No. 698, Chartist Trading in Exchange Rate Theory

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Abstract

This thesis consists of four papers, of which paper 1 and 4 are co-written with Mikael Bask. Paper [1] implements chartists trading in a sticky-price monetary model for determining the exchange rate. It is demonstrated that chartists cause the exchange rate to "overshoot the overshooting equilibrium" of a sticky-price monetary model. Chartists base their trading on a short-long moving average. The importance of technical trading depends inversely on the time horizon in currency trade. The exchange rate's perfect foresight path near long-run equilibrium is derived and it is demonstrated that the shorter the time horizon, the greater the exchange rate overshooting.

The aim of Paper [2] is to see how the dynamics of the basic target zone model changes when chartists and fundamentalists are introduced. Chartists use technical trading and the relative importance of technical and fundamental analyses depend on the time horizon in currency trade. The model also includes realignment expectations, which increase with the weight of chartists. The introduction of chartists may significantly reduce and reverse, the so-called "honeymoon effect" of a fully credible target zone. Further, chartists may cause the correlation between the exchange rate and the instantaneous interest rate differential to become either positive or negative.

Using a chartist-fundamentalist set-up, Paper [3] derives the effects on the current exchange rate of central bank intervention. Fundamentalists have rational expectations and chartists use so called support and resistance levels in their trading. This technique results in chartists having both bandwagon expectations and regressive expectations. Chartists may enhance or suppress the effect of intervention depending on their expectations. The results indicate that a chartist channel exists.

The aim of Paper [4] is threefold; (i) to investigate if there is a unique rational expectations equilibrium (REE) in a new Keynesian macroeconomic model augmented with technical trading, (ii), to investigate if the unique REE is adaptively learnable and, (iii), to investigate if this unique and adaptively learnable REE is desirable in an inflation rate targeting regime. The monetary authority is using a Taylor rule when setting the interest rate. A main conclusion is that a robust Taylor rule implies that the monetary authority should increase (decrease) the interest rate when the CPI inflation rate increases (decreases) and when the currency gets stronger (weaker).

JEL codes: E43; E52; F31; F33; F37; F41

Keywords: Chartist Trading, Foreign Exchange, Overshooting, Sterilized Intervention, Target zone, Taylor rules

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This thesis consists of an introductory part and four self-contained papers. The papers are:

[1] Bask, M. and Selander, C. (2006). Heterogeneous Beliefs in a Sticky-Price Foreign Exchange Model. Forthcoming in *Ekonomia*.

[2] Selander, C. (2006). Chartist Trading in an Exchange Rate Target Zone Model.

[3] Selander, C. (2006). Sterilized Intervention - A Chartist Channel?

[4] Bask, M. and Selander, C. (2006). Robust Taylor Rules in an Open Economy with Heterogeneous Expectations.

1 Introduction

Should foreign exchange models focus on the observed behavior of exchange rates or that of currency traders? As one of the main purposes of economic theory is to develop models that can explain observed regularities, there is an obvious advantage with taking the first point of departure. However, in order to develop an economic theory of exchange rate movements, one cannot, disregard the behavior of those who actually trade in the foreign exchange market. This is the premise on which the first paper in this thesis starts and it is the common theme running through the subsequent papers. Put simply the ambition of this thesis is to make a contribution to the further development of the theory of exchange rates by focusing on the observed behavior of currency traders.

Recent surveys of the foreign exchange market has shown the extensive use of chartism, i.e., technical analysis, on the foreign exchange market. Along with this finding, theoretical models of the exchange rate has been developed. One area of exchange rate models have focused on heterogenous expectations and technical trading, i.e., the behavior of foreign exchange traders. Technical trading involves the study of past exchange rates and extrapolating techniques are used to form expectations of future exchange rates. It refers to trading based on time charts of exchange rates. This has led to it being known as "chartism".

The implementation of chartist trading into theoretical models for the exchange rate potentially alters the basic results of these models and give rise to different dynamics for the exchange rate. Frankel and Froot [27] were the first to implement a chartist fundamentalist set-up in a traditional flexible price monetary model. They showed that chartists survived on the market and that the behavior of chartists could well explain the dynamics of exchange rates. In the footsteps of Frankel and Froot [27], different types of chartist-fundamentalist set-ups have been implemented into a wide range of different exchange rate models.

This introduction provides some insights into technical trading in the foreign exchange market as well as a theoretical background for the papers that follow. The four papers in this thesis incorporate heterogeneous expectations and technical trading into various theoretical models for the exchange rate.

Section 2 is based on survey results describing foreign exchange traders' beliefs and behavior in the foreign exchange market. It aims to introduce the reader to technical analysis, what it is, how and by whom it is used, and what type of market expectations chartists may generate. Section 3 presents a short literature survey of the more recent theoretical work within exchange rate theory that incorporates chartism in exchange rate modeling. It includes chartists in traditional sticky price models; in target zone models; in models of central bank intervention; and in new Keynesian models with Taylor rules. The purpose is to give the reader a background and an introduction to the type of models covered in this thesis. In Section 4 all four papers are summarized individually.

2 Chartism in reality

This section is based primarily on surveys conducted in the foreign exchange market and focuses in particular on chartism. Section 2.1 starts with a brief presentation of the different trading strategies used in the foreign exchange market. Section 2.2 continues with the relationship between the time horizon in currency trade and the use of chartism compared to other trading strategies. Section 2.3 focuses on where and by whom chartism is used, i.e., an analysis of whether there are any differences between trading locations and types of traders that use chartism. Finally, Section 2.4 discusses the relationship between chartism and exchange rate variability.

2.1 Chartism and other trading strategies

Traders in the foreign exchange market use different trading strategies. The surveys conducted in the foreign exchange markets define four different types of strategies used by traders: technical analysis, fundamental analysis, flow analysis, and jobbing. This section will briefly describe these different trading strategies, with particular focus on technical analysis, i.e., chartism. The description of the different tools within technical analysis that follow are mainly derived from Ludden [42], Neely [50], Osler [53] and Pring [54].

Technical analysis Technical analysis, i.e., chartism, does not rely on any underlying economic or fundamental analysis. Instead, technical analysis involves the study of past exchange rate movements, traded volumes, and the volatility of exchange rates. Chartists are traders who employ technical analysis to make forecasts and trading decisions about foreign exchange rates.

Technical trading includes a huge variety of different tools. However, most of these may be described as one of two types; trend or oscillating indicators. Both types of indicators are used to extract trends and predict trend reversals in the exchange rate. Oscillating indicators consist of an oscillator combined with so-called oversold and overbought limits, which indicate that the currency is undervalued or overvalued. A trend indicator is often comprised of a smoothed version of the exchange rate combined with so-called support and resistance levels to identify trend reversals. Alternatively, the trend indicator is comprised of two differently smoothed time series of the exchange rate, where crossovers between the two helps identify trend reversals.

Both types of indicators may be used when the market is trending or ranging. However, oscillating indicators are considered more suitable when the market is ranging and trend indicators when it is trending. When the market is trending, exchange rates move either downwards with lower and lower peaks and troughs, or upwards with higher and higher peaks and troughs. On the other hand when the market is ranging, exchange rates oscillate, with no clear up- or downward trend.

Trend indicators generally involve the use of some type of moving average. A moving average is a weighted average of past exchange rates, which smooths the exchange rate and makes it easier to detect a trend. One way to detect trends and trend reversals is to compare two moving averages of different lengths. The comparison of long- and short-period moving averages gives buy and sell signals according to the following; when the short-period moving average falls below (rises above) the long-period moving average, it is a signal of a falling (rising) exchange rate. Hence, it is a signal to sell (buy) the foreign currency. Another way to detect trends and trend reversals is to use the so-called support and resistance lines together with the time path of the exchange rate or a moving average of the exchange rate. These lines may, for example, be constructed by drawing a line between two (or more) previous troughs and two (or more) previous peaks, respectively. When the moving average falls outside the interval created by the support and resistance lines, it is a signal of a trend reversal. Thus, when the resistance (support) line is crossed it is a signal to buy (sell) the foreign currency.

The oscillating indicators include momentum, volume and volatility oscillators. Momentum oscillators are the same as so-called oversold and overbought signals. They measure the speed of exchange rate change. Some of the most common momentum oscillators are the Momentum, the Rate of Change (ROC), and the Relative Strength Index (RSI). The momentum is the simplest and measures the rate of change in closing rates for the currency (i.e., the difference between the closing price of one currency in terms of another, over a selected period of time). It is used to detect trend weaknesses and likely trend reversals. High momentums occur when the exchange rate is strongly trending. The momentum is positive when the exchange rate rises, and negative when it falls. A low and/or rising momentum is a buy signal and a high and/or falling momentum is a sell signal. The ROC is constructed using the momentum, and the oscillator gives buy and sell signals in the same way as the RSI below.

The RSI is constructed as an index that oscillates between 0 and 100. It compares upward movements in closing rates to downward movements in closing rates over a selected period of time. Thus, if the exchange rate has been mainly rising over the selected period of time, the RSI goes to 100 and if it has been mainly falling the RSI goes to zero. The way in which this oscillator is designed enables fixed oversold and overbought limits to be constructed. Usually, if the market is trending (ranging) the overbought and oversold limits are set at 80 (70) and 20 (30), respectively. Transgression of the oversold (overbought) limits by the oscillator constitutes buy (sell) signals for the foreign currency.

Volume and volatility oscillators are based on the traded volume or volatility of the exchange rate, and are used in conjunction with other indicators to confirm the signals given by the trend indicators or momentum oscillators. The idea is that different signals given by trend or oscillating indicators, or different chart patterns, are confirmed if they appear together with certain changes in traded volume and/or volatility. These indicators are, therefore, mainly used in conjunction with the trend and oscillating indicators. They may also be used together for the purpose of strengthening each others signals.

Fundamental analysis Fundamental analysis focuses exclusively on macro economic indicators such as interest rates, trade deficits, inflation rate, GDP, unemployment and other macroeconomic variables in predicting the exchange rate. Accordingly, fundamentalists are traders who predict the exchange rate and make trading decisions with the help of underlying economic fundamentals and the use of macroeconomic models (Taylor and Allen [65]).

Flow analysis Flow analysis, i.e., customer orders or order flows, describe a trading style in which the trader looks at who is doing what on the market, i.e., what orders exist on the market, and their size and source. The idea is that order flows contain private information. Identifying who is behind the order, and which orders contain most information, help traders to decipher underlying information. Thus, flowtists are traders who base their trading decisions upon the existing orders in the foreign exchange market (Lyons [44]).

Jobbing Jobbing describes a trading style in which the trader continuously buys and sells at high frequency in order to make many small profits. This type of trading is undertaken under very short time horizons. The strategy may be described as speculation at a very high frequency (Cheung and Chinn [12]).

2.2 How common is the use of chartism among traders?

There is a possibility that technical analysis is self-eliminating. In order to check for this Menkhoff [46] tested both whether professionals preferring technical analysis were younger than other traders in the survey, and whether they failed to reach senior positions as often as other traders. However, he found no statistical evidence for this. Menkhoff [46] also examined whether technical analysis could be considered an inferior strategy. He did this by testing whether those professionals who preferred technical analysis worked in smaller institutions and had a lower level of education. He found no evidence to support this idea.

Further, Menkhoff [47] found no difference in the use of technical analysis between the three groups; chief foreign exchange dealers, other foreign exchange dealers and fund managers that deal with foreign exchange. Menkhoff [47] discovered that foreign exchange dealers trade under significantly shorter time horizons than fund managers, and that the latter are more inclined to rely on fundamentals than foreign exchange dealers. Lui and Mole [43] argue that traders with the largest trading limits consider technical analysis to be less useful than traders with smaller trading limits, and Cheung and Chinn [12] suggest that the choice of trading technique is related to the type of business that traders employ; interbank or customer business. Traders for whom customer business accounts for a larger share of their foreign exchange transactions, tend to base their trading on fundamental analysis and customer orders (i.e., order flows). Oberlechner's [52] results differ from those of Cheung and Chinn [12] but are in line with those of Lui and Mole [43]. Thus, Oberlechner [52] does not find that the choice of trading technique is related to the type of business that traders employ; interbank or customer business. His results show, however, that traders with smaller trading limits are prone to a more chartist approach than traders with larger trading limits.

In testing the relevance of trading location, Oberlechner [52] compares the smaller trading locations (i.e., Vienna and Zurich) with larger trading locations (i.e., Frankfurt and London) and finds that for shorter time horizons, traders from larger trading centres use a significantly more fundamental approach than traders from smaller trading centres.

The general result is that chartism seem widespread among foreign exchange traders and that the position, age or education of traders does not seem to influence the choice to use chartism. In contrast, trading volume, trading limit and also trading location may be of some importance but the results regarding these determinants are mixed.

2.3 The time horizon in chartism

Taylor and Allen [65] conducted a questionnaire survey for the Bank of England in the foreign exchange market in London, in November 1988. This survey was the first to ask specifically about the use of technical trading among traders. The results of the survey were striking, with 90 percent of the respondents reporting using some form of chartist analysis in forming their expectations about the future exchange rate on time horizons, ranging from an intraday to a one week horizon. Two percent of the respondents reported that they never used fundamentals in forming their expectations about the exchange rate, for any time horizon. 60 percent of the respondents reported chartism to be at least as important as fundamentals for the time horizon of intraday to one week. For longer time horizons, the importance of fundamentals became more pronounced. When it came, for example, to a time horizon of one year or longer, 30 percent of the respondents relied purely on fundamentals, while 85 percent considered fundamentals to be more important than chartism.

