When the Joneses’ Consumption Hurts: Optimal Income Taxation and Public Good Provision in an OLG Model**

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Abstract
This paper considers a two-type, self-selection, overlapping generations model with nonlinear labor income and capital income taxation and public good provision, when people care about their relative consumption compared to others. In each case, the standard optimality expressions are modified by terms that reflect the extent to which people care about relative consumption. The modified tax formulas imply substantially higher marginal labor income tax rates than in the conventional case, under plausible assumptions and available empirical estimates regarding comparison consumption concerns. The extent to which the public good provision rule should be modified is shown to depend critically on the preference elicitation format. The effects of positionality on the marginal capital income tax rates are ambiguous.

Keywords: Optimal taxation, redistribution, public goods, relative consumption, status, positional goods.

JEL Classification: D62, H21, H23, H41

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1. Introduction

The literature on optimal taxation and public goods typically assumes that the utility of each individual depends only on his/her own consumption of goods, leisure and services. Yet, there is growing empirical evidence suggesting that this assumption may not be entirely appropriate. In particular, several recent empirical studies have focused on interdependence among individuals in the form of status effects, showing that individuals value their own consumption (or income) relative to that of others.¹ There are also recent evolutionary models that explain why selfish genes would prefer that the humans they belong to are motivated by relative concerns (Samuelson 2004; Rayo and Becker 2007). The present paper considers an overlapping generations (OLG) model where relative consumption matters, and where the consumers differ in ability and preferences. The set of tax instruments facing the government consists of nonlinear taxes on both labor and capital income; the revenues from which are used to redistribute and provide a public good. The overall purpose is to analyze how the appearance of relative consumption concerns modifies the optimal tax structure and public good provision, respectively, compared with the standard model for nonlinear taxation.

Earlier literature dealing with public policy and relative consumption or income addresses a variety of issues such as income tax policy (Boskin and Sheshinski 1978, Blomquist 1993, Persson 1995, Ireland 2001), public good provision (Ng 1987), social insurance (Abel 2005), growth (Corneo and Jeanne 1997, 2001), environmental externalities (Brekke and Howarth 2002; Wendner 2005; Howarth 2006) and stabilization policy (Ljungqvist and Uhlig 2000). However, most studies that analyze taxes are based on linear tax instruments in static models, and have in common that they neglect capital income taxation. To our knowledge, the only papers that deal with relative income under nonlinear taxation are Oswald (1983), Tuomala (1990) and

Ireland (2001). Each of these innovative studies use static models without the possibilities to tax capital income and provide public goods.2

Why is it interesting to extend the study of public policy under relative consumption concerns into a dynamic model where the government uses redistributive nonlinear taxation? First, nonlinear income taxes constitute a reasonably realistic description of the tax instruments that many countries have (or potentially have) at their disposal. In our case, therefore, the government’s decision to use distortionary income taxation will follow from optimization, given the available information, and not from any a priori restrictions on the set of available policy instruments. This means that our model provides a suitable framework for analyzing the basic question of whether the appearance of relative consumption concerns itself motivates the use of distortionary taxation. In addition, as we are able to show that such a motive indeed exists, our model also enables us to study how and why this corrective motive for taxation interacts with the redistributive (i.e. self-selection) motive for using distortionary taxation.3 Second, by using a dynamic model, we are able to consider capital income taxation. Since earlier research suggests that the capital income tax may be a useful tool for relaxing the self-selection constraint (see below), a natural question here is whether this tax is also useful for purposes of internalizing positional externalities. This research question is strengthened by the potential interaction between the redistributive and corrective motives for tax policy mentioned above. Third, the knowledge of public good provision in second best economies with relative consumption concerns is very scarce. As far as we know, there is only one earlier study dealing with this issue, namely Wendner and Goulder (2007), in a model with identical individuals where the public good is financed by linear taxation. As earlier studies have shown that policy rules for public goods are very sensitive to the tax

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2 There are also dynamic models dealing with relative income concerns, such as Abel (2005), which do not explicitly consider distributional issues.

3 Earlier literature on optimal nonlinear income taxation typically abstracts from corrective motives for taxation. Exceptions are the studies dealing with environmental externalities (e.g. Pirilä and Tuomala 1997; Cremer and Ghavari 1998; and Aronsson and Blomquist 2003) and unemployment (e.g. Marceau and Boadway 1994; Aronsson and Sjögren 2004; and Aronsson et al. 2007).
instruments available to the government, it appears worthwhile to analyze public goods in the context of our more general model.

There is a small yet growing literature dealing with redistribution and/or public provision under asymmetric information in dynamic economies. It extends the traditional static optimal income tax model to allow for both labor income and capital income taxation. The seminal contribution here is a paper by Ordover and Phelps (1979). In a model with a continuum of ability-types, they show (among other things) that if leisure is separable from private consumption in terms of the utility function (so the marginal rate of substitution between present and future consumption does not depend on the leisure choice other than via income), then the marginal capital income tax rate should be zero for each ability-type. Pirttilä and Tuomala (2001), in a generalization of the model in Brett (1997), consider an OLG model with two ability-types and endogenous before tax wage rates. Their results show that production inefficiency at the second best optimum (which is a consequence of the desire to relax the self-selection constraint) justifies capital income taxation, whereas the marginal labor income tax rates take the same general form as in Stiglitz (1982), i.e. a positive marginal labor income tax rate should be imposed on the low-ability type and a negative marginal labor income tax rate on the high-ability type. Finally, Boadway et al. (2000) analyze nonlinear labor income taxation and proportional capital income taxation in a model where both ability and initial wealth are unobserved by the government. In their framework, the capital income tax is interpretable as an indirect instrument to tax wealth.

Our study is based on a two-type model, where the preferences facing each ability-type in each generation are described by a general (nonseparable) utility function. In addition, as our model contains production and does not restrict the analysis to a linear technology, it follows that the before tax wage rates are endogenous. The government redistributes and collects revenues for a public good by using general labor income

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4 The literature dealing with public good provision under nonlinear income taxation explains, among other things, how the desire to relax the self-selection constraint motivates deviations from the first best Samuelson rule; this is further discussed below. See also e.g. Christiansen (1981) and Boadway and Keen (1993).
and capital income taxation. Therefore, our model largely resembles the one used by Pirttilä and Tuomala (2001), with the important exception that in our case the relative private consumption matters for individual utility.

The present paper contributes to the literature in at least two ways. First, we are able to characterize the optimal labor income and capital income tax structure in an economy, where each individual compares his/her own private consumption with that of others. As argued above, this is a relevant extension of earlier literature dealing with redistribution under asymmetric information, as this literature typically disregards the consequences of relative consumption concerns. It is also relevant for purposes of comparison with earlier literature on public policy and relative consumption, in which more restrictive tax instruments are used. Second, the paper contributes to the literature on public good provision. By analogy to the arguments discussed above, this is relevant both (1) as an extension to earlier literature dealing with public good provision in economies with asymmetric information and general income taxes (which does not consider relative consumption concerns), and (2) as a comparison with the study of public provision in economies with positional goods and linear tax instruments.

The outline of the study is as follows. Section 2 presents the model and the outcome of private optimization. Section 3 characterizes the optimal tax and expenditure problem of the government, whereas Section 4 presents the corresponding results in a format that aims to facilitate straightforward interpretations and comparisons with earlier literature. Section 5 provides some concluding remarks.

2. Positional preferences, firms and market equilibrium

Consider an OLG model where each agent lives for two periods. Following the convention in earlier literature, we assume that each individual works during the first period of life and does not work during the second period. There are two types of individuals, where the low-ability type (type 1) is less productive than the high-ability type (type 2), and where \( n'_i \) denotes the number of individuals of ability-type \( i \) that were born at the beginning of period \( t \). Such an individual cares about his/her
consumption when young and when old, \( c_t^i \) and \( x_{t+1}^i \); his/her leisure when young, \( z_t^i \), given by a time endowment, \( H \), less the hours of work, \( l_t^i \) (when old, all available time is leisure); and the provided amount of the public good when young and when old, \( G_t \) and \( G_{t+1} \).

