17), with the exception that the derivatives of the wage ratio and the number of employed persons, respectively, with respect to the public input good take other forms here than in Section 3. The intuition behind Proposition 4 is straightforward: an increase in \( G \) contributes to reduce \( M \) (as it leads to increased domestic employment), which is desirable by the results derived earlier.

5. Summary and Discussion

Based on the argument that the combination of outsourcing and involuntary unemployment may imply serious restraint on the opportunities available for the low-skilled, this paper analyses redistributive nonlinear taxation and provision of a public input good in an economy with equilibrium unemployment, where firms outsource part of their production to other countries. Our study is based on an extension of the two-type optimal income tax model here augmented with a minimum wage policy directed towards the low-ability type (which, if binding, gives rise to equilibrium unemployment among the low-skilled) as well as an option for the domestic firms to outsource part of the productive resources. The policy instruments facing the government consists of a nonlinear income tax, a public input good (which influences output as an externality production factor) as well as a direct tax on the resources subject to outsourcing, where the latter instrument is either operative, Case (i); or not operative, Case (ii).

In Case (i), where the direct tax on outsourcing is operative – and its level subject to choice by the government – the presence of outsourcing does not modify the policy rules for the marginal income tax rates and provision of the public input good. Instead, the results show that the government may both relax the self-selection constraint and increase employment among the low-skilled by implementing a positive marginal income tax rate for the low-ability type and a negative marginal income tax rate for the high-ability type. By a similar argument, the optimal tax on outsourcing is positive, since a lower level of outsourcing implies less wage inequality (which contributes to relax the self-selection...
constraint) and increased employment among the low-skilled. In other words, the appearance of equilibrium employment strengthens the argument for taxing low-ability labor and subsidizing high-ability labor at the margin as well as strengthens the motive for taxing outsourcing. For the public input good, however, the incentive to relax the self-selection constraint and the incentive to increase employment affect the optimal policy in opposite directions: the government will overprovide (underprovide) the public input good relative to the first best policy rule if the incentive to increase employment among the low-skilled dominates (is dominated by) the incentive to relax the self-selection constraint.

In Case (ii), where the direct tax on outsourcing is not operative (and set equal to zero), income taxation and public provision become indirect instruments for influencing the amount of resources spent on outsourcing. According to our results, the appearance of outsourcing will not directly affect the marginal income tax rate implemented for the low-ability type, while it may change the marginal income tax rate implemented for the high-ability type in either direction, depending on whether an increase in the labor supply by the high-ability type leads to more or less outsourcing. The qualitative contribution of the latter mechanism is ambiguous in general, because high-ability labor is complementary both with low-ability labor and outsourcing. On the other hand, the desire to reduce outsourcing provides an unambiguous incentive to increase the provision of the public input good.

Future research might take several new directions. We shall point out two of them. First, we have completely neglected the role of capital income taxation as a means to affect the resources spent on outsourcing. If domestic labor and capital are complements in production – and as long as the government lacks a direct instrument for controlling outsourcing – capital income taxation might be a useful tool to increase the productivity of domestic labor and, therefore, influence the employment, wage inequality and outsourcing simultaneously. Second, the resources that domestic firms spend on outsourcing will give rise to welfare effects in other countries. This suggests that uncoordinated policies might be inefficient from the perspective of society as a whole, and that outsourcing provides an argument for policy coordination. We leave these and other extensions for future research.
Appendices

Appendix 1

The first order conditions governing the optimal tax structure in Section 3 are

\[
\begin{align*}
- \frac{\partial W}{\partial (n'u')} u'_n n' + \lambda \hat{u}^2_z \left[ \phi + l^1 \frac{\partial \phi}{\partial l^1} \right] + \gamma [w'n' + (T^1 + b + \Delta) \frac{\partial n'}{\partial l^1}] &= 0 \quad (A1) \\
\frac{\partial W}{\partial (n'u')} u'_n n' - \lambda \hat{u}^2_c - \gamma n' &= 0 \quad (A2) \\
- \left[ \frac{\partial W}{\partial (\bar{n}^2 u^2)} \bar{n}^2 + \lambda \right] u^2_c + \lambda \hat{u}^2_z l^1 \frac{\partial \phi}{\partial l^1} + \gamma [w^2 \bar{n}^2 + (T^1 + b + \Delta) \frac{\partial n'}{\partial l^2}] &= 0 \quad (A3) \\
\left[ \frac{\partial W}{\partial (\bar{n}^2 u^2)} \bar{n}^2 + \lambda \right] u^2_c - \gamma \bar{n}^2 &= 0 \quad (A4) \\
\lambda \hat{u}^2_z l^1 \frac{\partial \phi}{\partial M} + \gamma [F_M(\cdot) - \psi_M(M) + (T^1 + b + \Delta) \frac{\partial n'}{\partial M}] &= 0. \quad (A5)
\end{align*}
\]

Appendix 2

The first order conditions governing the optimal tax structure in Section 4 can be written as

\[
\begin{align*}
- \frac{\partial W}{\partial (n'u')} u'_n n' + \lambda \hat{u}^2_z \left[ \phi + l^1 \frac{\partial \phi}{\partial l^1} \right]_{M=M^o} + \gamma [w'n' + (T^1 + b + \Delta) \frac{\partial n'}{\partial l^1} \bigg|_{M=M^o}] &= 0 \quad (A6) \\
\frac{\partial W}{\partial (n'u')} u'_n n' - \lambda \hat{u}^2_c - \gamma n' &= 0 \quad (A7) \\
- \left[ \frac{\partial W}{\partial (\bar{n}^2 u^2)} \bar{n}^2 + \lambda \right] u^2_c + \lambda \hat{u}^2_z l^1 \frac{\partial \phi}{\partial l^1} \bigg|_{M=M^o} + \gamma [w^2 \bar{n}^2 + (T^1 + b + \Delta) \frac{\partial n'}{\partial l^2} \bigg|_{M=M^o}] &= 0 \quad (A8) \\
+ \Lambda \frac{\partial M}{\partial l^2} &= 0 \\
\left[ \frac{\partial W}{\partial (\bar{n}^2 u^2)} \bar{n}^2 + \lambda \right] u^2_c - \gamma \bar{n}^2 &= 0. \quad (A9)
\end{align*}
\]
References