Menkhoff [46] confirms the common use of technical analysis for short term forecasts in the German foreign exchange market in 1992, as do Lui and Mole [43] for the Hong Kong foreign exchange market in 1995. Oberlechner [52], whose survey includes the Frankfurt, London, Vienna and Zurich foreign exchange markets, also confirms the use of chartism. He also finds that his sample of traders consider chartism to be more important at every time horizon than was the case for the traders in the Taylor and Allen [65] survey. Cheung and Chinn [12] confirm the use of chartism in the U.S. foreign exchange market and point out that technical trading has become more important over the five year period prior to their survey of the U.S. foreign exchange market in 1998.

However, both in the survey by Cheung and Chinn [12] and that by Menkhoff [46], the percentage of traders using technical analysis is lower. It seems that, when other trading strategies are included, such as customer orders (i.e., order flows) and jobbing, technical trading is given less weight than in surveys where these trading strategies are not included. In the Cheung and Chinn [12] survey, for example, the respondents could choose between four different strategies describing their trading techniques; technical analysis, customer order, fundamental analysis, and jobbing.

The results of these surveys also indicate that technical trading is not only used for short term forecasting but also plays a significant role in the medium, as well as, in the long-run. In the Oberlechner [52] survey, more than half of the respondents used a trading pattern where chartism dominates the short term trading and fundamentals become more pronounced as the time horizon increases. However, Oberlechner [52] also detected a group of respondents who constantly used a more chartist approach when trading, regardless of the time horizon, as well as, a group that actually increased their use of chartism when their time horizon increased.

The general conclusion is that technical analysis is most important at shorter time horizons but that it still plays a significant role when the time horizon increases.

2.4 Traders beliefs about chartism and the exchange rate

Cheung and Chinn [12] and Cheung and Wong [13] find that traders believe technical trading to be a major determining factor of exchange rate variability up to a time horizon of six months. In the case of time horizons longer than six months, a small proportion of the respondents still believe that technical trading is a significant decision factor. Short term variability in the exchange rate is not only attributed to technical trading, but also to other non-fundamental forces like speculation, over-reaction to news and bandwagon effects. News are interpreted as innovations in macroeconomic variables. In the medium-run, exchange rate movements are believed to depend on economic fundamentals, technical trading, and speculative forces. In the long-run, most of the respondents believed that exchange rate variability is determined almost solely by economic fundamentals.

Lui and Mole [43] examine the importance attached by traders to different types of fundamental factors influencing the exchange rate. They find that interest rates and related factors such as monetary aggregates and bond prices are considered to be the most important determining factors for the exchange rate at time horizons of intraday to one month. Whereas for time horizons above one month, greater importance is attached to balance of payments related factors. Lui and Mole [43] compare the perceived importance of fundamental factors with those of different technical analysis techniques, and find that traders regard interest rate related factors and technical analysis as the most important factors at the intraday time horizon in determining the exchange rate.

It would seem that traders believe that interest rates and technical trading are important determinants of the exchange rate and its variability in the short-run, and that technical trading may still be an important determinant of exchange rate variability in the medium-run. The general belief among traders about the long-run variability of the exchange rate seems to be that it reflects fundamental variability. These beliefs coincide well with the reported behavior of traders with regard to their choice of trading technique, i.e., traders use mainly technical analysis in the short-run and fundamental analysis in the long-run.

2.5 Chartism and exchange rate expectations

As technical trading is supposed to have a destabilizing effect on the exchange rate, some surveys have tried to asses the type of expectations held by chartists. Three surveys that examine traders' expectations in a more formal way are Allen and Taylor [1], Frankel and Froot [26] and Ito [34].

The type of expectations studied in Allen and Taylor [1] and Frankel and Froot [26] include static, bandwagon, extrapolative, adaptive and regressive expectations. Static expectations mean that traders do not expect the exchange rate to change

$$s^e_{t+1} = s_t,$$

where s is defined as the domestic price of foreign currency and the superscript e denotes expectations. Bandwagon expectations mean that traders expect that the exchange rate will continue to move in the same direction as it did in the prior period, but more strongly, and thus the most recent trend is extracted. Bandwagon expectations are defined as

$$\Delta s^e_{t+1} = \beta \Delta s_t,$$

where $\beta > 1$ and $\Delta s^e_{t+1} = s^e_{t+1} - s_t$ and $\Delta s_t = s_t - s_{t-1}$. Extrapolative expectations imply that traders believe that the expected future exchange rate is the weighted average of the current observed spot rate and the lagged exchange rate. Extrapolative expectations are defined as

$$s^e_{t+1} = (1-\gamma)\, s_t + \gamma s_{t-1},$$

where $0 < \gamma < 1$. In the case of adaptive expectations, the expected future spot exchange rate is formed adaptively by traders as a weighted average of the current observed spot rate and the lagged expected spot rate. Adaptive expectations are defined as

$$s^e_{t+1} = (1-\phi)s_t + \phi s^e_t,$$

where $0 < \phi < 1$. Regressive expectations mean that traders expect the exchange rate to regress to its long-run equilibrium value. Regressive expectations are defined as

$$\Delta s^e_{t+1} = -\delta\left(s_t - \overline{s}_t\right),$$

where $\delta > 0$, and $\overline{s}$ is the long-run equilibrium value of the exchange rate, often based on purchasing power parity (PPP). Bandwagon expectations are explosive and have a destabilizing effect on the exchange rate. Extrapolative, adaptive, regressive and static expectations are, contrary to bandwagon expectations, non-explosive.

Allen and Taylor [1] found that chartists in the London foreign exchange market have a tendency to "underpredict" the exchange rate when the market is rising and to "overpredict" it when the market is falling. According to the authors, these results can be interpreted as an *average* elasticity of expectations that is less than unity, i.e., that expectations are stabilizing. However, the tendency to "overpredict" when the market is falling, i.e., to expect a larger change than the one that occurs, may be interpreted as an indication of bandwagon expectations. The results of the Allen and Taylor [1] survey also indicate extrapolative expectations among chartists. Frankel and Froot [26] conclude that expectations seem to be either adaptive or regressive, and that for longer horizons (6-12 months), expectations seem to be regressive to a greater extent.

The general result in both Allen and Taylor [1] and Frankel and Froot [26] is that chartists' expectations do not seem to overreact systematically to changes in the exchange rate. However, it is unclear whether traders' expectations may be destabilizing in the sense that they cause the exchange rate to move away from its fundamental value.

While the results of the Frankel and Froot [26] study were based on the median responses, the study made by Ito [34] was based on individual responses, which may be more appropriate, since Frankel and Froot [26] also conclude that expectations appear to be heterogenous. Ito [34] actually tested for individual effects. He found expectations to be heterogenous, of bandwagon type in the short-run, and stabilizing in the long-run.

3 Chartism in theory

This section presents a number of different examples of theoretical exchange rate articles that take chartism into account when modeling the exchange rate. Here the focus will mainly be on the different ways in which chartists and fundamentalists are introduced in the different exchange rate models. The different areas of exchange rate modeling covered in this short survey are, in Section 3.1, traditional monetary models augmented with heterogeneous expectations and chartist traders; in Section 3.2, differently augmented target zone models developed from the basic Krugman model [37]; and in Section 3.3, models for sterilized central bank interventions that incorporate chartist trading. Finally, in Section 3.4, the new Keynesian macroeconomic models with Taylor rules that incorporate an exchange rate target, are presented.

The literature survey is limited to these areas because the papers in this thesis fall within these categories. The following sections will give some background and hopefully some insight into these four areas of exchange rate theory.

3.1 Chartism in monetary models

Since the early eighties, when Meese and Rogoff [45] showed that exchange rate dynamics could be as well described by a random walk, as other exchange rate models, new ideas about exchange rate behavior have been developed. A major topic has been to develop ideas on how to model the apparent heterogeneity among traders in the foreign exchange market. Frankel and Froot [27] were first to suggest a type of model in which the heterogeneity of market participants is taken into account according to a chartist-fundamentalist set-up. They use a flexible price monetary model with three actors; chartists, fundamentalists and portfolio managers. The fundamentalists' expectations are of the regressive type (i.e., the exchange rate will regress to its fundamental value) and the chartists' expectations are of the random walk type. The fundamentalists' and chartists' views are weighted by portfolio managers, who are the actual traders in this model, and the weight function evolves according to the previous success of the chartists' and fundamentalists' predictions. A whole literature of chartist-fundamentalist models has developed in the footsteps of Frankel and Froot [27].

De Grauwe and Dewachter [14] and [15] develop a model similar to that of Frankel and Froot [27]. Chartists' and fundamentalists' views are weighted endogenously, and the baseline model is the exchange rate overshooting model developed by Dornbusch [20]. The deviation of the exchange rate from its fundamental value determines the weight of chartists and fundamentalists. As in the Frankel and Froot [27] model, chartists' expectations are destabilizing because of the extrapolative nature of their expectations, and fundamentalists' expectations are stabilizing because of the regressive nature of their expectations. When the exchange rate is close to its fundamental value, the weight of chartists is high, and as the exchange rate deviates from this value, the weight of fundamentalists increases. De Grauwe and Dewachter [14] and [15] show, via simulations of their model, that the speculative dynamics added to the Dornbusch [20] exchange rate overshooting model create chaotic behavior in the exchange rate.

Levin [40] develops two models in his paper, which are both based on the Dornbusch [20] overshooting model. In the first model, there are two types of asset holders. These are chartists with extrapolative expectations and fundamentalists with regressive expectations. The dynamic behavior of the first model is that of the Dornbusch [20] model with exchange rate overshooting in the short-run, due to sticky prices, and convergence in the long-run to a new equilibrium. The second model, where there is one type of asset holder, who simultaneously has both chartist and fundamentalist expectations, is not necessarily stable, and gives rise to either a saddle path or an unstable path. In the case of a saddle path, the dynamics of the exchange rate are the same as in the model with two types of asset holders and, in the case of an unstable path, the dynamics of the exchange rate give rise to a bubble.

Hommes [31] developed a model called adaptive belief systems (ABS). This is a standard asset pricing model derived from mean-variance maximizing agents, extended to incorporate heterogenous beliefs among agents (developed with Brock in a series of papers: [6], [7], [8], [9]). The ABS is a deterministic, non-linear system that exhibits chaos. This makes it a suitable tool for examining financial markets where asset prices are persistent (have, or are close to having a unit root), asset returns are unpredictable (with no or little autocorrelation), trading volume is persistent, and there is positive cross correlation between volatility and volume. The ABS involves an evolutionary competition between trading strategies, in Hommes [31] between chartists and fundamentalists. The ABS was primarily developed for the stock market, but the results are easily transferable to the foreign exchange market and show that, in most cases, fundamentalists fail to drive chartists out of the market.

The set-up of Westerhoff [68] is a model with fundamentalists who have regressive expectations, where the perceived fundamental value of the exchange rate follows a jump process and changes with the arrival of news. Fundamentalists can, on average, determine the fundamental value but do make mistakes. Chartists form their expectations according to a moving average, and a random component designed to account for other technical trading tools that chartists may use. The decision of which trading rule to follow depends on the expected future performance possibilities, and the fundamentalist trading rule becomes more popular the more the spot rate deviates from its expected future value. There is, however, a minimum fraction of agents who are always fundamentalists. Simulations of the model give rise to bubbles, display unit roots in the exchange rates, fat tails of returns and volatility clustering, i.e., quite realistic exchange rate dynamics.

3.2 Chartism in target zone models

Along with the development of the EMS, the EMU and in the aftermath of the currency crisis in the international financial markets during the 1990s, questions concerning exchange rate regimes, optimal currency areas and the literature on speculative attacks, central bank interventions and currency crisis where reinvigorated. For example, the effectiveness of target zones as a means of achieving exchange rate stability has become an actively debated issue and spurred new research within this area of exchange rate modeling.

A target zone implies setting a central parity around which the exchange rate is allowed to fluctuate within an interval. When the exchange rate approaches the limits of the interval, the monetary authority intervenes in the domestic monetary market and prevents the exchange rate from deviating outside the interval. The ERM II (the Exchange Rate Mechanism of the European Monetary System) is an example of a broad target zone. Most of the more recent papers within this area of exchange rate modeling are based on the seminal work of Krugman [37], which first circulated in 1988.

Krugman [37] assumes a perfectly credible target zone, defended by authorities with infinitesimal interventions at the edges of the zone. The exchange rate is a function of monetary fundamentals that follow a Wiener process and the model is defined over fundamentals. The Krugman [37] model may be solved explicitly. Modelling interventions in this way and assuming perfect credibility, causes the exchange rate curve to bend at the edges of the zone, so that it forms a sigmoid shaped curve. The S-shaped curve is flatter than the 45-degree line (representing the free float exchange rate), i.e., the exchange rate in a target zone is less sensitive to changes in fundamentals than a free floating exchange rate. Hence, the target zone has a stabilizing effect on the exchange rate. A comparison of the S-shaped curve, representing the target zone exchange rate, with the linear 45-degree line representing the free-float exchange rate, demonstrates what Krugman called the "honeymoon effect". Although innovative and highly influential, Krugman's [37] model has some empirical implications that have been rejected by data from the ERM, the Bretton Woods system and the Gold standard, see for example, Flood et al. [25].