In addition, people care about their relative consumption when both young and old. The preferences for relative consumption, or positional preferences, can of course be modeled in many different ways. Here, we follow the dominating bulk of the literature and assume that each individual compares his/her own consumption in period \( t \) with a reference level determined by the average consumption in the economy as a whole at that time, \( \bar{c}_t \). We also follow e.g. Akerlof (1997), Corneo and Jeanne (1997), Ljungqvist and Uhlig (2000), Bowles and Park (2005) and Carlsson et al. (2007) in letting the relative consumption be described by the difference between the individual’s own consumption and the mean consumption in the economy as a whole.\(^5\)

The utility function of ability-type \( i \) born in period \( t \) can then be written as

\[
U_t^i = v_t^i(c_t^i, z_t^i, x_{t+1}^i, c_t^i - \bar{c}_t, x_{t+1}^i - \bar{c}_{t+1}, G_t, G_{t+1}) \\
= u_t^i(c_t^i, z_t^i, x_{t+1}^i, \bar{c}_t, \bar{c}_{t+1}, G_t, G_{t+1}),
\]

where the function \( v_t^i(\cdot) \) is increasing in each argument, implying that \( u_t^i(\cdot) \) is decreasing in \( \bar{c}_t \) and \( \bar{c}_{t+1} \) (a property that Dupor and Liu 2003 denote “jealousy”) and increasing in the other arguments; both \( v_t^i(\cdot) \) and \( u_t^i(\cdot) \) are assumed to be twice continuously differentiable in their respective arguments and strictly concave. The

\(^5\) Alternative approaches include ratio comparisons (Boskin and Sheshinski 1978; Layard 1980) and comparisons of the ordinal rank (Frank 1985; Hopkins and Kornienko 2004). Dupor and Liu (2003) consider a specific flexible functional form that includes the difference comparison and ratio comparison approaches as special cases. It is, of course, also possible that people compare themselves more to some people than to others, although, as noted by Clark et al. (2006), empirical evidence is scarce. Still, we believe that most qualitative results hold for many generalizations along those lines. Nevertheless, such generalizations constitute worthwhile extensions for future research.
reference consumption levels in periods \( t \) and \( t+1 \), respectively, measured by the mean consumption level of all people alive, are given by

\[
\overline{c}_t = \frac{n_1^1c_1^1 + n_2^1c_2^1 + n_{t-1}^1x_{t-1}^1 + n_t^2x_t^2}{N_t},
\]

(2)

\[
\overline{c}_{t+1} = \frac{n_1^{t+1}c_{t+1}^1 + n_2^{t+1}c_{t+1}^2 + n_1^{t+1}x_{t+1}^1 + n_2^{t+1}x_{t+1}^2}{N_{t+1}},
\]

(3)
in which \( N_t = n_1^t + n_2^t + n_{t-1}^t + n_t^2 \) and \( N_{t+1} = n_1^{t+1} + n_2^{t+1} + n_1^{t+1} + n_2^{t+1} \). Thus, this means that each individual compares his/her own consumption with the average consumption in each period. We also assume that each individual treats the reference levels, \( \overline{c}_t \) and \( \overline{c}_{t+1} \), as exogenous.

Since much of the subsequent analysis is focused on relative consumption concerns, it is useful to introduce measures of the degree to which such concerns matter for each individual. By defining \( \Delta_t^i = c_t^i - \overline{c}_t \) and \( \Delta_{t+1}^i = x_{t+1}^i - \overline{c}_{t+1} \), we can rewrite the first part of equation (1) as

\[
U_t^i = v_i^\prime(c_t^i, z_t^i, x_t^i, \Delta_t^i, \Delta_{t+1}^i, G_t, G_{t+1}).
\]

We can then define the degree of consumption positionality when young and old, respectively, based on the utility function in equation (1) as follows:

\[
\alpha_t^i = \frac{v_{t,c}}{v_{t,c} + v_{t,\Delta}},
\]

(4)

\[
\beta_t^i = \frac{v_{t,\Delta}}{v_{t,\Delta} + v_{t,c}},
\]

(5)

where \( v_{t,c} = \partial v_t^i / \partial c_t^i \) and similarly for the other variables. The term \( \alpha_t^i \) can then be interpreted as the fraction of the overall utility increase from the last dollar spent in period \( t \) that is due to the increased relative consumption. For instance, if \( \alpha_t^i = 0 \), then relative consumption does not matter at all on the margin, whereas in the other extreme case where \( \alpha_t^i = 1 \), absolute consumption does not matter at all (i.e. all that
matters is relative consumption). The interpretation of $\beta_i^t$ is analogous except that $\beta_i^t$ reflects the degree of consumption positionality when being old instead of when being young. From the assumptions about the utility functions, we have $0 < \alpha_i^t, \beta_i^t < 1$. In addition, let us denote the average degree of consumption positionality in period $t$ by

$$\rho_t = \sum_i \beta_i^{t-1} \frac{n_i^{t-1}}{N_t} + \sum_i \alpha_i^t \frac{n_i^t}{N_t} \in [0,1].$$

Thus, $\rho_t$ reflects the mean value of the degree of consumption positionality for all people alive in period $t$.

Following earlier comparable literature, we assume that leisure is completely non-positional, meaning that people only care about the absolute level of $z$. It is, nevertheless, possible to think of a situation where people also care about their relative leisure compared to others. In addition, although $G$ is a pure public good in our study, one can easily think of publicly provided goods that have a spatial distribution that makes them less valuable for some people, and where people care about the relative possibility to use these goods. For example, people may derive utility not only from having access to a publicly provided recreation area, but also from having better access to such an area than others. Although these assumptions are crucial for most of the subsequent results, it is straightforward to adjust the model in order to take relative concerns with respect to $z$ and $G$ into account. The qualitative insights will still hold as long as private consumption is more positional than leisure and the public good. The (scarce) available empirical evidence is consistent with our approach. Carlsson et al. (2007) found that leisure is, in fact, much less positional than private consumption/income, and that it may even be completely non-positional. Solnick and Hemenway (2005) found that (spatially distributed) public goods too are considerably less positional than private consumption/income.

Otherwise the utility function in equation (1) is quite general and may vary both between types and over time and is furthermore not necessarily time-separable, meaning for example that the marginal rate of substitution between relative and absolute consumption when being old is not necessarily independent of the consumption level when being young. Thus, the model is flexible enough to encompass habit formation in private consumption. Note also that the version of the
utility function in the second line is more general than in the first line. We start the
analysis with the more general case that resembles a classical externality problem e.g.
in terms of pollution associated with private consumption. Some more restrictive
special cases of equation (1) will be discussed subsequently.

The individual budget constraint is given by

\[ w'_i l'_i - T_i (w'_i l'_i) - s'_i = c'_i, \]

\[ s'_i (1 + r_{t+1}) - \Phi_{t+1} (s'_{t+1}) = x'_{t+1}, \]

where \( s'_i \) is savings, \( r_{t+1} \) is the market interest rate, while \( T_i (\cdot) \) and \( \Phi_{t+1} (\cdot) \) denote the
payments of labor income and capital income taxes, respectively. The first order
conditions for the hours of work and savings can be written as

\[ u'_{i,c} w'_i \left[ 1 - T_i (w'_i l'_i) \right] - u'_{i,z} = 0, \]

\[ -u'_{i,c} + u'_{i,x} \left[ 1 + r_{t+1} \left[ 1 - \Phi'_{t+1} (s'_{t+1}) \right] \right] = 0, \]

in which \( u'_{i,c} = \partial u'_i / \partial c'_i \), \( u'_{i,z} = \partial u'_i / \partial z'_i \) and \( u'_{i,x} = \partial u'_i / \partial x'_{t+1} \), whereas \( T_i (w'_i l'_i) \) and
\( \Phi'_{t+1} (s'_{t+1}) \) are the marginal labor income tax rate and the marginal capital income tax
rate, respectively.

The production sector consists of identical competitive firms producing a
homogenous good with constant returns to scale. Given these characteristics, the
number of firms is not important and will be normalized to one for notational
convenience. The production function is given by \( F(L^1_i, L^2_i, K_i) \), where \( L^1_i = n^1_i l^i_i \) is the
total number of hours of work supplied by ability-type \( i \) in period \( t \). The firm
obeys the necessary conditions

\[ F'_e (L^1_i, L^2_i, K_i) - w'_i = 0 \text{ for } i=1,2, \]

\[ F'_k (L^1_i, L^2_i, K_i) - r_i = 0, \]
where subindices attached to the production function denote partial derivatives.

The capital market equilibrium condition becomes

\[ \sum_{i=1}^{2} n_i s_i' = K_{t+1}, \]

meaning that the aggregate savings in period \( t \) will form the capital stock used in the production in period \( t+1 \).

3. The government’s problem

We assume that the government faces a general social welfare function as follows:

\[ W = W(n_0^1 U_0^1, n_0^2 U_0^2, n_1^1 U_1^1, n_1^2 U_1^2, \ldots), \]

which is increasing in each argument. Since the optimum conditions are expressed for any such SWF, they are thus necessary optimum conditions for a Pareto efficient allocation.\(^6\) A similar formulation was used by Pirttilä and Tuomala (2001), although they in addition assumed that the social welfare function was utilitarian within each generation.

The informational assumptions are conventional. The government is able to observe income, although ability is private information. As in most of the earlier literature on the self-selection approach to optimal taxation, we assume that the government wants to redistribute from the high income to the low income earners.\(^7\) This means that the most interesting aspect of self-selection is to prevent the high-ability type from

\(^{6}\) All results obtained here that are independent of the social welfare function (i.e. basically all results that we comment upon) could have been obtained by instead explicitly solving for the Pareto efficient allocation by maximizing the utility of one ability-type born in a certain period, while holding the utility constant for all other agents (the other ability-type born in the same period and both ability-types born in all other periods). The chosen strategy is motivated by convenience, as it simplifies the presentation.

\(^{7}\) This of course implies restrictions on the utility functions beyond what is stated above.
pretending to be a low-ability type. The self-selection constraint that may bind then becomes

\[ U_i^2 = u_i^2(c_i^2, \bar{c}_i^2, \bar{x}_{i+1}, \bar{c}_{i+1}, G_i, G_{i+1}) \]
\[ \geq u_i^2(c_i^1, H - \phi_i^1, x_{i+1}, \bar{c}_{i+1}, G_i, G_{i+1}) = \hat{U}_i^2, \]  

(14)

where \( \phi_i = w_i^2 / w_i^1 \) is the wage ratio (relative wage rate) in period \( t \). The expression on the right-hand side of the weak inequality is the utility of the mimicker. Although the mimicker enjoys the same consumption as the low-ability type in each period, he/she enjoys more leisure (as the mimicker is more productive than the low-ability type).\(^8\)

Note that \( T_i(\cdot) \) is a general labor income tax, which can be used to implement any desired combination of \( l_i^1, c_i^1, l_i^2 \) and \( c_i^2 \) given the savings chosen by each ability-type. Therefore, we will use \( l_i^1, c_i^1, l_i^2 \) and \( c_i^2 \), instead of the parameters of the labor income tax function, as direct decision variables in the optimal tax and expenditure problem. Note also that the general capital income tax, \( \Phi_{i+1}(\cdot) \), can be used to implement any desired combination of \( c_i^1, x_{i+1}, c_i^2, x_{i+1}^2, x_{i+1}^2 \), and \( K_{i+1} \), given the labor income of each individual. Therefore, instead of choosing the parameters of the capital income tax function directly, we formulate the optimization problem such that \( x_{i+1}, x_{i+1}^2 \) and \( K_{i+1} \) are also used as direct decision variables. The resource constraint is given by

\[ F(L_i^1, L_i^2, K_i) + K_i - \sum_{i=1}^2 \left[ n_i^1 c_i^1 + n_i^2 x_i^1 \right] - K_{i+1} - G_i = 0. \]  

(15)

\(^8\) The set of policy instruments facing the government in our framework means that it is able to control the present and future consumption as well as the hours of work of each ability-type (this is discussed more thoroughly below). As a consequence, in order to be a mimicker, the high-ability type must mimic the point chosen on each tax function (both the labor income tax and the capital income tax) by the low-ability type, and thus consume equally much in both periods.
Equation (15) means that output is used for private consumption, net investments and public consumption.

The Lagrangian is written as

\[
\mathcal{L} = W(n_0^1 U_0^1, n_0^2 U_0^2, n_1^1 U_1^1, n_1^2 U_1^2, \ldots) + \sum_i \lambda_i \left[ U_i^2 - \hat{U}_i^2 \right] + \sum_i \gamma_i \left[ F(I_i, L_i, K_i) + K_i - \sum_{i=1}^2 [n_i^1 c_i^1 + n_i^1 x_i^1] - K_{i+1} - G_i \right] .
\]

(16)

Let \( \hat{u}_i^2 = u_i^2 (c_i^1, H - \phi_i^1, x_i^1, \bar{c}_i, \bar{c}_{i+1}, G_i, G_{i+1}) \). For further use, note that

\[
\frac{\partial \mathcal{L}}{\partial \bar{c}_i} = \sum_{i=1}^2 \frac{\partial W}{\partial (n_i^1 U_i^1)} n_{i-1, i}^1 u_{i-1, i}^1 + \sum_{i=1}^2 \frac{\partial W}{\partial (n_i^1 U_i^1)} n_i^1 u_{i, i}^1 + \lambda_{i-1} \left[ u_{i-1, i}^2 - \hat{u}_{i-1, i}^2 \right] + \lambda_i \left[ u_{i, i}^2 - \hat{u}_{i, i}^2 \right] .
\]

(17)

The first order conditions for \( l_i^1, c_i^1, x_i^1, l_i^2, c_i^2, x_i^2, K_{i+1} \) and \( G_i \) are given by

\[
- \frac{\partial W}{\partial (n_i^1 U_i^1)} n_i^1 u_{i, i}^1 + \lambda_i \hat{u}_{i, i}^2 \left[ \phi_i^1 + l_i^1 \frac{\partial \phi_i^1}{\partial l_i^1} \right] + \gamma_i n_i^1 w_i^1 = 0 ,
\]

(18)

\[
\frac{\partial W}{\partial (n_i^1 U_i^1)} n_i^1 u_{i, i}^1 - \lambda_i \hat{u}_{i, i}^2 - \gamma_i n_i^1 + n_i^1 \frac{\partial \mathcal{L}}{\partial \bar{c}_i} = 0 ,
\]

(19)

\[
\frac{\partial W}{\partial (n_i^1 U_i^1)} n_i^1 u_{i, i}^1 - \lambda_i \hat{u}_{i, i}^2 - \gamma_i n_i^1 + n_i^1 \frac{\partial \mathcal{L}}{\partial \bar{c}_{i+1}} = 0 ,
\]

(20)

\[- \frac{\partial W}{\partial (n_i^1 U_i^1)} n_i^2 u_{i, i}^2 + \lambda_i \hat{u}_{i, i}^2 \left[ l_i^2 \frac{\partial \phi_i^2}{\partial l_i^2} + \gamma_i n_i^2 w_i^2 \right] = 0 ,
\]

(21)

\[\text{Note that there is a potential time inconsistency problem involved here (since the government may have incentives to modify the second period taxation facing each generation) once the individuals have revealed their true types. Although we acknowledge this potential problem, we follow earlier comparable literature by only considering situations where the government commits to its tax and expenditure policies. This approach is motivated by the observation that lack of commitment from the point of view of the government opens a spectrum of possibilities for modeling both public policy and the response by the private sector, which would be beyond the scope of this paper.}\]
\[
\frac{\partial W}{\partial (n_i^2 U_{t_i}^2)} n_i^2 + \lambda_i \right] u_{i,t}^2 - \gamma_i n_i^2 + \frac{n_i^2 \partial \xi}{N_i} \frac{\partial }{\partial c_i} = 0, \tag{22}
\]

\[
\frac{\partial W}{\partial (n_i^2 U_{t_i}^2)} n_i^2 + \lambda_i \right] u_{i,t}^2 - \gamma_i n_i^2 + \frac{n_i^2 \partial \xi}{N_{i+1}} \frac{\partial }{\partial c_{i+1}} = 0, \tag{23}
\]

\[\gamma_{i+1} (1 + r_{i+1}) - \gamma_i + \lambda_{i+1} \hat{u}_{i+1,t+1} \frac{\partial \phi_{i+1}}{\partial K_{i+1}} = 0, \tag{24}\]

\[
\sum_{i=1}^{2} \left[ \frac{\partial W}{\partial (n_i^{U_t})} n_i^{u_{i,t,G_i}} + \frac{\partial W}{\partial (n_{i-1}^{U_t})} n_i^{u_{i-1,t,G_i}} \right] + \lambda_i \left[ u_{i,t,G_i} - \hat{u}_{i,t,G_i} \right] + \lambda_{i-1} \left[ u_{i-1,t,G_i} - \hat{u}_{i-1,G_i} \right] - \gamma_i = 0 \tag{25}\]

in which we have used the first order conditions for the firm, i.e. \( w_i = F_i (L_i^1, L_i^2, K_i) \) for \( i=1,2 \), and \( r_i = F_K (L_i^1, L_i^2, K_i) \).

### 3.1 The positionality effect

Before turning to the optimal tax and expenditure expressions, let us first focus on equation (17), where \( \frac{\partial \xi}{\partial c_i} \) will be referred to as the positionality effect, since it reflects the overall welfare effects of a change in the level of reference consumption, ceteris paribus. This effect can be rewritten in terms of the individual degrees of consumption positionality, implying the following:

**Lemma 1.** The welfare effect of increased reference consumption in period \( t \) can be written as

\[
\frac{\partial \xi}{\partial c_i} = -\frac{\rho_i}{1 - \rho_i} \gamma_i N_i + \frac{1}{1 - \rho_i} \left[ \lambda_{i-1} \hat{u}_{i-1,t} \left[ \hat{\beta}_{i-1} - \beta_{i-1} \right] + \lambda_i \hat{u}_{i,t} \left[ \hat{\alpha}_{i} - \alpha_{i} \right] \right]. \tag{26}\]

Therefore, if \( \beta_i \geq \hat{\beta}_i \) and \( \alpha_i \geq \hat{\alpha}_i \), meaning that the low-ability type is at least as positional as the mimicker, so that \( \frac{\partial \xi}{\partial c_i} < 0 \), then increased reference consumption in period \( t \) reduces the welfare.