The literature following Krugman [37] has tried, in various ways, to develop this basic model to fit data better. Tristani [66] and Werner [67], for example, include realignment expectations as dependant on the level of the exchange rate within the zone in their models. This improves the empirical fit of the model with regard to the much debated "honeymoon effect", which has received little support empirically (Flood et al. [25]). Including realignment expectations as depending on the level of the exchange rate within the zone, makes the slope of the exchange rate curve steeper than the 45-degree line, and completely erases the "honeymoon effect".

Bertola and Caballero [3] and Bertola and Svensson [4], for example, not only relax the assumption of a fully credible target zone but also introduce actual realignments in their models. This also increases the mean reversion in the exchange rate creating a hump-shaped distribution, which is another empirical characteristic of a target zone exchange rate. Beetsma and Van Der Ploeg [5] and Lindberg and Söderlind [41] introduce intramarginal interventions by central banks into the Krugman [37] model. Empirical data suggests that interventions may be both marginal and intramarginal. Because of the increased mean reversion, intramarginal interventions also create a hump-shaped exchange rate distribution. Miller and Weller [48] and Neely et al. [51] introduce price rigidity which also increases mean reversion and causes a hump-shaped exchange rate distribution.

Driffill and Sola [21] extend the target zone model by allowing for changes in the stochastic process driving fundamentals, i.e., monetary policy alternates between a broad and narrow target zone. Allowing for these policy changes creates a better empirical fit with regard to the exchange rate distribution and the correlation between the interest rate differential and the exchange rate within the band.

However, these extensions of the Krugman [37] model have continued to assume homogeneity among traders, although, heterogeneity among traders in the foreign exchange market has been well established in the surveys conducted in the foreign exchange market. Heterogeneity among traders has been introduced into target zone models in the form of stop-loss trading by Krugman and Miller [38] and Kempa and Nelles [36]. There are very few target zone models that include chartist behavior and most theoretical papers on target zones have focused on other issues when developing new models. However, Reitz et al. [56] have a different approach and combine target zone type interventions with an empirical exchange rate model that has a chartist-fundamentalist set-up. Target zone interventions are defined as "buying (selling) an undervalued (overvalued) currency when the distance between the exchange rate and its fundamental value exceeds a critical threshold value" (p 453). The purpose of their paper is to see whether this type of interventions are effective in stabilizing the exchange rate. Through simulations of their model Reitz et al. [56] find that these types of interventions have a stabilizing effect on the exchange rate which is prolonged by the trade of both chartists and fundamentalists. However, they do not use a theoretical target zone model of the type described above, and to our knowledge, there has been no attempt to try and theoretically implement a chartist-fundamentalist set-up within a target zone model with the purpose of attaining an explicit solution for the exchange rate.

3.3 Chartism in models of sterilized central bank interventions

After the break down of the Bretton Woods exchange rate system, a period of floating exchange rates arose. The question of how to stabilize the exchange rate without a system of fixed exchange rates spurred new research. Specifically, the question of sterilized central bank interventions as an independent exchange rate policy tool received much attention.

Sterilized intervention is most easily described as central banks buying (selling) domestic currency on the foreign exchange market, and at the same time, making an offsetting sell (buy) of domestic currency on the domestic monetary market, leaving the monetary base unchanged. The two traditional theories that explain why sterilized interventions have an effect on the exchange rate level, although monetary fundamentals remain unchanged, are the signalling theory and the portfolio balance theory. The signalling hypothesis was first proposed by Mussa [49] and, according to this hypothesis, sterilized intervention is a signal of future monetary policy. Thus, assuming that expectations about the future spot exchange rate affect the current spot exchange rate, and that the market believes in the central bank's intervention signal, sterilized interventions have an effect on the spot exchange rate. This is because it alters the foreign exchange market's expectations about the future spot exchange rate, which then has an immediate effect on the current spot exchange rate. The portfolio balance channel suggests that, if foreign and domestic bonds are considered imperfect substitutes and that portfolio managers are risk averse and choose their optimal portfolio according to a mean-variance behavior, sterilized intervention will affect the exchange rate when portfolio managers readjust their portfolios according to changes in the relative supply of foreign and domestic bonds.

The empirical support for the signaling and portfolio balance channels are mixed. Neither Humpage [32] nor Kaminsky and Lewis [35] find support for the signaling channel, and Gosh [30] finds no support for the portfolio balance channel. However, fairly strong support for the portfolio balance channel has been presented by Dominguez and Frankel [19], and Evans and Lyons [24] have found support for a portfolio balance price effect on the exchange rate market. They claim that the results are also applicable to sterilized covert central bank interventions.

Another channel through which sterilized intervention could have an effect on the exchange rate has been proposed by Hung [33]; the noise trading channel, or the chartist channel. Hung [33] concludes that the presence of non-fundamentalist traders, i.e., noise traders, whose behavior is more or less predictable constitutes a channel through which sterilized intervention can be effective. Hung [33] does not formally derive a model for this proposed channel. Instead Hung [33] argues that, on the basis of noise trader behavior, central banks can, by means of covert intervention, manage the exchange rate. If the central bank uses covert sterilized intervention and a "leaning with the wind strategy" (i.e., buy if the currency is appreciating and sell if the currency is depreciating), they can cause noise traders to believe in a trendreversal (i.e., to push the exchange rate just enough to hit a buy or sell signal for noise traders) and make them act accordingly. The trading behavior that Hung [33] describes here is consistent with chartists trading rules. Henceforth, this channel will be referred to as the chartist channel.

Hung [33] tested whether or not sterilized intervention has resulted in increased volatility in the exchange rate, indicating the use of a "leaning with the wind strategy" and found supportive empirical evidence for this hypothesis during parts of the sample period. Dominguez [18], although not testing the same hypothesis, also found that covert central bank intervention increased volatility. These results could also be interpreted as supportive of the chartist channel. In addition, Dominguez [17] shows that covert sterilized interventions affect both the volatility and the level of the exchange rate. This speaks against the signalling channel and for the chartist channel.

Sarno and Taylor [58] were the first to suggest a coordination channel. They suggested that sterilized intervention may be effective through its role in coordinating fundamentalist traders' reentrance in the market and correct the exchange rate misalignment. However, Sarno and Taylor [58] call their proposed channel the coordination channel but do not formally derive a model for it. In testing the effectiveness of intervention, Reitz and Taylor [55] and Taylor [64] find supportive evidence for this channel using, in the former case, a Markow-switching model, and in the latter, a non-linear microstructural analysis in the form of a STAR GARCH model for the real exchange rate.

There are a few papers that adopt a chartist-fundamentalist set-up in modeling central bank interventions. However, they rely more on a type of fundamentalist channel than on the chartist channel proposed by Hung [33] or the coordination channel proposed by Sarno and Taylor [58]. De Grauwe and Grimaldi [16] derive a chartist-fundamentalist model where the sterilized central bank intervention strategy is known to the market and implemented by fundamentalists in their trading strategy. When fundamentalists implement central bank behavior, they increase the mean reversion in the exchange rate and thereby cause chartist trading strategies to become less profitable. The weight between chartists and fundamentalists depends on the profitability in trading strategies and, hence, the weight of chartists will decrease with intervention. In this way, intervention becomes effective in reducing misalignments and decreasing volatility, as it is conducted according to a "leaning against the wind" strategy (i.e., to sell (buy) if the currency is appreciating (depreciating)). Kubelec [39] uses a similar model to that of De Grauwe and Grimaldi [16] where central bank interventions are sterilized and follows a "leaning against the wind" strategy. However, Kubelec [39] uses of the evolutionary fitness measure, developed by Brock and Hommes ([6], [7], [8], [9]) based on the profitability of the different trading strategies for determining the weight between chartists and fundamentalists. Thus, in both of these papers, the central bank does not explicitly try to exploit chartists, instead their intervention strategies lessen the profitability of chartist trading strategies and, hence, drive chartists out of the market.

Schmidt and Wollmershäuser [59] also model sterilized interventions in a setting with both chartists and fundamentalists. In their model, the weight between chartists and fundamentalists is fixed and divided equally. They assume that the central bank tries to exploit chartists through covert intervention. They test different strategies that the central bank can use for their interventions. These strategies are: a targeting strategy, a trend reverting strategy, and a smoothing strategy. The targeting strategy involves intervening when the exchange rate deviates from its fundamental value, the trend reverting strategy uses a moving average rule which means to sell (buy) when the exchange rate is above (below) the moving average. The smoothing strategy involves going against the wind on an everyday basis. The moving average strategy proves to be most effective followed by the targeting strategy. Schmidt and Wollmershäuser [59] find that their results support the existence of a chartist channel.

3.4 Chartism in new Keynesian models with Taylor rules

During the last two decades, a new paradigm in monetary policy has evolved. This concerns independent central banks, openness and inflation rate targeting. In other words, monetary policy is conducted by the central bank, without political influence, with the purpose of creating price-stability and credibility in order to avoid the time inconsistency problem. Monetary policy is conducted through interest rate management with an explicit target for the inflation rate. The success of this type of monetary policy rule hinges on the central bank's ability to shape market expectations of future interest rates, inflation rates and income levels. It is, therefore, important for the central bank to commit to the rule, to be as transparent as possible in its decision making, and to make the right policy decisions as often as possible.

This practise is nowadays well established among central banks of the industrialized countries and the literature within this area is flourishing. Much of the literature regarding interest rate targeting has been influenced by the work of Taylor [61]. In 1993, John B. Taylor [61] suggested that the monetary policy of the Federal Reserve could be described by a fairly simple interest rate rule, where the central bank sets the interest rate in response to changes in inflation and the output gap (i.e., the difference between actual GDP and potential GDP). This rule has been the center of attention within the monetary policy literature since it was presented and is often referred to as a Taylor rule. Since Taylor presented this rule as descriptive of how the Federal Reserve conducts monetary policy, a literature has evolved regarding whether or not this type of interest rate rule, which does not incorporate a target path for the monetary aggregates, can control the price level and create price-stability.

It is well-known that models in economics and finance, in which agents have rational expectations regarding some of the variables in the model, may exhibit a multiplicity of rational expectations equilibria (REE). This is problematic. For instance, without imposing additional restrictions into such a model, it can not be known in advance whether agents will coordinate on an REE and, if so, which REE they will coordinate on. One way to reduce the number of equilibria attainable, is to focus on those REE that are possible results of an adaptive learning process for the agents.

The assumption of rational expectations among agents in an economy has long been criticized for being too strong. Rational expectations imply that agents often have an outstanding capacity when it comes to deriving equilibrium outcomes of variables in a model. This assumption has, therefore, in the more recent literature, been complemented by an analysis of the possible convergence to REE. For example, Evans and Honkapohja [22] suggest that market participants learn a model by continuously updating their expectations about the future. It can be assumed that the agents' expectations are formed by a correctly specified model, i.e., a model that corresponds to the REE, but without having perfect knowledge about the parameters in the model. Using past and current values of the variables in the model, the parameters are learned over time since the beliefs are revised as new information is gained, according to a recursive least squares technique.

Recursive learning have been introduced, for example, by Bullard and Mitra [10] in the context of a new Keynesian model with Taylor rules. The model used in Bullard and Mitra [10] is for a closed economy with price staggering derived from an optimizing micro framework by Rotemberg and Woodford [57]. Bullard and Mitra [10] find that when the monetary authority uses a Taylor rule that includes contemporaneous data for the output gap and inflation rate, there are parameter regions that give rise to a unique stable REE that is characterized by recursive least squares learnability. Moreover, the same result holds when the monetary policy rule reacts to contemporaneous expectations for the output gap and inflation rate. However, the same desirable properties do not hold for Taylor rules based on lagged data or on forward looking expectations for the output gap and inflation rate. Evans and McGough [23] investigate which types of Taylor rules, generating stable sunspot equilibria (i.e., an equilibrium that depends on an intrinsic random variable or sunspots that matters only because the market believes so), are learnable. Like Bullard and Mitra [10], they find that Taylor rules based on forward looking variables may well generate sunspot equilibria.

Galí and Monacelli [29] provide an optimizing framework for the same type of model but for a small open economy and without including learning. However, the Taylor rule in Galí and Monacelli [29] does not include an exchange rate variable. Taylor [63] points out that it is likely that current models understates the exchange rate effects in small open economies, and in Taylor [62] he finds that monetary policy rules that react directly to the exchange rate work better for a few countries in Europe than policy rules that react indirectly. Ball [2] and Svensson [60] also investigate the effects of including the exchange rate in the Taylor rule. The general suggestion from these papers is that there might be a small improvement from including the exchange rate.

Bullard and Schaling [11] derive the conditions under which a worldwide equilibrium in a two country model is determinate and learnable with the use of Taylor rules. One of the conclusions is that when monetary authorities respond to international variables (including the foreign output gap and the real exchange rate) in the Taylor rule, determinacy and learnability conditions for worldwide equilibrium are met.