Proof: see the Appendix.
Two mechanisms are worth noticing. First, in the absence of the self-selection constraint, i.e. if ability-type specific lump-sum taxes were possible to implement, an increase in the reference consumption would unambiguously decrease the welfare, since the reference consumption enters the utility function of each individual via the arguments $\Delta_i = c_i - \bar{c}_i$ and $\Delta_{i+1} = x_{i+1} - \bar{c}_{i+1}$. Thus, the reference consumption constitutes a negative externality for each type in each period. This explains the first term on the right-hand side of equation (26), which relates the positionality effect to the average degree of positionality without any reference to differences in the degree of positionality between ability-types. Second, if the low-ability type is more positional than the mimicker, then an increase in the reference consumption means a larger utility loss for the low-ability type than for the mimicker; as such, it contributes to an additional welfare loss via the self-selection constraint. However, if the mimicker is more positional than the low-ability type, then an increase in the reference consumption contributes to relax the self-selection constraint, implying that the second right-hand side term of equation (26) is positive; this mechanism will be discussed in more detail subsequently. In this case, the sign of $\frac{\partial \xi}{\partial \bar{c}}$ can be either positive or negative.

A consequence of the above discussion is that in the special case where the degree of positionality does not depend on type, equation (26) reduces to

$$\frac{\partial \xi}{\partial \bar{c}} = -\gamma_i N_i \frac{\rho_i}{1 - \rho_i} < 0.$$  

We will return to this special case in the analysis of optimal taxation and public expenditures.

### 4. Tax and expenditure results

In this section, we will present the optimality conditions for the marginal labor income tax rates, the marginal capital income tax rates and the public good provision in a format that facilitates straightforward economic interpretations and comparisons with the benchmark case with no relative consumption concerns.
4.1 Labor Income Taxation

Define the marginal rate of substitution between leisure and private consumption for ability-type $i$ as

$$MRS_{z,c}^{i,j} = \frac{u_{l,c}^i}{u_{l,c}^i},$$

and similarly for the mimicker. The marginal labor income tax rate for the low-ability type is derived by combining equations (8), (18) and (19), while the marginal labor income tax rate for the high-ability type is derived by combining equations (8), (21) and (22). We show in the Appendix that

$$T_1'(w_i^1l_i^1) = \frac{\lambda^*_1}{w_i^1n_i^1} \left[ MRS_{z,c}^{1,j} - \hat{MRS}_{z,c}^{2,j} \left[ \phi_i + \frac{\partial \phi_i}{\partial l_i^1} l_i^1 \right] \right] - \frac{MRS_{z,c}^{1,j}}{\gamma_iw_i^1N_i} \frac{\partial E}{\partial c_i},$$

$$T_2'(w_i^2l_i^2) = -\frac{\lambda^*_2}{w_i^2n_i^2} \hat{MRS}_{z,c}^{2,j} \frac{\partial \phi_i}{\partial l_i^2} l_i^2 - \frac{MRS_{z,c}^{2,j}}{\gamma_iw_i^2N_i} \frac{\partial E}{\partial c_i},$$

where $\lambda^*_i = \lambda_i\hat{u}_{l,c}^2 / \gamma_i$. The first part of each tax formula is analogous to results derived in earlier literature and is due to the self-selection constraint. With $MRS_{z,c}^{1,j} > \hat{MRS}_{z,c}^{2,j}$ (which applies if the preferences do not differ between ability-types), and if we assume (by analogy to earlier comparable literature) that $\partial \phi_i / \partial l_i^j < 0$, the contribution of the self-selection constraint is to increase the marginal labor income tax rate of the low-ability type. Similarly, if $\partial \phi_i / \partial l_i^j > 0$, the self-selection constraint contributes to decrease the marginal labor income tax rate of the high-ability type. These effects are well understood from earlier research; see Stiglitz (1982).

On the other hand, the final part of each formula is novel, and is due to the relative consumption concerns. As indicated above, although an increase in the reference consumption reduces the utility of each ability-type, the derivative $\partial E / \partial c_i$ can be either positive or negative. This is so because it affects the self-selection constraint and, therefore, also the utility of the mimicker. The following result is a consequence of combining Lemma 1 with equations (27) and (28):
**Proposition 1.** If the low-ability type is at least as positional as the mimicker, so that \( \partial \mathcal{E}/\partial \xi_i < 0 \), then the positionality effect contributes to increase the marginal labor income tax rate facing each ability-type, ceteris paribus.

Note that although Proposition 1 gives a sufficient condition for the final part of both equation (27) and (28) to be positive, it is not necessary, since \( \partial \mathcal{E}/\partial \xi_i \) can be negative even if the mimicker is more positional than the low-ability type.

To go further, we make use of the positionality effect definition described above. This enables us to address more thoroughly how the concern for positionality affects the marginal labor income tax rates. Let us use the short notations

\[
\sigma_i^1 = \frac{\lambda_i^1}{w_i^1 n_i^1} \left[ \text{MRS}^{1,1}_{\xi,\xi} - \hat{\text{MRS}}^{2,2}_{\xi,\xi} \left[ \phi_i + \frac{\partial \phi_i}{\partial l_i^1} \right] \right],
\]

\[
\sigma_i^2 = -\frac{\lambda_i^2}{w_i^2 n_i^2} \hat{\text{MRS}}^{2,2}_{\xi,\xi} \frac{\partial \phi_i}{\partial l_i^2},
\]

\[
\Gamma_i = \frac{\lambda_i^1}{\gamma_i N_i} \left[ \beta_i^1 - \beta_i^2 \right] + \frac{\lambda_i^2}{\gamma_i N_i} \left[ \alpha_i^2 - \alpha_i^1 \right],
\]

where \( \sigma_i^1 \) and \( \sigma_i^2 \) reflect the optimal marginal labor income tax rates without relative consumption concerns, i.e. the first term on the right-hand side of equation (27) and (28), respectively. The term \( \Gamma_i \) reflects positionality differences between the mimicker and the low-ability type, where \( \Gamma_i > 0 \) (\(< 0\)) if the mimicker is always (i.e. as both young and old) more (less) positional than the low-ability type. By combining equations (26), (27) and (28), we can then rewrite the formulas for the marginal labor income tax rates such that the contribution of positionality is decomposed into two effects as follows:

**Proposition 2.** The optimal marginal labor income tax rate for each ability-type can be written in the following additive form (for \( i=1,2 \)):

\[
T_i'(w_i l_i') = \sigma_i^1 + [1 - \sigma_i^1] \rho_i - [1 - \sigma_i^1] [1 - \rho_i] \frac{\Gamma_i}{1 - \Gamma_i}.
\]
Proof: See the Appendix.

To interpret Proposition 2, let us start with the simplest first-best case where there is no cost of fulfilling the self-selection constraint, i.e. \( \lambda_t = 0 \) for all \( t \). Then, \( \sigma^1_t = \sigma^2_t = \Gamma_t = 0 \), implying that \( T'_t(w_i^1 l_i^1) = T'_t(w_i^2 l_i^2) = \rho_t \). In this (unrealistic) case, there is no value of further redistribution of income from the high-ability to the low-ability type, and the marginal labor income tax rates reflect a pure efficiency effect. This exemplifies a straightforward Pigouvian tax, where people are taxed for their consumption causing negative (positional) externalities on others, whereas leisure does not. One additional dollar for all in the economy, ceteris paribus, implies that the average utility increase is only \( 1 - \rho_t \) of the sum of each individual’s utility increase in isolation. Therefore, the “loss” \( \rho_t \) reflects the marginal external cost of private consumption.\(^{10}\)

Consider now the more general and realistic second-best formula in Proposition 2. The intuition is again straightforward. The first term on the right-hand side of equation (29) is the tax expression that would follow without any positional concern. The second term reflects the marginal external cost of consumption, which is now modified compared with the first-best. Consider first the low-ability type. The choice of additional work corresponding to a one dollar gross wage increase causes negative external costs as in the first-best case. However, these external costs are now smaller than in first-best, provided that \( \sigma^1_t > 0 \). Indeed, the part of the income increase that is paid in taxes will not imply any positional externalities. By analogy, if \( \sigma^2_t < 0 \), the term \( [1 - \sigma^2_t] > 1 \) means that the government attaches greater weight to the corrective part of the tax formula for the high-ability type than in the first-best. The intuition is that the self-selection component in the formula for the high-ability type is a subsidy, which strengthens the positional externality.

\(^{10}\) This case resembles the identical consumption tax derived by Dupor and Liu (2003) in a first-best economy.
Before discussing the final part of equation (29) in more detail, let us briefly consider the special case where \( \Gamma = 0 \), which is of relevance for comparison with earlier literature on optimal income taxation. In this case, equation (29) reduces to

\[
T_i'(w_i'l_i') = \sigma_i' + (1 - \sigma_i')\rho_i. \tag{30}
\]

Equation (30) is particularly striking from the perspective of the marginal labor income tax rate of the high-ability type. A common assumption in earlier literature on general income taxation is that the wage rates are fixed. In this case, \( \sigma_i^2 = 0 \) and equation (30) simplifies to \( T_i'(w_i'^2l_i'^2) = \rho_i \), as externality correction would be the only motive for taxing the high-ability type at the margin.