However, none of these papers include heterogeneous expectations among traders in the foreign exchange market, and thus, do not take chartist trading into account. Nor do they combine the search for Taylor rules that are associated with a unique and adaptively learnable equilibrium, as well as a desirable equilibrium evaluated with a loss function for the monetary authority.

4 Summary of papers

Paper [1].

Heterogeneous Beliefs in a Sticky-Price Foreign Exchange Model

This paper introduces a chartist-fundamentalist set-up in a sticy-price monetary based exchange rate model, due originally to Dornbusch [20]. As in Frankel and Froot [27] portfolio managers are the actual traders in this paper. Portfolio managers use both chartist and fundamentalist trading techniques. The weight between chartists and fundamentalists depends on the traders' time horizon which is largely supported by the empirical surveys presented in this introduction. For shorter time horizons, more weight is placed on technical analysis, while in the case of longer time horizons, more weight is placed on fundamental analysis. Chartists use a moving average technique in forming their expectations and fundamentalists have regressive expectations where the fundamental value of the exchange rate is the PPP value of the exchange rate.

The time horizon is determined endogenously in the model and thus, the portfolio managers choose to use chartism as well as fundamentalism. Note that fundamentalists do not have rational expectations and that only in the long-run do their expectations conform to rational expectations.

It is demonstrated that the exchange rate "overshoots the overshooting equilibrium" when chartists are introduced into a sticky-price monetary model. The "overshooting equilibrium" refers, of course, to Dornbusch [20]. The exchange rate's perfect foresight path near long-run equilibrium is derived. It is also demonstrated that the shorter the time horizon is, the larger the magnitude of exchange rate overshooting. Finally, the effects on the exchange rate's time path of changes in the model's structural parameters are derived. The extent of the exchange rate overshooting, given perfect foresight, is smaller the more flexible the price of goods. With regard to changes in the other structural parameters in the model, the effect on the magnitude of exchange rate overshooting is ambiguous.

Paper [2].
Chartist Trading in an Exchange Rate Target Zone Model

This paper develops a target zone model that follows the Krugman [37] approach, but introduces heterogeneity in the market in the form of two different types of traders, chartists and fundamentalists. Empirical surveys have revealed the vast use of technical analysis in the foreign exchange market, and that the relative importance of technical and fundamental analysis depends on the time horizon in currency trade. This is implemented in the model. The weight of chartists and fundamentalists is determined by the length of the time horizon in currency trade (i.e., the length of time that traders intend to trade). The Krugman [37] model is also augmented with realignment expectations where these depend on the weight of chartists in the market. When the time horizon in trade is infinitely long, i.e., when the weight of chartists is zero, the model is equivalent to the Krugman [37] model. Thus, the augmented model will nest that of Krugman [37].

The introduction of both chartist and fundamentalist traders in the Krugman [37] model does create a better empirical fit with regard to the empirically observed correlation between the interest rate differential and the exchange rate, the exchange rate volatility and the expectations of realignments. However, due to the use of infinitesimal marginal interventions in the model, the relationship between the exchange rate and fundamentals is still, even with the market completely dominated by chartists, somewhat S-shaped and the exchange rate distribution is still strongly U-shaped. The "honeymoon effect" does not completely disappear in the model when the market is completely dominated by chartists, but it is diminished for larger deviations of the fundamentals and has a reversed effect for fundamentals equal to central parity.

The paper also contains an attempt to produce a more explicit and refined mathematical derivation of the solution for the exchange rate path within the target zone than the one offered by Krugman [37]. In particular, the mathematical derivations of the solution for the model suggest another notation for the expected depreciation of the exchange rate than the one commonly used in the literature on this subject. The typical interpretation of this term as the expected depreciation the exchange rate may also be misleading. It should instead be interpreted as a "deterministic *measure* of the expected depreciation of the exchange rate".

Paper [3].
Sterilized Intervention - A Chartist Channel?

This paper develops a theoretical model for central bank intervention in which it is possible to explicitly derive the effects of different types of interventions on the current exchange rate in the case of both homogeneous and heterogeneous expectations (i.e., without and with chartists), and to compare the effects of intervention on the current exchange rate in the different cases. The heterogeneous expectations model takes a portfolio balance framework approach and incorporates a chartist channel into a monetary model for the foreign exchange market. The portfolio balance approach is, in this context, necessary to theoretically derive any effects of sterilized intervention through a chartist channel.

Traders in the benchmark model have rational expectations, whereas in the heterogeneous expectations model, they are based on fundamental and technical analysis. Using a chartist-fundamentalist set-up, the effects on the current exchange rate of central bank intervention are derived. Fundamentalists have rational expectations and chartists use a technical trading strategy which involves the use of so-called support and resistance levels for the exchange rate. The chartists' trading technique results in them having two different types of expectations; bandwagon and regressive expectations. These give rise to a variety of effects on the current exchange rate when the central bank intervenes in the foreign exchange market. Chartists may enhance or suppress the effects of intervention depending on their expectations.

The results indicate that there is a chartist channel which central banks may be able to exploit when intervening in the foreign exchange market. However, the stability restrictions on the model, in the heterogeneous expectations case, dampen the effect of chartists. Although they may indeed enhance or suppress the effect of sterilized intervention on the current exchange rate, they cannot reverse the effect of intervention. The effect of sterilized intervention on the current exchange rate may be greater in the heterogeneous expectations model than in the benchmark model when chartists act to reinforce the effect of intervention, i.e., when chartists have bandwagon expectations. The effect of sterilized intervention on the current exchange rate may also be less in the heterogeneous expectations model than in the benchmark model when chartists act to oppose intervention, i.e., when chartists have regressive expectations.

Paper [4].

Robust Taylor Rules in an Open Economy with Heterogeneous Expectations

The aim of this paper is to investigate the extent to which monetary policy is affected when currency trade is partly driven by technical trading. Specifically, we augment the small open economy model in Galí and Monacelli [29] with technical trading in the form of extrapolation of trends, where the determinacy and learnability of the rational expectations equilibriums are in focus. In the present paper, we assume that expectations are formed by a correctly specified model, i.e., a model that nests the REE, but without perfect knowledge about the parameter values in the model. However, by using past and current values of all variables in the model, the parameter values are learned over time. This is because the beliefs are revised as new information is gained.

We use the analysis in Bullard and Mitra [10] as our benchmark and investigate whether the results in their paper still hold in our augmented model. The monetary authority uses a Taylor rule when setting the nominal interest rate, and we investigate the properties of the model developed numerically. Specifically, we investigate whether there is a unique REE in the augmented Galí and Monacelli model [29]. We also investigate whether the unique REE is adaptively learnable in a recursive least squares sense, and finally whether it is desirable in the sense that a low and not too variable CPI inflation rate is achieved.

Since the model developed in this paper is too complicated for theoretical policy analysis, we provide a numerical illustration of our findings and contrary to what Taylor [63] claims, we find an interest rate rule with desirable properties that includes the change in an exchange rate index. Moreover, this rule does not include the output gap, which might be an advantage as it comes closer to the reality of central banking. Thus, the value of the currency is a better response variable than the output gap in the most desirable parametrization of the interest rate rule.

To find a robust Taylor rule, we would like the rule to be relatively independent of the proportion of technical trading in the foreign exchange market as the extent to which technical trading is used at each moment in time is not obvious even though questionnaire surveys reveal that it is extensively used in the foreign exchange market. We conclude that in order to have a rule that is robust with respect to the degree of technical trading in the foreign exchange market the, monetary authority should increase (decrease) the interest rate when the CPI inflation rate increases (decreases) and when the currency gets stronger (weaker).

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Heterogeneous Beliefs in a Sticky-Price Foreign Exchange Model*

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Abstract

It is demonstrated in this paper that the exchange rate "overshoots the overshooting equilibrium" when chartists are introduced into a sticky-price monetary model due originally to Dornbusch [5]. Chartists are introduced since questionnaire surveys reveal that currency trade to a large extent is based on technical trading, where moving averages is the most commonly used technique. Moreover, the surveys also reveal that the importance of technical trading depends inversely on the time horizon in currency trade. Implementing these observations theoretically, and deriving the exchange rate's perfect foresight path near long-run equilibrium, it is also demonstrated in this paper that the shorter the time horizon is, the larger the magnitude of exchange rate overshooting. Finally, the effects on the exchange rate's time path of changes in the model's structural parameters are derived.

JEL codes: F31; F41.

Keywords: Exchange Rates; Moving Averages; Overshooting; Technical Analysis; Time Horizon.

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1 Introduction

Should foreign exchange models focus on observed behavior of exchange rates, or should the focus be on observed behavior of currency traders? Since a main purpose of economic theory is to develop models that can explain observed regularities, there is an obvious advantage of the first point of departure. Nevertheless, in order to develop an *economic* theory of exchange rate movements, one cannot disregard the behavior of those who actually trade in the foreign exchange market. Modeling observed behavior of foreign exchange traders is, however, not sufficient in order to obtain an economic theory since one must also explain *why* traders act as they do. The level of ambition in the present paper is to take a first step towards developing an economic theory of exchange rate movements by taking into account observed behavior of currency traders.

In November 1988, Taylor and Allen [14] conducted a questionnaire survey for the Bank of England on the foreign exchange market in London. The survey covered 353 banks and financial institutions, with a response rate of over 60 per cent, and was among the first to ask specifically about the use of *technical analysis*, or *chartism*, among currency traders. The results of the survey were striking, with two per cent of the respondents reported never to use fundamental analysis in forming their exchange rate expectations, while 90 per cent reported placing some weight on technical analysis at the intraday to one week horizon. At longer *time horizons*, however, Taylor and Allen [14] found that the importance of technical analysis became less pronounced. For a theoretical description of technical trading techniques used in the foreign exchange market, the reader can turn to Neely [11].

That technical analysis is extensively used in currency trade has also been confirmed by Menkhoff [10], who conducted a survey in August 1992 on the German market, by Lui and Mole [9], who conducted a survey in February 1995 on the Hong Kong market, by Oberlechner [12], who conducted a survey in the spring 1996 on the markets in Frankfurt, London, Vienna and Zurich, and, as a final example, by Cheung and Chinn [1], who conducted a survey between October 1996 and November 1997 on the U.S. market. A general observation in these surveys is that a skew towards reliance on technical, as opposed to fundamental, analysis at shorter time horizons was found, which became gradually reversed as the length of the time horizon considered was increased. See also Oberlechner [13], who explores the psychology of the currency market from a variety of perspectives. Oberlechner [13] is based on surveys conducted in the European and in the North American markets.

Frankel and Froot [6] were the first to use a *chartist-fundamentalist* setup in a foreign exchange model, where the heterogeneous behavior of currency traders was taken into account. In their model, a bubble in the exchange rate takes off and collapses since the portfolio managers[1] learn more slowly about the model than they are changing it by revising the weights given to the chartists' and fundamentalists' exchange rate expectations. A chartist-fundamentalist model is also developed by De Grauwe and Dewachter [3], where the weights given to the chartists' and fundamentalists' expectations depend on the deviation of the exchange rate from its fundamental value. Specifically, more (less) weight is given to the chartists' expectations when the exchange rate is close to (far away from) its fundamental value. Related references are De Grauwe and Dewachter [2], and De Grauwe et al. [4], where a factor in common in all these references is a chaotic behavior of the exchange rate. See also Hommes [8], a chapter in a forthcoming volume in the *Handbook of Computational Economics*, for an excellent survey of the literature on heterogeneous agent models in economics and finance.

The specific purpose of this paper is to implement theoretically, the aforementioned observation that the relative importance of technical versus fundamental analysis in the foreign exchange market depends on the time horizon in currency trade. For shorter time horizons, more weight is placed on technical analysis, while more weight is placed on fundamental analysis for longer time horizons. In the model developed, technical analysis is based on moving averages since it is the most commonly used technique among currency traders using chartism (e.g., Taylor and Allen [14], and Lui and Mole [9]). Further, fundamental analysis is based on a sticky-price monetary foreign exchange model

[1] Without affecting the theoretical results in this paper, we assume that it is the chartists and fundamentalists, and not the portfolio managers, who trade in currencies. Therefore, we leave the portfolio managers out of account in the model.

due originally to Dornbusch [5].

The main questions in focus are: how is the dynamics of the exchange rate affected when technical analysis is introduced into a sticky-price monetary model? Specifically, will the exchange rate "overshoot the overshooting equilibrium"? This phrase was coined by Frankel and Froot [7] when discussing possible explanations to the dramatic appreciation of the U.S. dollar in the mid-1980's. The "overshooting equilibrium" refers, of course, to Dornbusch [5]. Further, how is the time horizon and the overshooting effect affected when market expectations are characterized by perfect foresight, where market expectations are the weighted average of the chartists' and fundamentalists' expectations[2]? It should be emphasized that perfect foresight is assumed in the derivations throughout the whole paper.

The remainder of this paper is organized as follows. The benchmark model and the expectations formations are presented in Section 2. The formal analysis of the model is carried out in Section 3, and Section 4 contains a short concluding discussion of the main results in this paper.

2 Theoretical framework

The benchmark model is presented in Section 2.1, and the expectations formations are formulated and discussed in Section 2.2.

2.1 Benchmark model

Basically, the model is a two-country model with a money market equilibrium condition, an international asset market equilibrium condition, a price adjustment mechanism since goods prices are assumed to be sticky, and market expectations that are formed by the relative weights given to the chartists' and fundamentalists' exchange rate expectations. The formal structure of the model is presented below, where Greek letters denote positive structural parameters.

The money market is in equilibrium when

[2] That is, in a model with portfolio managers, like in Frankel and Froot [6], market expectations coincide with the portfolio managers' expectations. See also footnote 1.

$$m[t] - p[t] = \overline{y} - \alpha i[t], \tag{1}$$

where m, p, $\overline{y}$ and i are (the logarithm of) the relative money supply[3], (the logarithm of) the relative price level, (the logarithm of) the relative real income, and the relative nominal interest rate, respectively. Moreover, m and $\overline{y}$ are exogenously given[4]. Thus, according to (1), real money demand depends positively on real domestic income and negatively on nominal domestic interest rate. The money market is assumed to be permanently in equilibrium, i.e., disturbances are immediately intercepted by a perfectly flexible domestic interest rate.

The international asset market is in equilibrium when

$$i[t] = s^e[t+1] - s[t], \tag{2}$$

where s is (the logarithm of) the spot exchange rate, which is defined as the domestic price of the foreign currency. Moreover, the superscript e denotes expectations. The equilibrium condition in (2), also known as uncovered interest rate parity, is based on the assumption that domestic and foreign assets are perfect substitutes, which only can be the case if there is perfect capital mobility. Since the latter is assumed, only the slightest difference in expected yields would draw the entire capital into the asset that offers the highest expected yield. Thus, the international asset market can only be in equilibrium if domestic and foreign assets offer the same expected yield. According to (2), a positive (negative) relative nominal interest rate means that the exchange rate is expected to depreciate (appreciate). The equilibrium condition is maintained by the assumption of a perfectly flexible exchange rate.

The price adjustment mechanism is

$$p[t+1] - p[t] = \beta(s[t] - p[t]), \tag{3}$$

where $0 \leq \beta \leq 1$ and $s - p$ is (the logarithm of) the real spot exchange rate. According to (3), goods prices are assumed to be sticky. Thus, goods prices

[3] That is, the difference between the domestic and foreign money supplies. The other macroeconomic variables in the model are defined in a similar way.

[4] In the simulations of the model in Section 3.5, m will follow a stochastic process.

respond to market disequilibria, but not fast enough to eliminate the disequilibria instantly. Two extremes are obtained by setting $\beta = 0$, which is the case of completely rigid goods prices, and by setting $\beta = 1$, which is the case of perfectly flexible goods prices.

2.2 Expectations formations

According to questionnaire surveys (see cited references in Section 1), the relative importance of technical versus fundamental analysis in the foreign exchange market depends on the time horizon in currency trade. For shorter time horizons, more weight is placed on technical analysis, while more weight is placed on fundamental analysis for longer time horizons. In the present paper, we formulate this observation as

$$s^e\left[t+1\right] = \omega\left(\tau\right) s_f^e\left[t+1\right] + \left(1-\omega\left(\tau\right)\right) s_c^e\left[t+1\right], \tag{4}$$

where s^e, s_f^e and s_c^e denote market expectations and expectations formed by fundamental analysis and chartism, respectively. Moreover, $\omega\left(\tau\right)$ is a weight function that depends on the time horizon, τ

$$\omega\left(\tau\right) = 1 - \exp\left(-\tau\right). \tag{5}$$

The specific choice of functional form of the weight function in (5) is made to simplify the formal analysis of the model. Of course, other functions may be used as long as $\omega\left(0\right) = 0$ and $\lim_{\tau\to\infty} \omega\left(\tau\right) = 1$.

Technical analysis, or chartism, utilizes past exchange rates in order to detect patterns that are extrapolated into the future. Focusing on past exchange rates is not considered as a shortcoming for currency traders using this technique since a primary assumption behind chartism is that all relevant information about future exchange rate movements is contained in past movements. Further, fundamental analysis is based on a model that consists of macroeconomic fundamentals only, which in the present paper is a sticky-price monetary foreign exchange model due originally to Dornbusch [5] (see the benchmark model in Section 2.1).

The most commonly used technique among currency traders using chartism is the moving average model (e.g., Taylor and Allen [14], and Lui and Mole [9]). According to this model, buying and selling signals are generated by two moving averages; a short-period moving average and a long-period moving average, where a buy (sell) signal is generated when the short-period moving average rises above (falls below) the long-period moving average. In its simplest form, the short-period moving average is the current exchange rate and the long-period moving average is an exponential moving average of past exchange rates.

Thus, when chartism is used, it is expected that the exchange rate will increase (decrease) when the current exchange rate is above (below) an exponential moving average of past exchange rates

$$s_c^e\left[t+1\right] = s\left[t\right] + \gamma\left(s\left[t\right] - MA\left[t\right]\right), \tag{6}$$

where MA is an exponential moving average of past exchange rates, i.e., the long-period moving average. Moreover, the long-period moving average can be written as

$$MA\left[t\right] = \left(1-\exp\left(-\upsilon\right)\right)\sum_{k=0}^{\infty}\exp\left(-k\upsilon\right)s\left[t-k\right], \tag{7}$$

where the weights given to current and past exchange rates sum up to 1

$$\left(1-\exp\left(-\upsilon\right)\right)\sum_{k=0}^{\infty}\exp\left(-k\upsilon\right) = \left(1-\exp\left(-\upsilon\right)\right)\cdot\frac{1}{1-\exp\left(-\upsilon\right)} = 1. \tag{8}$$

Finally, when fundamental analysis is used, it is expected that the exchange rate will adjust to its fundamental value according to a regressive adjustment scheme

$$s_f^e\left[t+1\right] = s\left[t\right] + \delta\left(\overline{s} - s\left[t\right]\right), \tag{9}$$

where $0 \leq \delta \leq 1$ and $\overline{s}$ is (the logarithm of) the spot exchange rate in long-run equilibrium, i.e., the exchange rate's fundamental value. Note that when $\delta = 1$, it is expected that the exchange rate will be in long-run equilibrium the next time period.

3 Formal analysis of the model

The long-run effects in the model are derived in Section 3.1. Thereafter, in Section 3.2, the exchange rate overshooting phenomenon is investigated. Specifically, we will examine whether the exchange rate "overshoot the overshooting equilibrium", i.e., if the magnitude of exchange rate overshooting in the present model is larger than in the Dornbusch [5] model? The adjustment path to long-run equilibrium, when market expectations are characterized by perfect foresight, is derived in Section 3.3, and the *perfect foresight time horizon* and the *perfect foresight overshooting effect* are scrutinized in Section 3.4.

It should again be emphasized that all results in Sections 3.2-3.5 also holds for the perfect foresight time horizon, $\tau = \tau_{pf}$, even if it is only in Section 3.4 that it is highlighted in the notations. The reason for the latter is that the time horizon's dependence on the model's structural parameters is in focus in Section 3.4.

Since the long-period moving average in (7)-(8) is a function of all past exchange rates, the model is not easy to analyze formally. But by assuming that the economy has, for a long time, been in long-run equilibrium before a monetary disturbance occurs, the moving average in (7)-(8) is (approximately) equal to the long-run equilibrium exchange rate

$$\begin{aligned} MA[t] &\approx (1-\exp(-v))\sum_{k=0}^{\infty}\exp(-kv)\,\overline{s} \\ &= \overline{s}\,(1-\exp(-v))\sum_{k=0}^{\infty}\exp(-kv) = \overline{s}. \end{aligned} \tag{10}$$

The assumption in (10) simplify the analysis considerably, but will be relaxed in Section 3.5, where a small simulation study is accomplished in order to illustrate the behavior of the model.

3.1 Long-run equilibrium

Since it is assumed in this section that the exchange rate is in long-run equilibrium,

$$s[t] = \overline{s}. \tag{11}$$

Substituting (10) (assuming equality in the equation) and (11) into the expectations formation in (6), the expectations formed by technical analysis become

$$s_c^e\,[t+1] = \overline{s} + \gamma\,(\overline{s} - \overline{s}) = \overline{s}, \tag{12}$$

i.e., it is expected that the exchange rate is in long-run equilibrium. Moreover, substituting (11) into the expectations formation in (9), the expectations formed by fundamental analysis become

$$s_f^e\,[t+1] = \overline{s} + \delta\,(\overline{s} - \overline{s}) = \overline{s}, \tag{13}$$

i.e., it is expected that the exchange rate is in long-run equilibrium. Then, substitute the expectations formations in (12)-(13) into market expectations in (4)

$$s^e\,[t+1] = \omega\,(\tau)\,\overline{s} + (1 - \omega\,(\tau))\,\overline{s} = \overline{s}, \tag{14}$$

i.e., the market expects that the exchange rate is in long-run equilibrium. Recall that the results in (12)-(14) are based on the assumption that the economy has, for a long time, been in long-run equilibrium before a monetary disturbance occurs.

(The logarithm of) the relative price level[5] in long-run equilibrium, $\overline{p}$, can be solved for by using the equations that describe the money and the international asset markets in equilibrium, i.e., (1)-(2)

$$\overline{p} = m\,[t] - \overline{y}, \tag{15}$$

since, according to (11) and (14),

$$s\,[t] = s^e\,[t+1] = \overline{s}. \tag{16}$$

Thus, the quantity theory of money holds in the long-run since, according to (15),

$$\frac{d\overline{p}}{dm\,[t]} = 1. \tag{17}$$

[5] Henceforth, it will not be emphasized that a macroeconomic variable is expressing the difference between, for example, the domestic and foreign price levels.

Moreover, since (16) as well as

$$p[t] = p[t+1] = \overline{p}, \tag{18}$$

hold in long-run equilibrium, the price adjustment mechanism in (3) reduces to

$$\overline{s} = \overline{p}. \tag{19}$$

Thus, purchasing power parity holds in the long-run since, according to (19),

$$\frac{d\overline{s}}{d\overline{p}} = 1. \tag{20}$$

Finally, the quantity theory of money and purchasing power parity, i.e., (17) and (20), implies that

$$\frac{d\overline{s}}{dm[t]} = \frac{d\overline{s}}{d\overline{p}} \cdot \frac{d\overline{p}}{dm[t]} = 1, \tag{21}$$

which is the long-run effect on the exchange rate of a change in money supply.

It should be stressed that the quantity theory of money and purchasing power parity results are dependent on the simplifying assumption that the economy has, for a long time, been in long-run equilibrium before a monetary disturbance occurs[6]. But even if the quantity theory of money and purchasing power parity hold in the long-run, given this simplifying assumption, there are short-run deviations from these one-to-one relationships. This is demonstrated in the next section on exchange rate overshooting.

3.2 Exchange rate overshooting

Using (10) (assuming equality in the equation) in the expectations formation in (6), and substituting the resulting equation as well as the expectations formation in (9) into market expectations in (4), we have that

$$\begin{aligned} s^e[t+1] &= \omega(\tau)(s[t] + \delta(\overline{s} - s[t])) + \\ &\quad (1-\omega(\tau))(s[t] + \gamma(s[t] - \overline{s})) \\ &= s[t] + \gamma(s[t] - \overline{s}) + \omega(\tau)(\gamma + \delta)(\overline{s} - s[t]). \end{aligned} \tag{22}$$

[6] This is because, in general, $s_c^e[t+1] \neq \overline{s}$ when $s[t] = \overline{s}$. To see this, substitute (11) into (6).

Then, combine the equations that describe the money and the international asset markets in equilibrium, i.e., (1)-(2), and substitute market expectations in (22) into the resulting equation

$$\begin{aligned} m[t] - p[t] &= \overline{y} - \alpha \left(\begin{array}{l} s[t] + \gamma (s[t] - \overline{s}) + \\ \omega(\tau)(\gamma + \delta)(\overline{s} - s[t]) - s[t] \end{array} \right) \\ &= \overline{y} - \alpha (\gamma (s[t] - \overline{s}) + \omega(\tau)(\gamma + \delta)(\overline{s} - s[t])) . \end{aligned} \tag{23}$$

Differentiating (23) with respect to $m[t]$, $s[t]$ and $\overline{s}$ gives

$$dm[t] = -\alpha (\gamma (ds[t] - d\overline{s}) + \omega(\tau)(\gamma + \delta)(d\overline{s} - ds[t])) , \tag{24}$$

or, if (21) is substituted into (24),

$$\frac{ds[t]}{dm[t]} = \frac{d\overline{s}}{dm[t]} + \frac{1}{\alpha (\omega(\tau)(\gamma + \delta) - \gamma)} = 1 + o(\tau) . \tag{25}$$

The current price level is held constant when deriving (25) since it is assumed to be sticky. Thus, (25) is the short-run effect on the exchange rate, near long-run equilibrium, of a change in money supply. A sticky price level also means that the quantity theory of money, i.e., (17), does not hold in the short-run since the price level is not affected by a monetary disturbance. Moreover, purchasing power parity, i.e., (20), does not either hold in the short-run since the exchange rate is affected by a monetary disturbance while the price level is not.