Returning once again to Proposition 2, the third term on the right-hand side of equation (29) reflects self-selection effects of positional concerns. Suppose first that \( \Gamma > 0 \), in which case the mimicker is more positional than the low-ability type. This means that a given decrease in private consumption (due to higher taxation) causes a larger utility loss for the low-ability type than for the mimicker. Therefore, mimicking becomes more attractive and the relevant self-selection constraint tightens. This provides an incentive for the government to implement a lower marginal labor income tax rate than it would otherwise have done, which means that the third term contributes to decrease the marginal labor income tax rate. On the other hand, if \( \Gamma < 0 \), then the opposite argument applies, as a higher marginal labor income tax rate in this case tends to relax the self-selection constraint. Consider also the two factors that are proportional to \( \Gamma_i \). The factor \( [1 - \sigma_i'] \) is interpretable in a way similar to its effect on the second term of equation (29): if \( [1 - \sigma_i'] \) of an additional dollar is already taxed away, it does not give rise to positional externalities. Similarly, the factor \( [1 - \rho_i] \) appears because the induced self-selection effects are due to the non-positional part of the marginal income.

In summary, the mechanism behind the third term can explain why it is theoretically possible that relative income concerns work to reduce the marginal labor income tax
rate. If $\Gamma > 0$, and if increased marginal income taxation creates a sufficiently strong
incentive to become a mimicker, then this effect may dominate the externality-correcting component.

Consider now the order of magnitudes. Since concerns for relative consumption are
difficult to measure, it is not surprising that the available estimates of $\rho$ vary
considerably in the literature, although almost all estimates are substantially above
zero. For example, according to Alpizar et al. (2005) and Carlsson et al. (2007), $\rho$
is typically in the order of magnitude of 0.5, whereas Luttmer (2005) obtained larger
estimates close to one. There is little evidence regarding the size of $\Gamma$; perhaps a
value of zero is a reasonable first approximation. Overall, the results then suggest that,
given the framework, the optimal marginal labor income tax rates may be
substantially higher when taking relative consumption effects into account.

4.2 Capital Income Taxation

Let us then turn to the marginal capital income tax structure. Define the marginal rate
of substitution between consumption in periods $t$ and $t+1$ for ability-type $i$,

$$MRS_{i,t} = \frac{u_{i,t}}{u_{i,t+1}},$$

and similarly for the mimicker. The marginal capital income tax rate for the low-
ability type can be derived by combining equations (9), (19), (20) and (24), whereas
the marginal capital income tax rate for the high-ability type can be derived by
combining equations (9), (22), (23) and (24). We show in the Appendix that the
marginal capital income tax rates can be written as

$$\Phi_{i,t+1}(s_{i,t}^{1}, r_{i,t}^{1}) = \frac{\hat{\lambda}_{i,t}^{2}}{\gamma_{i,t} n_{i,t}^{1} r_{i,t}^{1}} \left[ MRS_{i,t} - \hat{MRS}_{i,t}^{2} \right] - \frac{\hat{\lambda}_{i,t}^{2} \hat{u}_{i,t}^{2} f_{i,t}^{1}}{\gamma_{i,t} r_{i,t}^{1}} \frac{\partial \Phi_{i,t}}{\partial K_{i,t}},$$

(31)
Let us start by discussing the marginal capital income tax rate of the low-ability type. Note that the first row is due to the appearance of the self-selection constraints. The first term reflects the self-selection constraint in period $t$. It means that if the relative valuation of current consumption by the low-ability type exceeds (falls short of) the relative valuation by the mimicker, there is an incentive for the government to stimulate (discourage) current consumption via a higher (lower) marginal capital income tax rate. As such, this incentive effect serves to relax the self-selection constraint by making mimicking less attractive. There is a similar purpose behind the second term in the first row, although this effect is associated with the self-selection constraint in period $t+1$. It arises here because the savings in period $t$ determines the capital stock in period $t+1$. If an increase in the capital stock increases (decreases) the wage ratio, then mimicking becomes less (more) attractive, providing an incentive for the government to stimulate (discourage) savings by choosing a lower (higher) marginal capital income tax rate than it would otherwise have done. Note also that the first row of the formula for the high-ability type is analogous to, and has the same interpretation as, the second term in the first row of the formula for the low-ability type. These effects are well understood from earlier research.

The second row of each tax formula is novel and refers to the assumption that the private consumption good is, in part, a positional good. As the marginal capital income tax rates reflect a desired tradeoff between present and future consumption, each such term is decomposable into two parts. The intuition is, of course, that each individual values relative consumption both when young and old. By combining Lemma 1 with equations (31) and (32), we can derive the following result:

**Proposition 3.** If the low-ability type is at least as positional as the mimicker in all periods, then the positionality effect in period $t$, $\partial \lambda / \partial \varepsilon < 0$, contributes to decrease the marginal capital income tax rates in period $t+1$, whereas the positionality effect in
period $t+1$, $\partial \xi / \partial \tau_{t+1} < 0$, contributes to increase the marginal capital income tax rates in period $t+1$, ceteris paribus.

The intuition behind Proposition 3 is straightforward. The positionality effect in period $t$ means that an increase in the average consumption in period $t$ gives rise to a welfare loss. This provides an incentive for the government to choose lower marginal capital income tax rates than it would otherwise have done, which in turn stimulates savings and discourages consumption in period $t$. By analogy, the positionality effect in period $t+1$ means that an increase in the average consumption in period $t+1$ results in a welfare loss. As a consequence, there is an incentive for the government to reduce the average consumption in period $t+1$, which means that the government chooses higher marginal capital income tax rates than it would otherwise have done. The relative size of these two effects determines whether the appearance of positional goods constitutes an incentive to tax or subsidize the capital income, ceteris paribus.

So far, we have not used the decomposition of the positionality effect given by equation (26). In general, since two such effects are involved, this decomposition does not give results that are as easy to interpret as the corresponding expressions for the marginal labor income tax rates in Proposition 2. Nevertheless, it is instructive to combine equation (26) with equations (31) and (32) in the special case where the degree of positionality does not vary over time. Consider Proposition 4:

**Proposition 4.** If the average degree of positionality as well as the positionality differences between the mimicker and the low-ability type remain constant over time, so that $\rho_{t+1} = \rho_t$ and $\Gamma_{t+1} = \Gamma_t = \Gamma$, then the marginal capital income tax rates reduce to

$$
\Phi_{t+1}(s_i^1, r_{t+1}) = \frac{\hat{\lambda}_{i,x} \hat{\mu}_{i,x}}{\gamma_{t+1} r_{t+1}^1} \left[ MRS_{c,x}^{1,i} - MRS_{c,x}^{2,i} \right] \frac{1-\rho}{1-\Gamma} - \frac{\hat{\lambda}_{t+1,i,x} \hat{\mu}_{t+1,i,x}}{\gamma_{t+1} r_{t+1}^1} \frac{\partial \phi_{t+1}}{\partial K_{t+1}}, \quad (33)
$$

$$
\Phi_{t+1}(s_i^2, r_{t+1}) = -\frac{\hat{\lambda}_{i,x} \hat{\mu}_{i,x}}{\gamma_{t+1} r_{t+1}^1} \frac{\partial \phi_{t+1}}{\partial K_{t+1}}. \quad (34)
$$

Proof: See the Appendix.
Two aspects of Proposition 4 are worth emphasizing. First, there is no direct effect of positionality in the tax formulas and, second, there is no need to modify the effects of the self-selection constraint that are common in the two tax formulas (the relationship between each marginal capital income tax rate of the aggregate capital stock). Therefore, in this special case, the appearance of positionality does not change the way in which we measure the marginal capital income tax rate of the high-ability type (compared with an economy without positional goods). The intuition is that under the conditions in the proposition, the current and future aspects of positionality cancel out to a large extent, suggesting that the incentives underlying capital formation are similar to those that would apply in economies without positional goods. However, this does of course not mean that the effect of positionality that still remains is necessarily unimportant.

Note that leisure is not generally weakly separable from private consumption. As a consequence, the low-ability type and the mimicker will differ with respect to the relative value attached to current consumption: the contribution of this difference to the marginal capital income tax rate of the low-ability type is still affected by concern for positionality. To interpret the “positionality-weight” \([1 - \rho]/[1 - \Gamma]\), consider first the situation where \(MRS_{c,x}^{1,t} > MRS_{c,x}^{2,t}\), meaning that the first term on the right hand side of equation (33) contributes to increase the marginal capital income tax rate of the low-ability type. As such, this term works to increase the current (first period) consumption of the low-ability type and, as a consequence, also the reference consumption in period \(t\). The expression \(1 - \rho\) serves to modify this effect, as increased reference consumption gives rise to positional externalities. In other words, if we (for the moment) were to abstract from differences in the degree of positionality between the mimicker and the low-ability type, implying that \(\Gamma = 0\), the positionality-weight works to decrease the marginal capital income tax rate. This effect is counteracted (further strengthened) by \(\Gamma > 0\) \((< 0)\), as increased references consumption, in this case, relaxes (tightens) the self-selection constraint in period \(t\). The interpretation is analogous if \(MRS_{c,x}^{1,t} < MRS_{c,x}^{2,t}\).
It is worth emphasizing once again that there is no direct effect of positionality in equations (33) and (34) that is independent of the self-selection constraint. The following result is a direct consequence of Proposition 4.11

**Corollary 1.** Suppose that the average degree of positionality as well as the positionality differences between the mimicker and the low-ability type remain constant over time. Then, if leisure is weakly separable from private consumption in the sense that $U^i_t = q^i_t(f^i_t(c^i_t, x^i_t, G^i_t, G_{t+1}, c^i_t, G^i_{t+1}), z_t, e_t, G_{t+1}, G^i_{t+1})$ describes the utility function, and the wage ratio is constant so that $\frac{\partial \phi^i_{t+1}}{\partial K_{t+1}} = 0$, then both marginal capital income tax rates are zero.