In order to have exchange rate overshooting, it must be true that

$$o(\tau) = \frac{1}{\alpha (\omega(\tau)(\gamma + \delta) - \gamma)} > 0, \tag{26}$$

which means that the time horizon must satisfy

$$\tau > \log \left(1 + \frac{\gamma}{\delta} \right) , \tag{27}$$

where the weight function in (5) is utilized in the derivation. Thus, in the short-run, before goods prices have time to react, the exchange rate will rise (fall) more than money supply, and, consequently, more than is necessary to bring the exchange rate to long-run equilibrium. It will turn out in the next two sections that (27) is also the stability condition for the model when it is assumed that market expectations are characterized by perfect foresight.

By letting $\tau \to \infty$ in (25), an equation describing exchange rate overshooting that corresponds to Dornbusch [5] is obtained

$$\left.\frac{ds\,[t]}{dm\,[t]}\right|_{Dornbusch\,(1976)} = 1 + \frac{1}{\alpha\delta}. \tag{28}$$

(28) corresponds to Dornbusch [5] since, by letting $\tau \to \infty$ in (4)-(5), market expectations coincide with the expectations formed by fundamental analysis. In this case, the magnitude of exchange rate overshooting depends on the nominal interest rate response of real money demand (α), and the expected adjustment speed of the exchange rate according to fundamental analysis (δ).

Moreover, the magnitude of exchange rate overshooting depends inversely on the time horizon

$$\frac{do\,(\tau)}{d\tau} = \frac{do\,(\tau)}{d\omega\,(\tau)} \cdot \frac{d\omega\,(\tau)}{d\tau} = -\frac{\alpha\,(\gamma+\delta)}{\alpha^2\,(\omega\,(\tau)\,(\gamma+\delta)-\gamma)^2} \cdot \exp\,(-\tau) \leq 0, \tag{29}$$

i.e., for shorter time horizons, more weight is placed on technical analysis, and since technical analysis is a destabilizing force[7] in the foreign exchange market, the extent of exchange rate overshooting depends inversely on the time horizon. This also means that the magnitude of overshooting is even larger in this model than in the Dornbusch [5] model

$$\frac{ds\,[t]}{dm\,[t]} \geq \left.\frac{ds\,[t]}{dm\,[t]}\right|_{Dornbusch\,(1976)}, \tag{30}$$

i.e., the exchange rate "overshoots the overshooting equilibrium".

Finally, the magnitude of exchange rate overshooting depends on the structural parameters α, β, γ and δ in the following way[8]

[7] Technical analysis is a destabilizing force since chartists expect that the exchange rate will diverge from long-run equilibrium. To see this, substitute (10) into (6).

[8] The time horizon in currency trade (τ) is given below in (31)-(34), but is endogenously determined when market expectations are characterized by perfect foresight. See the corresponding equations (69)-(72) below.

$$\left.\frac{do(\tau)}{d\alpha}\right|_{\tau \text{ given}} = -\frac{\omega(\tau)(\gamma+\delta)-\gamma}{\alpha^2(\omega(\tau)(\gamma+\delta)-\gamma)^2} < 0, \tag{31}$$

if exchange rate overshooting is assumed, i.e., if it is assumed that (27) holds,

$$\left.\frac{do(\tau)}{d\beta}\right|_{\tau \text{ given}} = 0, \tag{32}$$

$$\left.\frac{do(\tau)}{d\gamma}\right|_{\tau \text{ given}} = -\frac{\alpha(\omega(\tau)-1)}{\alpha^2(\omega(\tau)(\gamma+\delta)-\gamma)^2} \geq 0, \tag{33}$$

and

$$\left.\frac{do(\tau)}{d\delta}\right|_{\tau \text{ given}} = -\frac{\alpha\omega(\tau)}{\alpha^2(\omega(\tau)(\gamma+\delta)-\gamma)^2} \leq 0. \tag{34}$$

Thus, the extent of exchange rate overshooting is larger, the less sensitive real money demand is to changes in the nominal interest rate (α), the faster the expected adjustment speed of the exchange rate is according to technical analysis (γ), and the slower the expected adjustment speed of the exchange rate is according to fundamental analysis (δ). The magnitude of exchange rate overshooting is not affected by changes in the degree of stickiness of goods prices (β).

3.3 Adjustment path to long-run equilibrium

To see how the exchange rate and the price level adjust to long-run equilibrium after a monetary disturbance, it is necessary to incorporate the price adjustment mechanism in (3). In order to do this, start with defining the expected change of the exchange rate as

$$\Delta^e s[t] \equiv s^e[t+1] - s[t], \tag{35}$$

and, then, substitute market expectations in (22) into the definition in (35)

$$\begin{aligned} \Delta^e s[t] &= \gamma(s[t]-\overline{s}) + \omega(\tau)(\gamma+\delta)(\overline{s}-s[t]) \\ &= (\gamma-\omega(\tau)(\gamma+\delta))(s[t]-\overline{s}). \end{aligned} \tag{36}$$

(36) means that the market expect that the exchange rate will adjust to long-run equilibrium, if

$$\gamma - \omega(\tau)(\gamma + \delta) < 0, \tag{37}$$

which reduces to (27), i.e., if it is expected that the exchange rate will adjust to long-run equilibrium after a change in money supply, the exchange rate will also overshoot its long-run equilibrium level in the short-run.

Then, combine the equations that describe the money and the international asset markets in equilibrium, i.e., (1)-(2), and use the definition in (35)

$$m[t] - p[t] = \overline{y} - \alpha \Delta^e s[t]. \tag{38}$$

Thereafter, substitute the long-run equilibrium price level in (15) into (38)

$$\Delta^e s[t] = \frac{p[t] - \overline{p}}{\alpha}, \tag{39}$$

or, if (36) is substituted into (39),

$$s[t] = \overline{s} + \frac{p[t] - \overline{p}}{\alpha(\gamma - \omega(\tau)(\gamma + \delta))}. \tag{40}$$

By using the relationship between the exchange rate and the price level in (40), its long-run counterpart in (19), and the price adjustment mechanism in (3), we can derive an equation that describes the adjustment path for the price level to long-run equilibrium.

In order to do this, start with defining the change of the price level as

$$\Delta p[t] \equiv p[t+1] - p[t], \tag{41}$$

and, then, substitute the definition in (41) into the price adjustment mechanism in (3)

$$\Delta p[t] = \beta(s[t] - p[t]). \tag{42}$$

Thereafter, substitute the relationship between the exchange rate and the price level in (40), and its long-run counterpart in (19), into (42)

$$\Delta p[t] = \left(\frac{\beta}{\alpha(\gamma - \omega(\tau)(\gamma + \delta))} - \beta\right)(p[t] - \overline{p}). \tag{43}$$

(43) means that the price level will adjust to long-run equilibrium, if

$$\frac{\beta}{\alpha\left(\gamma-\omega\left(\tau\right)\left(\gamma+\delta\right)\right)}-\beta<0, \tag{44}$$

or

$$\begin{cases} \tau<\log\frac{\alpha(\gamma+\delta)}{1+\alpha\delta} \\ \tau>\log\left(1+\frac{\gamma}{\delta}\right), \end{cases} \tag{45}$$

where the weight function in (5) is utilized in the derivation. The second equation in (45) is the same as (27). Moreover, the first equation in (45) implies that the market does not expect that the price level (nor the exchange rate) will adjust to long-run equilibrium, because (37) is not satisfied, even if the price level (and the exchange rate) will do that[9]. Consequently, this case is ruled out when market expectations are characterized by perfect foresight.

The difference equation in (43) can be written as

$$\begin{aligned} & p\left[t+1\right]-\left(1+\frac{\beta}{\alpha\left(\gamma-\omega\left(\tau\right)\left(\gamma+\delta\right)\right)}-\beta\right)p\left[t\right] \\ = & -\left(\frac{\beta}{\alpha\left(\gamma-\omega\left(\tau\right)\left(\gamma+\delta\right)\right)}-\beta\right)\overline{p}, \end{aligned} \tag{46}$$

if the definition in (41) is utilized. Then, the solution of the difference equation in (46) is

$$p\left[t\right]=\overline{p}+\left(1+\frac{\beta}{\alpha\left(\gamma-\omega\left(\tau\right)\left(\gamma+\delta\right)\right)}-\beta\right)^{t}\left(p\left[0\right]-\overline{p}\right), \tag{47}$$

and, after twice substituting the relationship between the exchange rate and the price level in (40)[10] into (47), the exchange rate's adjustment path is derived

$$s\left[t\right]=\overline{s}+\left(1+\frac{\beta}{\alpha\left(\gamma-\omega\left(\tau\right)\left(\gamma+\delta\right)\right)}-\beta\right)^{t}\left(s\left[0\right]-\overline{s}\right). \tag{48}$$

---

[9] There is no exchange rate overshooting after a monetary disturbance in this case. See (25) and note that $-1<o\left(\tau\right)<0$ since the first equation in (45) implies that $\alpha\left(\omega\left(\tau\right)\left(\gamma+\delta\right)-\gamma\right)<-1$.

[10] (40) at time t is substituted first into (47), and then is (40) at time 0 substituted. Note that since (40) holds at any point in time, it also holds at time 0.

Thus, the exchange rate and the price level will adjust to long-run equilibrium after a monetary disturbance, if

$$\left|1+\frac{\beta}{\alpha\left(\gamma-\omega\left(\tau\right)\left(\gamma+\delta\right)\right)}-\beta\right|<1. \tag{49}$$

Moreover, the adjustment process is oscillating, if

$$-1<1-\frac{\beta}{\alpha\left(1-\gamma-\left(\gamma+\delta\right)\exp\left(-\tau\right)\right)}-\beta<0, \tag{50}$$

and non-oscillating, if

$$0<1-\frac{\beta}{\alpha\left(1-\gamma-\left(\gamma+\delta\right)\exp\left(-\tau\right)\right)}-\beta<1, \tag{51}$$

where the weight function in (5) is utilized in the derivations. For example, since the expression within the inequalities in (50)-(51) is larger when the time horizon is longer, the adjustment process of the exchange rate (and the price level) to long-run equilibrium after a monetary disturbance is more likely to be oscillating for shorter than for longer time horizons. Also recall that the magnitude of exchange rate overshooting depends inversely on the time horizon. Thus, which is an interesting characteristic of the model, a comparably short time horizon in currency trade is consistent with a highly variable and oscillating exchange rate.

3.4 Perfect foresight path

It is important that market expectations about future exchange rate movements are not arbitrary, and, given the model, do not involve (persistent) prediction errors. This is to say that the market should have perfect foresight, which means that the first equation in (45) can be ruled out since it implies that the market does not expect that the exchange rate will adjust to long-run equilibrium, even if the exchange rate will do that. Thus, since the second equation in (45) is the same as (27), (27) is both the stability condition for the model as well as the condition for exchange rate overshooting.

In order to characterize the perfect foresight adjustment path to long-run equilibrium, start with defining the change of the exchange rate as

$$\Delta s[t] \equiv s[t+1] - s[t]. \tag{52}$$

Then, (48) evaluated one time period ahead is

$$s[t+1] = \overline{s} + \left(1 + \frac{\beta}{\alpha(\gamma - \omega(\tau)(\gamma+\delta))} - \beta\right)^{t+1} (s[0] - \overline{s}), \tag{53}$$

which, together with (48), is substituted into the definition in (52)

$$\Delta s[t] = \left(\begin{array}{l} \left(1 + \frac{\beta}{\alpha(\gamma-\omega(\tau)(\gamma+\delta))} - \beta\right)^{t+1} - \\ \left(1 + \frac{\beta}{\alpha(\gamma-\omega(\tau)(\gamma+\delta))} - \beta\right)^{t} \end{array} \right) (s[0] - \overline{s}). \tag{54}$$

Finally, using (48) in (54) gives the difference equation

$$\Delta s[t] = \left(\frac{\beta}{\alpha(\gamma - \omega(\tau)(\gamma+\delta))} - \beta\right)(s[t] - \overline{s}), \tag{55}$$

which, of course, is similar to the difference equation in (46).

Clearly, for the market to have perfect foresight, it must be true that

$$\Delta^e s[t] = \Delta s[t], \tag{56}$$

or, according to (36) and (55),

$$\gamma - \omega(\tau_{pf})(\gamma+\delta) = \frac{\beta}{\alpha(\gamma - \omega(\tau_{pf})(\gamma+\delta))} - \beta, \tag{57}$$

where the left-hand side of (57) is the expected adjustment speed of the exchange rate and the right-hand side of (57) is the actual adjustment speed of the exchange rate. Recall that τ_{pf} is the perfect foresight time horizon in currency trade. The general solution of (57) is

$$\tau_{pf} = f(\alpha, \beta, \gamma, \delta). \tag{58}$$

Thus, the perfect foresight time horizon is a function of the structural parameters in the model, and, therefore, endogenously determined within the model. Recall that as long as the perfect foresight time horizon is finite, the magnitude of exchange rate overshooting is larger in this model than in the Dornbusch [5] model, i.e., the exchange rate "overshoots the overshooting equilibrium".