Note that the function $f(\cdot)$ is the same for both ability-types, while the function $q^i_t(\cdot)$ may still vary across ability-types. Although the above result is based on assumptions that may not seem entirely realistic, it is, nevertheless, interesting from the perspective of comparison with earlier literature. Corollary 1 implies that the important result derived by Ordover and Phelps (1979), for when capital taxation is not needed, carries over to our more general case that includes relative consumption concerns.

### 4.3 Public good provision

Define the marginal rate of substitution between public and private consumption for ability type $i$, when young and old respectively, in period $t$ as

$$MRS^{ij}_{G,c} = \frac{u_{t,i,G,c}}{u^i_{t,c}},$$

$$MRS^{ij}_{G,x} = \frac{u_{t,i+1,G,x}}{u^i_{t-1,x}},$$

and similarly for the mimicker. Using these definitions and substituting the social optimum conditions for private consumption, i.e. equations (19), (20), (22) and (23), into equation (25), we show in the Appendix that

11 From the separability assumption follows that $MRS^{ij}_{c,x} = M\dot{R}S^{2j}_{c,x}$. 

24

\[ MB_{t,G} = 1 + \lambda_t u_{t,x}^2 \left[ M\hat{R}S_{G,x}^{1,t} - MRS_{G,x}^{1,t} \right] + \frac{\lambda_{t-1} u_{t-1,x}^2}{\gamma_t} \left[ M\hat{R}S_{G,x}^{2,t} - MRS_{G,x}^{1,t} \right] + \frac{MB_{t,G}}{\gamma_t N_t} \frac{\partial \mathcal{L}}{\partial \mathcal{C}_t} \]  

(35)

where \( MB_{t,G} = n_t^{1R} MRS_{G,x}^{1,t} + n_t^{2R} MRS_{G,x}^{2,t} + n_{t-1}^{1R} MRS_{G,x}^{1,t} + n_{t-1}^{2R} MRS_{G,x}^{2,t} \) is the sum of each individual’s (alive in period \( t \)) marginal willingness to pay (WTP) for the public good in period \( t \) expressed in terms of his/her private consumption in this period. Note that this marginal WTP is defined while holding the consumption of everybody else fixed, which will be discussed further below. The first term on the right-hand side of equation (35) reflects the marginal rate of transformation between public and private goods, which is by assumption normalized to unity, and the second and third terms reflect self-selection effects that are well understood from earlier research. The fourth term is novel. We have:

**Proposition 5.** If the low-ability type is at least as positional as the mimicker, so that \( \partial \mathcal{L} / \partial \mathcal{C}_t < 0 \), then the positionality effect contributes to increase the optimal provision of the public good.

The intuition is straightforward. When private consumption causes negative (positional) externalities and public consumption does not, it is optimal to provide relatively more of the public good.12

In order to express the optimality condition in terms of individual degrees of positionality, note that \( [\partial \mathcal{L} / \partial \mathcal{C}_t] / [\gamma_t N_t] = [\Gamma_t - \rho_t] / [1 - \rho_t] \). Using the short notation

\[ \Omega = \lambda_t u_{t,x}^2 \left[ M\hat{R}S_{G,x}^{2,t} - MRS_{G,x}^{1,t} \right] + \frac{\lambda_{t-1} u_{t-1,x}^2}{\gamma_t} \left[ M\hat{R}S_{G,x}^{2,t} - MRS_{G,x}^{1,t} \right] \]

12 Note that Proposition 5 should of course not be interpreted to mean that the amount provided in an economy where people care about relative income should necessarily be larger than in an economy where they do not. Rather, the appropriate interpretation is that given that people do care about relative consumption, the provision of the public good should be extended beyond the level that corresponds to the optimality rule without considering the positionality effect.
for the self-selection terms that would result without any positional concerns, we obtain:

**Proposition 6.** The optimal provision of the public good is given by

\[
MB_{i,G} = (1 + \Omega_i) \frac{1 - \rho_i}{1 - \Gamma_i}.
\]

(36)

Before interpreting each factor, we can observe the following result for a special case:

**Corollary 2.** If the mimickers’ degree of positionality is the same as the one of the low-ability type, as both young and old, and if leisure is weakly separable from private and public consumption when young and when old, so that the utility function can be written as

\[
U_t^i = q'_i(f(c_t^i, G_t, c_{i+1}), g(x_{i+1}^t, G_{i+1}, c_{i+1}, c_{i+1}^t), z_t^i, c_{i+1}, c_{i+1}^t),
\]

then the optimal provision of \( G \) is given by

\[
MB_{i,G} = 1 - \rho_i.
\]

Note that the functions \( f(\cdot) \) and \( g(\cdot) \) are the same for both types, while the function \( q'_i(\cdot) \) can still vary. Although the assumptions underlying Corollary 2 are strong, they will, nevertheless, provide a natural benchmark case. The interpretation is straightforward. Given suitable separability assumptions and assumptions about the positionality distribution, we cannot relax the self-selection constraint via provision of public goods. Now, remember that \( \rho \) is the average degree of positionality, meaning that, on average, \( \rho \) is the fraction of the utility increase from a one dollar increase in private consumption that comes from increased relative consumption. If each person

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13 If the mimickers’ degree of positionality is the same as that of the low-ability type, as both young and old, then clearly \( \Gamma_i = 0 \). From the separability assumption follows that \( MRS_{c^{ij}_{i,0}} = MRS_{c^{ij}_{i,i}} \) and \( MRS_{c^{ij}_{i,0}} = MRS_{c^{ij}_{i,i}} \). Note also that the separability structure is general enough to allow the intertemporal marginal rate of substitution of private consumption to depend on leisure. For example, it may seem plausible that consumption when being young (compared to when being old) becomes relatively more valuable when leisure increases. A utility function such as the one in Corollary 1 would not allow this.
receives one additional dollar, then the relative consumption is held constant, and what remains is the absolute, or the non-positional, utility effect. Assume, for example, that \( \rho = 0.8 \) and that everybody pays one additional dollar for increased provision of the public good. Then the utility decrease of this payment is only 20\% (i.e. 1-0.8) as large as it would have been had we aggregated the utility losses of each dollar payment in isolation. For the public good there is by definition no such leakage.

Consider now the interpretation of equation (36) more generally. In the absence of positional effects, the term \( 1 + \Omega_i \) would equal the sum of marginal rates of substitution on the left-hand side, where \( \Omega_i \) reflects effects on the self-selection constraint of a marginal substitution of private consumption for public consumption. Since a tighter self-selection constraint implies a social welfare loss, it implies a social cost if it becomes more attractive for the high-ability type to mimic the low-ability type. Consider now the effect of \( \Gamma_i \). An additional public good provision clearly implies a reduction in private consumption. If the mimicker is more positional than the low-ability type, then \( \Gamma_i > 0 \). The reduction in private consumption for all implies a relatively larger utility loss for the less positional. Thus, in this case, it becomes relatively more attractive to become a mimicker, in turn implying that the relevant self-selection constraint tightens and a corresponding marginal social cost.

Let us now return to the issue of the interpretation of the benefit side, i.e. of \( MB_{i,G} \). How to measure the benefit of a public good is a classic problem in economics at least since Samuelson (1954). There are different practical methods that we will not dig deeper into here. However, we briefly discuss the implications of different ways to measure the benefit given that the method works as intended. In principle, i.e. given that people respond truthfully according to their preferences, \( MB_{i,G} \) reflects the sum of all people’s marginal WTP for \( G \), ceteris paribus, i.e. while holding everything else fixed. However, an increase in \( G \) typically comes together with other changes, notably that other people’s taxes or charges are increased. In one frequently used method, the survey-based so-called contingent valuation (CV) method, it is typically recommended (see Arrow et al. 1993) that a realistic payment vehicle is used when asking people about their maximum WTP. One often used payment vehicle is to ask
the subjects how they would vote in a referendum where everybody would have to pay a certain amount, the same for all, through increased taxes (or charges) for the improvement. In the standard case where people do not care about relative consumption, this formulation has no important theoretical implication given that people respond truthfully (although it may of course make the exercise more realistic). Here, however, it does. Consider the case where others will have to pay the same amount as an individual $i$ for the increment. This implies that $i$’s relative consumption will be unaffected, so $c_i' - c_i$ and $x_{i+1}' - c_{i+1}$ are constant. This, in turn, implies that the relevant marginal WTP measures will instead be given by

$$CVMRS_{G,c}^{i,t} = \frac{v_i^t}{v_i^c},$$

$$CVMRS_{G,x}^{i,t} = \frac{v_i^{t-1}}{v_i^{t-1,x}},$$

where $CVMRS_{G,c}^{i,t}$ can be interpreted as ability type $i$’s marginal WTP for $G$ in terms of $c$, provided that everybody else alive at this moment in time will have to pay the same amount at the margin. We then have a corresponding aggregated benefit measure as

$$CVMB_{i,G} = n_1CVMRS_{G,c}^{i,t} + n_2CVMRS_{G,c}^{2,t} + n_{t-1}CVMRS_{G,x}^{i,t} + n_{t-1}CVMRS_{G,x}^{2,t}.$$  