In order to derive how the perfect foresight time horizon is affected by changes in the structural parameters α, β, γ and δ, define

$$x_0 \equiv \gamma - \omega\left(\tau_{pf}\right)(\gamma+\delta) = (\gamma+\delta)\exp\left(-\tau_{pf}\right) - \delta, \tag{59}$$

where the weight function in (5) is utilized in the second step, which means that (57) can be written as

$$x_0 = \frac{\beta}{\alpha x_0\left(\tau_{pf}\right)} - \beta, \tag{60}$$

which has the solution

$$x_0 = -\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \frac{\beta}{\alpha}}. \tag{61}$$

Since (27) implies that $x_0 < 0$, we must have that

$$x_0 = -\frac{\beta}{2} - \sqrt{\frac{\beta^2}{4} + \frac{\beta}{\alpha}}. \tag{62}$$

Then, the perfect foresight time horizon can be solved for by combining (59) and (62)

$$\tau_{pf} = -\log \underbrace{\left(1 - \frac{\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \frac{\beta}{\alpha}} + \gamma}{\gamma+\delta}\right)}_{\equiv x_1}, \tag{63}$$

where $0 < x_1 \leq 1$ since $\tau_{pf} \geq 0$, i.e.,

$$\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \frac{\beta}{\alpha}} + \gamma < \gamma + \delta, \tag{64}$$

which is soon utilized. Consequently,

$$\frac{d\tau_{pf}}{d\alpha} = -\frac{1}{x_1}\cdot\frac{dx_1}{d\alpha} = -\frac{1}{x_1}\cdot\frac{\beta}{\alpha^2}\cdot\frac{1}{2\sqrt{\frac{\beta^2}{4}+\frac{\beta}{\alpha}}}\cdot\frac{1}{\gamma+\delta} < 0, \tag{65}$$

$$\frac{d\tau_{pf}}{d\beta} = -\frac{1}{x_1} \cdot \frac{dx_1}{d\beta} = \frac{1}{x_1} \cdot \left(\frac{1}{2} + \left(\frac{\beta}{2} + \frac{1}{\alpha} \right) \cdot \frac{1}{2\sqrt{\frac{\beta^2}{4} + \frac{\beta}{\alpha}}} \right) \cdot \frac{1}{\gamma + \delta} > 0, \quad (66)$$

$$\frac{d\tau_{pf}}{d\gamma} = -\frac{1}{x_1} \cdot \frac{dx_1}{d\gamma} = \frac{1}{x_1} \cdot \frac{\gamma + \delta - \left(\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \frac{\beta}{\alpha}} + \gamma \right)}{(\gamma + \delta)^2} > 0, \quad (67)$$

where (64) is utilized in the last step, and

$$\frac{d\tau_{pf}}{d\delta} = -\frac{1}{x_1} \cdot \frac{dx_1}{d\delta} = -\frac{1}{x_1} \cdot \frac{\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \frac{\beta}{\alpha}} + \gamma}{(\gamma + \delta)^2} < 0. \quad (68)$$

The interpretation of (65)-(68) is that the perfect foresight time horizon is longer, the less sensitive real money demand is to changes in the nominal interest rate (α), the more flexible goods prices (β) are, the faster the expected adjustment speed of the exchange rate according to technical analysis (γ) is, and the slower the expected adjustment speed of the exchange rate according to fundamental analysis (δ) is.

Finally, the effect on the magnitude of exchange rate overshooting of a change in the structural parameters, given perfect foresight, can be derived

$$\frac{do(\tau_{pf})}{d\alpha} = \underbrace{\left.\frac{do(\tau_{pf})}{d\alpha}\right|_{\tau_{pf}\ \text{given}}}_{<0} + \underbrace{\frac{do(\tau_{pf})}{d\tau_{pf}} \cdot \frac{d\tau_{pf}}{d\alpha}}_{\geq 0}, \quad (69)$$

$$\frac{do(\tau_{pf})}{d\beta} = \underbrace{\left.\frac{do(\tau_{pf})}{d\beta}\right|_{\tau_{pf}\ \text{given}}}_{=0} + \underbrace{\frac{do(\tau_{pf})}{d\tau_{pf}} \cdot \frac{d\tau_{pf}}{d\beta}}_{\leq 0} \leq 0, \quad (70)$$

$$\frac{do(\tau_{pf})}{d\gamma} = \underbrace{\left.\frac{do(\tau_{pf})}{d\gamma}\right|_{\tau_{pf}\ \text{given}}}_{\geq 0} + \underbrace{\frac{do(\tau_{pf})}{d\tau_{pf}} \cdot \frac{d\tau_{pf}}{d\gamma}}_{\leq 0}, \quad (71)$$

and

$$\frac{do(\tau_{pf})}{d\delta} = \underbrace{\left.\frac{do(\tau_{pf})}{d\delta}\right|_{\tau_{pf}\ \text{given}}}_{\leq 0} + \underbrace{\frac{do(\tau_{pf})}{d\tau_{pf}} \cdot \frac{d\tau_{pf}}{d\delta}}_{\geq 0}, \quad (72)$$

where (29), (31)-(34) and (65)-(68) are utilized in the derivations. Of course, (29) and (31)-(34) also hold for the perfect foresight time horizon. Thus, the extent of exchange rate overshooting, given perfect foresight, is smaller the more flexible goods prices (β) are. Concerning changes in the other structural parameters in the model, the effects on the magnitude of exchange rate overshooting are ambiguous.

It is important to distinguish between (31)-(34) and (69)-(72). The former set of equations, i.e., (31)-(34), describes the *direct effect* on exchange rate overshooting of a change in the structural parameters, while the latter set of equations, i.e., (69)-(72), describes the *total effect* on exchange rate overshooting of a change in the structural parameters. For example, the direct effect of an increase in the nominal interest rate response of real money demand (α) is to decrease the magnitude of exchange rate overshooting (see the first term on the right-hand side of (69)). However, the *indirect effect* of an increase in this semi-elasticity is to increase the extent of exchange rate overshooting (see the second term on the right-hand side of (69)), and this is because a shorter perfect foresight time horizon (see (65)) increases the magnitude of exchange rate overshooting (see (29)). Consequently, the total effect of an increase in this semi-elasticity is ambiguous (see (69)).

3.5 Simulations of the model

The simplifying assumption that the economy has, for a long time, been in long-run equilibrium before a monetary disturbance occurs, is not made in this section, and this is because it is a bit restrictive. Thus, the long-period moving average in (7)-(8) is no longer necessarily equal to the long-run equilibrium exchange rate as in (10). Instead, it is a function of all past exchange rates, but since this complicates the formal analysis, a small simulation study is accomplished in order to illustrate the behavior of the model. Basically, the equations that are used in the simulations below are the equations that describe the benchmark model, i.e., (1)-(3), the expectations formations, i.e., (4)-(9), and the exchange rate in long-run equilibrium, i.e., (15) and (19). All these equations are reduced to two equations, where (73) below is derived in the Appendix, and

(74) below is (3) slightly rewritten. Thus, the first equation is

$$
\begin{aligned}
s[t] \quad = \quad & \frac{1+\alpha\delta-\alpha\delta\exp(-\tau)}{\alpha(\delta-\delta\exp(-\tau)-\gamma\exp(-\tau-\upsilon))}\cdot(m[t]-\overline{y})- \\
& \frac{1}{\alpha(\delta-\delta\exp(-\tau)-\gamma\exp(-\tau-\upsilon))}\cdot p[t]- \\
& \frac{\gamma(\exp(-\tau)-\exp(-\tau-\upsilon))}{\delta-\delta\exp(-\tau)-\gamma\exp(-\tau-\upsilon)}\cdot\sum_{k=1}^{\infty}\exp(-k\upsilon)\,s[t-k],
\end{aligned}
\tag{73}
$$

and the second equation is

$$
p[t+1]=\beta s[t]+(1-\beta)\,p[t]. \tag{74}
$$

Even if (8) is not explicitly utilized in the derivation of (73), the weights given to current and past exchange rates are still constrained by (8).

In the previous sections, money supply was exogenously given. This assumption is now replaced with the assumption that money supply follows a stochastic process

$$
\begin{cases}
m[t]=m[t-1]-1 & \text{with probability } \varepsilon \\
m[t]=m[t-1] & \text{with probability } 1-2\varepsilon \\
m[t]=m[t-1]+1 & \text{with probability } \varepsilon.
\end{cases}
\tag{75}
$$

Of course, money supply may follow a more realistic stochastic process, like a trend stationary AR(1) process. However, since the aim of the simulations is not to mimic the actual behavior of exchange rates, we use a process that is as simple as possible when illustrating the behavior of the model. Obviously, it is not possible to illustrate the behavior of the model from all possible aspects. Therefore, we restrict the illustrations to three cases; a change in the degree of stickiness of goods prices (β), a change in the time horizon in currency trade (τ), and a change in the distribution of weights given to current and past exchange rates (υ). Specifically, the degree of stickiness of goods prices is assumed to be $\beta = 0.001$ and $\beta = 0.5$, respectively, the time horizon in currency trade is assumed to be $\tau = 2$ and $\tau = 100$, respectively, and the parameter that determines the distribution of weights given to current and past exchange rates is assumed to be $\upsilon = 0.001$ and $\upsilon = 100$, respectively. In all three cases, the same time path of money supply is used, where the probability of a change in money supply is $2\varepsilon = 0.2$. See Figure 1.

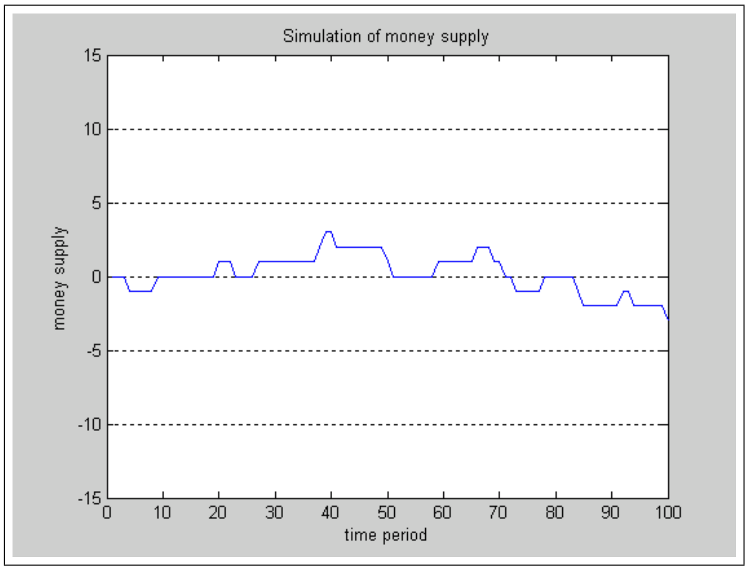

Figure 1.

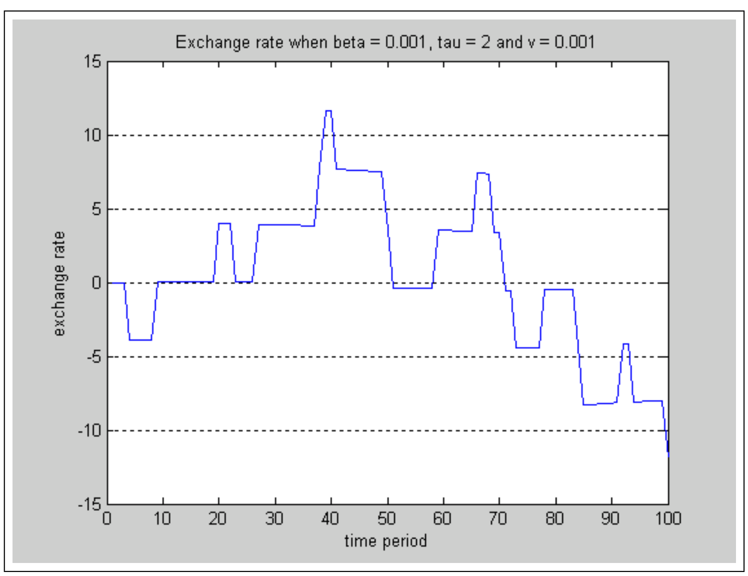

Figure 2.

The values of all other structural parameters in the model do not change in the simulations, and the number of time periods is 100. Thus, when deriving the exchange rate's time path in the simulations, the infinity symbol (∞) in (73) is replaced with 100. See Table 1 for the values of all structural parameters in the simulations, and see Figures 2-9 for the time paths of the exchange rate.

Table 1: The values of the structural parameters in Figures 2-9

Figure	β	τ	v	α	γ	δ
2	0.001	2	0.001	1	0.5	0.5
3	0.001	2	100	1	0.5	0.5
4	0.001	100	0.001	1	0.5	0.5
5	0.001	100	100	1	0.5	0.5
6	0.5	2	0.001	1	0.5	0.5
7	0.5	2	100	1	0.5	0.5
8	0.5	100	0.001	1	0.5	0.5
9	0.5	100	100	1	0.5	0.5

Note: β is the stickiness of goods prices, τ is the time horizon in currency trade, v is the parameter that determines the distribution of weights given to current and past exchange rates, α is the nominal interest rate response of real money demand, γ is the expected adjustment speed of the exchange rate according to technical analysis, and δ is the expected adjustment speed of the exchange rate according to fundamental analysis.