It can then be shown (see the Appendix) that equation (36) can be rewritten as

$$CVMB_{i,G} = \frac{1 + \Omega_t}{(1 - \Gamma_t)(1 + \Psi_t)},$$  \hspace{1cm} (37)

where $\Psi_t$ reflects the normalized covariance between the degree of non-positionality of private consumption and the marginal WTP as reflected by a contingent valuation study of a referendum type. Thus,

$$\Psi_t = \text{cov} \left( \frac{1 - \alpha_t}{1 - \rho_t}, \frac{CVMRS_{G,c}^{i,t}}{CVMRS_{G,c}^{i,t}} \right),$$
where a bar denotes mean value, and $\alpha_i$ and $CVMRS_{G,c}^{i}$, irrespective of type and age, denote the degree of positionality and marginal WTP for the public good conditional on the payments of others, respectively. We can then derive:  

Proposition 7. If (i) the mimickers’ degree of positionality is the same as that of the low-ability type, as both young and old, (ii) leisure is weakly separable from private and public consumption, when young and when old, so that the utility function can be written as $U_i^t = q_t^i (f(c_t^i, G_t, \bar{c}_t, \bar{c}_{t+1}), g(x_{t+1}^i, G_{t+1}, \bar{c}_{t+1}, \bar{c}_t), z_t^i, \bar{c}_t, \bar{c}_{t+1})$, and (iii) the individual maximum WTP for the public good is elicited with a payment vehicle where all individuals have to pay the same amount, then the optimal provision of the public good is given by $CVMB_{t,G} = 1$.

Therefore, given that all individuals have to pay the same amount at the margin, the marginal benefit measured as the sum of marginal WTP should equal the marginal production cost. In other words, we are back to the basic cost-benefit rule where marginal benefit in terms of people’s aggregate marginal WTP equals marginal cost. Alternatively speaking, the basic Samuelson (1954) rule holds; the aggregate marginal rate of substitution between the public good and the private consumption is equal to the corresponding marginal rate of transformation when the relative consumption is

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14 The self-selection effects have the same interpretation in equation (36) as in equation (37). The factor $\Psi_i$ appears to have no important economic interpretation. It reflects the fact that the condition stated in the WTP question is in general impossible to realize in practice. To see this, consider a situation where the sum of people’s conditional aggregate WTP equals the relative production price ($=1$). Now, assume that there are equally many low- and high-ability types, and that high-ability individuals have a higher conditional WTP than low-ability individuals (12 instead of 10 USD). Also, let the good be financed at the margin so that the high-ability individuals pay 12 USD and the low-ability type 10 USD. However, since the average conditional marginal WTP is 11 USD, the average actual payment for the good by others is also 11 USD. This means that the relative income of the high-ability individuals has decreased, and that low-ability individuals have obtained an equally large relative income increase. The net welfare effect is then given by the positive relative income effect times the positionality of the low-ability individuals minus the negative relative income effect times the positionality degree of the high-ability individuals. Thus, the net welfare effect is positive if those with a lower marginal WTP are more positional than those with a higher marginal WTP. In other words, the net welfare effect is positive if $\Psi_i > 0$. 
held constant. The intuition is again straightforward. If others too have to pay, there is no “leakage” of the change in private consumption through relative consumption effects, implying that there are no reasons to correct for positional effects. However, what is perhaps less clear is whether people really manage to see through all effects while responding to WTP questions in practice. Still, before taking possible cognitive limitations or other deviations from rationality into account, it is important to know the point of departure.

5. Conclusion

This paper has analyzed the importance of relative consumption concerns for optimal nonlinear labor income and capital income taxation as well as for public good provision in an OLG framework. The results are possible to express in terms of straightforward modifications of the standard optimality results. Under reasonable assumptions, relative consumption concerns work in the direction of increasing the marginal labor income tax rates. Moreover, linking the results to available empirical evidence on the degree to which people care about relative consumption suggests that the marginal labor income tax rates may be substantially higher when such concerns are taken into account. The results on capital income taxation are less clear-cut, and relative consumption concerns can affect the marginal capital income tax rates in either direction, at least within the current framework. An important result derived by Ordover and Phelps (1979) for when capital taxation is not needed is also shown to carry over to our more general case.

The importance of relative consumption concerns for public good provision depends, in principle, on how people’s marginal WTP for a public good increase is elicited. If people are asked about their WTP independently of others, then relative consumption concerns may imply that substantially more of the public good should be provided compared to the choice rule without such concerns. However, if people’s maximum WTPs are elicited conditional on a payment vehicle where others have to pay too, e.g. through a tax increase, then there is little effect of relative consumption concerns on the appropriate choice rule for public good provision. Whether people in reality manage to see through all interdependent effects is of course less clear, and is an issue for future research also more generally.
Although our framework is more general than earlier comparable literature, several strong assumptions remain. First, we assume (as do other comparable studies) that individuals only work during the first period of life. This is an assumption of importance for the structure of the marginal labor income tax rates. Second, our study is based on a closed economy, which means that we disregard the possibility that cross-country interactions affect the measure of reference consumption used at the individual level. A relaxation of the latter assumption is particularly interesting, as it opens up for the study of public policy and relative consumption in a multi-country setting, where people also to some extent compare their consumption with the consumption of people in other countries. This means that the economy is characterized by transboundary positional externalities, suggesting that the optimal tax and expenditure policies derived in the context of a non-cooperative Nash equilibrium may differ substantially from those corresponding to a (second-best) cooperative equilibrium. Therefore, relative consumption concerns may also be an argument for international policy coordination. We leave these and other possible extensions for future research.

Appendix

Proof of Lemma 1

From equation (1) we have that \( u'_{t,c} = u'_{t,x} + v'_{t,s} \), \( u'_{t,\pi} = -v'_{t,s} \), \( u'_{t,x} = v'_{t,s} + v'_{t,\pi} \), and \( u'_{t,\pi,\pi} = -v'_{t,\pi,\pi} \), so

\[
\begin{align*}
    u'_{t,\pi} &= -\alpha_i u'_{t,c} , \quad \text{(A1)}
    \\
    u'_{t,\pi,\pi} &= -\beta_i u'_{t,x} . \quad \text{(A2)}
\end{align*}
\]

Corresponding expressions hold for the mimicker. By combining equations (17), (A1) and (A2), and the corresponding expressions for the mimicker, we obtain
\[ \frac{\partial E}{\partial c_t} = -\sum_{i=1}^{2} \frac{\partial W}{\partial (n_{i}^{1}u_{i,x})} n_{i-1}^{1}\beta_{i-1}^{1}u_{i-1,x}^{1} - \sum_{i=1}^{2} \frac{\partial W}{\partial (n_{i}^{2}u_{i,x})} n_{i}^{2}\alpha_{i}^{2}u_{i,x}^{2}. \]  

(A3)

Note that equations (19), (20), (22) and (23) imply

\[ \frac{\partial W}{\partial (n_{i}^{1}u_{i,x})} n_{i}^{1}u_{i,x}^{1} = \lambda_{i}^{1}\hat{u}_{i,x}^{1} + \gamma_{i}^{1}n_{i}^{1} - \frac{n_{i}^{1}\partial E}{N_{i}^{1}}, \]  

(A4)

\[ \frac{\partial W}{\partial (n_{i}^{2}u_{i,x})} n_{i}^{2}u_{i,x}^{2} = -\lambda_{i}^{2}u_{i,x}^{2} + \gamma_{i}^{2}n_{i}^{2} - \frac{n_{i}^{2}\partial E}{N_{i}^{2}}, \]  

(A5)

\[ \frac{\partial W}{\partial (n_{i-1}^{1}u_{i,x})} n_{i-1}^{1}u_{i,x}^{1} = \lambda_{i-1}^{1}\hat{u}_{i,x}^{1} + \gamma_{i-1}^{1}n_{i-1}^{1} - \frac{n_{i-1}^{1}\partial E}{N_{i}^{1}}, \]  

(A6)

\[ \frac{\partial W}{\partial (n_{i-1}^{2}u_{i,x})} n_{i-1}^{2}u_{i,x}^{2} = -\lambda_{i-1}^{2}u_{i,x}^{2} + \gamma_{i-1}^{2}n_{i-1}^{2} - \frac{n_{i-1}^{2}\partial E}{N_{i}^{2}}. \]  

(A7)

Substituting equations (A4), (A5), (A6) and (A7) into equation (A3) gives equation (26).

**The Marginal Labor Income Tax Rates**

Consider the tax formula for the low-ability type. By combining equations (18) and (19), we obtain

\[ \frac{n_{i}^{1}u_{i,x}^{1}}{n_{i}^{1}u_{i,x}} \left[ \lambda_{i}^{1}\hat{u}_{i,x}^{1} + \gamma_{i}^{1}n_{i}^{1} - \frac{n_{i}^{1}\partial E}{N_{i}^{1}} \right] = \lambda_{i}^{1}\hat{u}_{i,x}^{1} \left[ \phi_{i}^{1} + l_{i}^{1} \frac{\partial T^{i}}{\partial l_{i}^{1}} \right] + \gamma_{i}^{1}n_{i}^{1}w_{i}^{1}. \]

(A8)

By substituting \( T'(w_{i}^{1})w_{i}^{1} = w_{i}^{1} - u_{i,x}^{1}/u_{i,x}^{1} \) into equation (A8) and rearranging we obtain equation (27). The marginal labor income tax rate of the high-ability type, equation (28), can be derived in a similar way.

To derive equation (29), we combine equations (26) and (27) to obtain
\[
T'(w_{i,t}^I) = \frac{\lambda^*_t}{w_{i,t}^{I}} \left[ MRS_{z,e}^{\alpha,t} - \hat{MRS}_{z,e}^{\alpha,t} \left[ \phi_t + \frac{\partial \phi_t}{\partial l^I_t} \right] \right] \\
- \frac{1}{\gamma_t w_i^{I} N_t} \left[ -\hat{\lambda}_{t-1}(\beta_{t-1}^{2} - \hat{\beta}_{t-1}^{2})\hat{u}_{t-1,t}^{2} - \hat{\lambda}_t(\hat{\alpha}_t^{2} - \hat{\alpha}_t^{2})u_{t,e}^{2} \right].
\] (A9)

Then, by using \( MRS_{z,e}^{\alpha,t}/w_i^I = 1 - T'(w_{i,t}^I) \) and rearranging, we obtain equation (29) for the low-ability type. That equation (29) also holds for the high-ability type can be verified by instead combining equations (26) and (28) and arranging in a similar way as for the low-ability type.

**The Marginal Capital Income Tax Rates**

Let us consider the marginal capital income tax rate of the low-ability type. By combining equations (19) and (20), we obtain

\[
MRS_{e,x}^{\alpha,t} \left[ \lambda_t \hat{u}_{t,x}^{2} - \gamma_{t} n_{t}^{I} + \frac{n_{t}^{I}}{N_{t+1}^{I}} \frac{\partial \xi_t}{\partial c_t} \right] = \lambda_t \hat{u}_{t,x}^{2} + \gamma_{t} n_{t}^{I} - \frac{n_{t}^{I}}{N_{t}^{I}} \frac{\partial \xi_t}{\partial c_t}.
\] (A10)

Then use equations (9) and (24) to derive \( MRS_{e,x}^{\alpha,t} = 1 + r_{t+1} - r_{t+1} \Phi(s_{t+1} r_{t+1}) \) and \( \gamma_t = \gamma_{t+1}(1 + r_{t+1}) + \lambda_t \hat{u}_{t+1,x}^{2} l_{t+1}^{I} \left[ \partial \phi_{t+1} / \partial K_{t+1} \right] \), respectively. Substituting into equation (A10) and rearranging, we obtain equation (31). Equation (32) can be derived in a similar way.

To derive equations (33) and (34), let us substitute \( [\partial \xi_t/\partial c_t]/[\gamma_t N_t] = [\Gamma_t - \rho^2]/[1 - \rho^2] \) as well as the corresponding expression for period \( t+1 \) into equations (31) and (32). We shall also use the short notations

\[
\delta^1_t = \frac{\lambda_t \hat{u}_{t,x}^{2}}{\gamma_{t+1} n_{t+1}^{I} r_{t+1}} \left[ MRS_{e,x}^{\alpha,t} - \hat{MRS}_{e,x}^{\alpha,t} \right] - \frac{\lambda_t \hat{u}_{t+1,x}^{2} l_{t+1}^{I}}{\gamma_{t+1} r_{t+1}^{I}} \frac{\partial \phi_{t+1}}{\partial K_{t+1}}
\]

\[
\delta^2_t = -\frac{\lambda_t \hat{u}_{t+1,x}^{2} l_{t+1}^{I}}{\gamma_{t+1} r_{t+1}^{I}} \frac{\partial \phi_{t+1}}{\partial K_{t+1}}
\]
to represent the self-selection terms in equations (31) and (32). The marginal capital income tax rates can then be rewritten as

\[ \Phi'_{t+1}(s_{t+1}^i) = \delta_i^t + \frac{\gamma_i}{\gamma_{t+1}} \frac{1}{1 - \rho_t} \frac{1 + r_{t+1}}{1 - \Gamma_{t+1}} - \frac{1 + r_{t+1}}{1 - \Gamma_{t+1}} \rho_{t+1} - \Gamma_{t+1} \]

for \( i = 1, 2 \). Now, using \( MRS_{c,x}^{ij} = 1 + r_{t+1} - r_{t+1} \Phi'_{t+1}(s_{t+1}^i) \) and rearranging, we obtain

\[ \Phi'_{t+1}(s_{t+1}^i) = \delta_i^t \frac{1 - \rho_{t+1}}{1 - \Gamma_{t+1}} - \frac{\gamma_i}{\gamma_{t+1}} \frac{1}{1 - \rho_t} \frac{1 - \rho_{t+1}}{1 - \Gamma_{t+1}} + \frac{1 + r_{t+1}}{r_{t+1}} \rho_{t+1} - \Gamma_{t+1} \]

Assuming \( \rho_{t+1} = \rho_t = \rho \) and \( \Gamma_{t+1} = \Gamma_t = \Gamma \), and then substituting into equation (A11) gives

\[ \Phi'_{t+1}(s_{t+1}^i) = \delta_i^t \frac{1 - \rho}{1 - \Gamma} + \frac{(1 + r_{t+1}) - \gamma_i / \gamma_{t+1}}{r_{t+1}} \rho - \Gamma. \]

By substituting \((1 + r_{t+1}) - \gamma_i / \gamma_{t+1} = -[\lambda_{t+1} / \gamma_{t+1}] \hat{\alpha}_{t+1} \hat{\lambda}_{t+1} \hat{\lambda}_{t+1} [\partial \phi_{t+1} / \partial K_{t+1}]\) into equation (A12) and using the definition of \( \delta_i^t \), we obtain equations (33) and (34).

**Public Good Provision**

Using the MRS definitions we can rewrite equation (25) as

\[ \sum_{j=1}^{2} \left[ \frac{\partial W}{\partial (n_i u_i^j)} n_i u_i^j MRS_{G,c}^{ij} + \frac{\partial W}{\partial (n_{j-1} u_{j-1}^i)} n_{j-1} u_{j-1}^i MRS_{G,x}^{ij} \right] + \lambda_i \left[ u_{i,G}^2 - \hat{u}_{i,G}^2 \right] + \lambda_{i-1} \left[ u_{i-1,G}^2 - \hat{u}_{i-1,G}^2 \right] - \gamma_i = 0 \]

By substituting equations (A4)-(A7) into equation (A13), we obtain
Using again the $MRS$ definitions and the definition of $MB_{t,G}$, we obtain equation (35).

To derive equation (37), note first that

$$MRS_{G,c}^{ij} = CVMRS_{G,c}^{ij} \frac{v_{t,c}^j}{u_{t,c}^j},$$

$$MRS_{G,x}^{ij} = CVMRS_{G,x}^{ij} \frac{v_{t-1,x}^j}{u_{t-1,x}^j}.$$  

Then, from equation (1) we have

$$\frac{v_{t,c}^j}{u_{t,c}^j} = 1 - \alpha_t^j,$$

$$\frac{v_{t-1,x}^j}{u_{t-1,x}^j} = 1 - \beta_{t-1}^j,$$

so

$$MRS_{G,c}^{ij} = (1 - \alpha_t^j)CVMRS_{G,c}^{ij},$$  

$$MRS_{G,x}^{ij} = (1 - \beta_{t-1}^j)CVMRS_{G,x}^{ij}.$$  

By substituting equations (A15) and (A16) into equation (36), we obtain

$$\sum_i (1 - \alpha_t^i)CVMRS_{G,c}^{ij} + (1 - \beta_{t-1}^i)CVMRS_{G,x}^{ij} = (1 + \Omega)\frac{1 - \rho}{1 + \Gamma}.$$  

The left-hand side of equation (A17) can be rewritten by using a covariance expression:
\[
\sum_{i} (1 - \alpha'_{i}) CVMRS^{i}_{G,x} + (1 - \beta'_{i,x}) CVMRS^{i}_{G,s}
= (1 - \rho_{i}) CVMB_{i,\alpha} \left[ 1 + \text{cov} \left( \frac{1 - \alpha}{1 - \rho}, CVMRS^{i}_{G} \right) \right].
\]

(A18)

Substituting equation (A18) into equation (A17) gives equation (37).

References