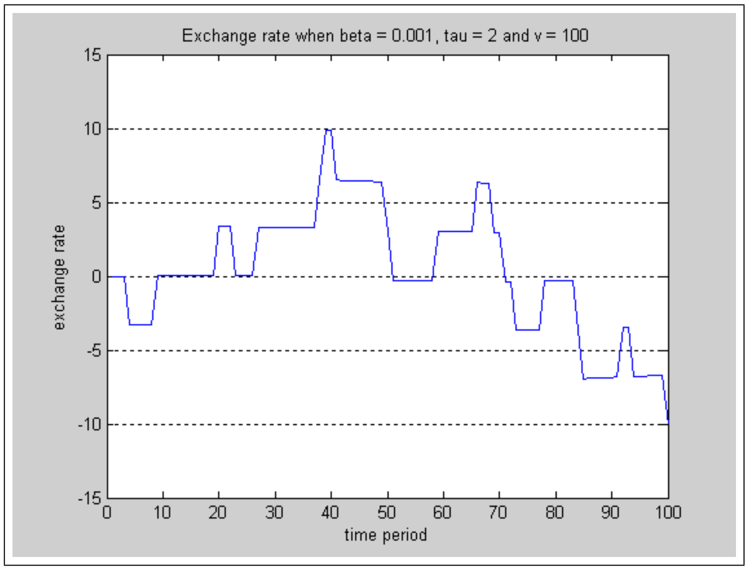

Figure 3.

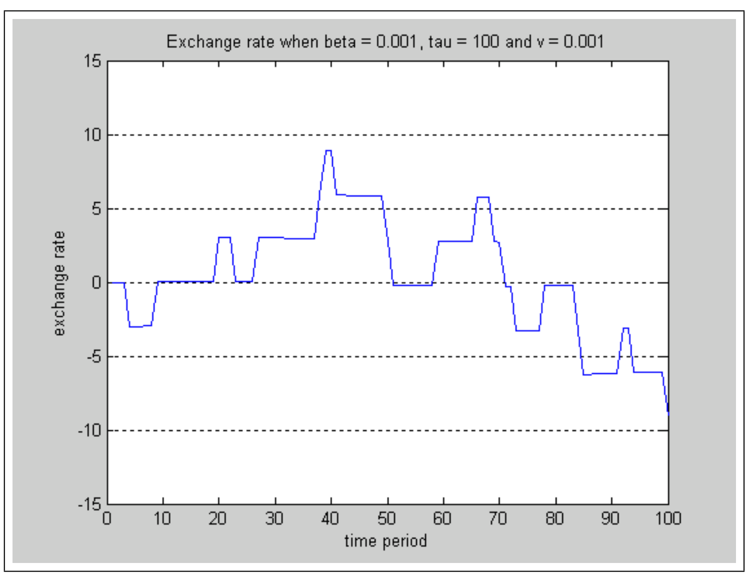

Figure 4.

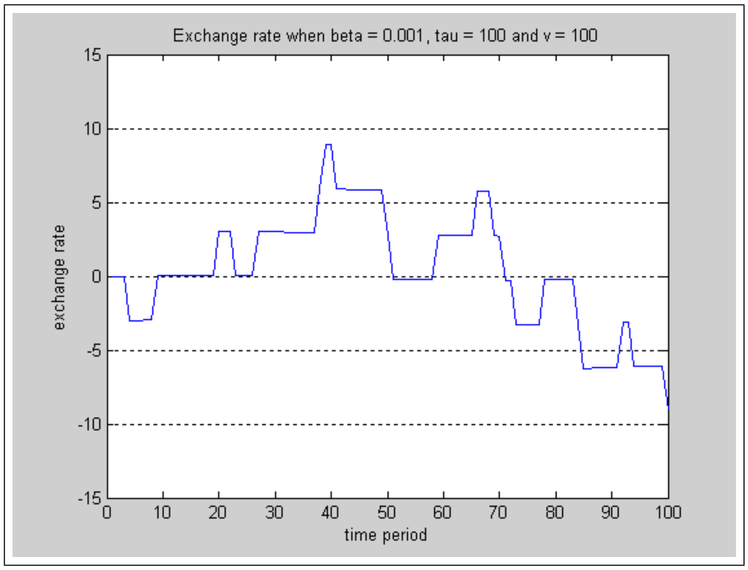

Figure 5.

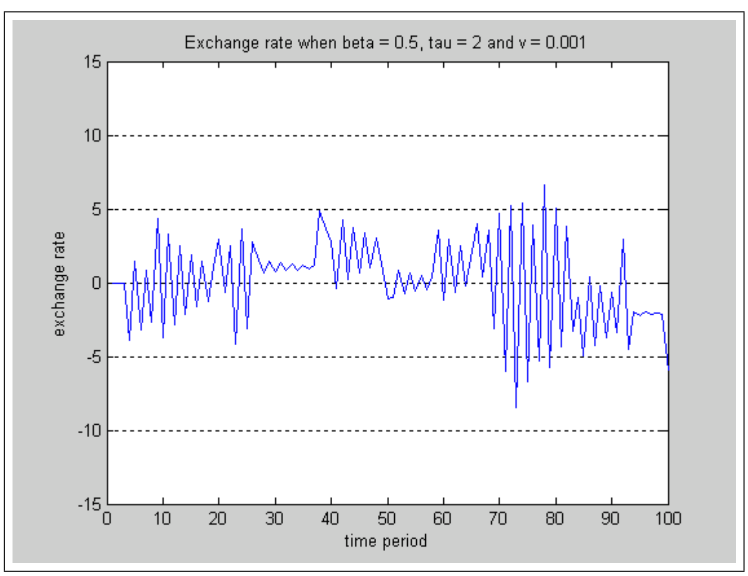

Figure 6.

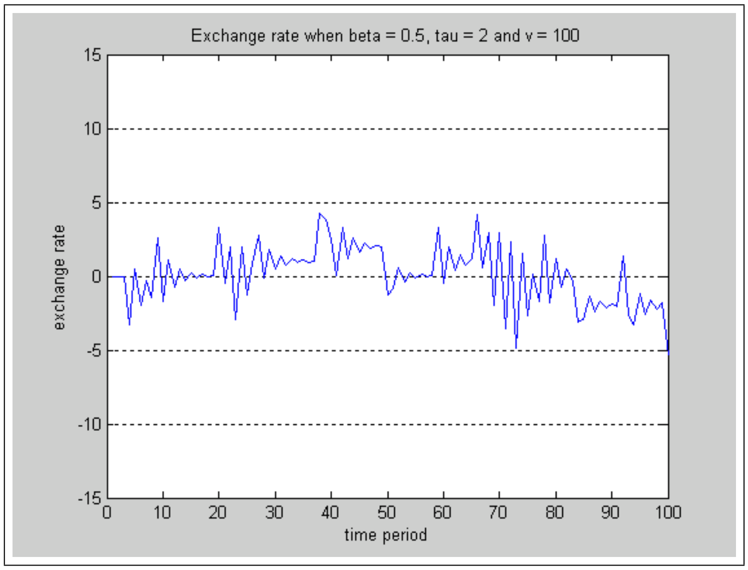

Figure 7.

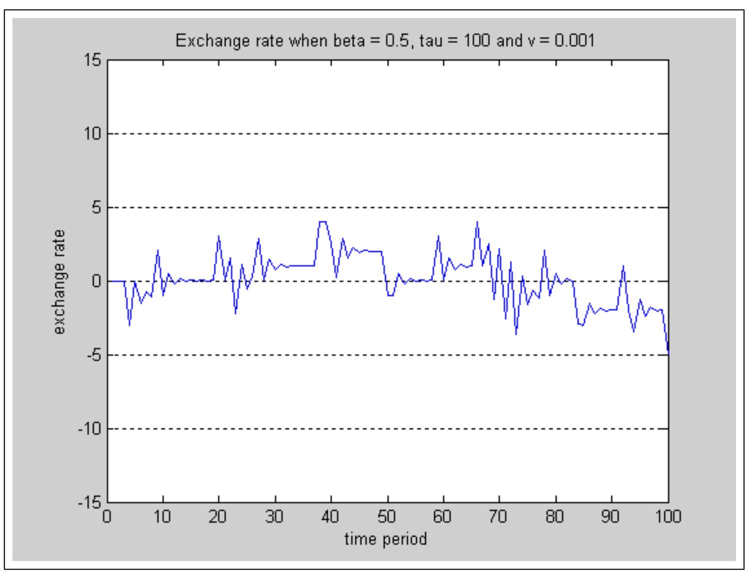

Figure 8.

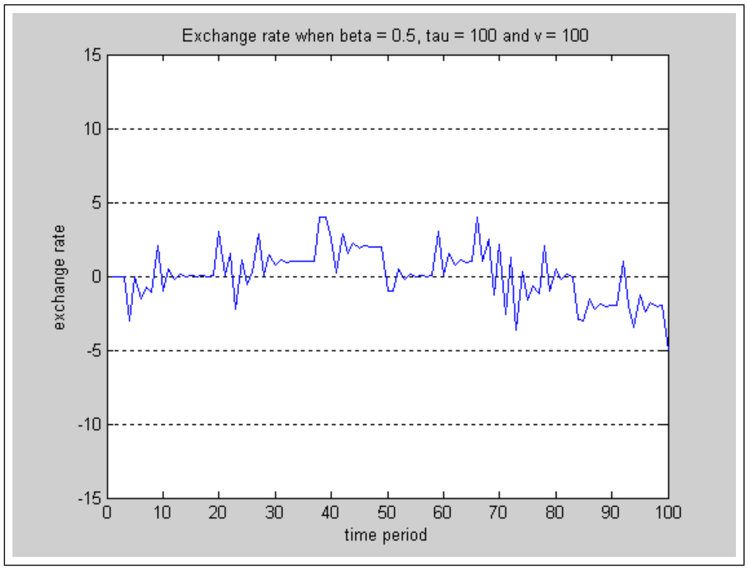

Figure 9.

Firstly, by visual inspection of Figures 1-9, the exchange rate's variability is larger than the variability of money supply. This behavior of the model is also consistent with the model's behavior in the previous sections, where (10) did hold (assuming equality in the equation), since the conditions for exchange rate overshooting and stability were the same. Secondly, an increase in the flexibility of goods prices (β) has two effects on the exchange rate's time path. The first effect is a decrease in the exchange rate's variability, and the second effect is that the exchange rate's adjustment path is non-oscillating when goods prices are almost completely rigid and oscillating when they are less rigid (e.g., compare Figures 2 and 6). The first effect is rather intuitive since goods prices absorb more of the monetary disturbance and, therefore, the exchange rate need not adjust as much for the economy to reach long-run equilibrium. The second effect is consistent with the model's behavior in the previous sections (see the discussion in connection with (50)-(51)).

Thirdly, a longer time horizon in currency trade (τ) decreases the exchange rate's variability (e.g., compare Figures 2 and 4), and this, of course, is because less weight is placed on technical analysis when forming market expectations. Fourthly, an increase in the parameter that determines the distribution of weights given to current and past exchange rates (v) decreases the exchange rate's variability (e.g., compare Figures 2-3), and this is because an increase in the parameter has the same effect, in principle, as a longer time horizon in currency trade (compare τ and v in (73)).

The model's behavior when the remaining structural parameters (i.e., α, γ and δ) change is not discussed in this paper. Of course, changing these parameters may also generate interesting results.

4 Concluding discussion

It was demonstrated in this paper that the exchange rate "overshoots the overshooting equilibrium" when chartists were introduced into a sticky-price monetary model due originally to Dornbusch [5]. Chartists were introduced since questionnaire surveys reveal that currency trade to a large extent is based on technical trading, where moving averages is the most commonly used technique. Moreover, the surveys also reveal that the importance of technical trading depends inversely on the time horizon in currency trade. By implementing these observations theoretically, and deriving the exchange rate's perfect foresight path near long-run equilibrium, it was also demonstrated that the shorter the time horizon is, the larger the magnitude of exchange rate overshooting.

The effects on the exchange rate's time path of changes in the model's structural parameters were also derived. To give one example of the predictions of the model developed, consider a change in the degree of stickiness of goods prices (β). According to (70), there are two effects on the magnitude of exchange rate overshooting; a direct effect and an indirect effect. More flexible goods prices will increase the perfect foresight time horizon, which in turn decreases the extent of exchange rate overshooting. This is the indirect effect of a change in the structural parameter, and since there is no direct effect, the total effect of an increased flexibility of goods prices is a decrease in the magnitude of exchange rate overshooting.

Thus, if we refer to the first paragraph in the introductory section of this paper, a first step has been taken in developing an economic theory of exchange rate movements by taking into account observed behavior of currency traders.

Of course, the present paper is not the only paper that takes this step, but in our opinion there are too few papers that use a chartist-fundamentalist setup in a foreign exchange model. Moreover, the chartists' expectations in those papers are very simple with, for example, no consideration taken for the role of the time horizon in currency trade. Therefore, more research within this area is needed, where the chartists' expectations are based on other technical trading techniques than moving averages, and the fundamentalists' expectations are based on other macroeconomic models than the model used in this paper.

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Appendix

Firstly, combine the equations that describe the money and the international asset markets in equilibrium, i.e., (1)-(2), and solve for the exchange rate

$$s[t] = s^e[t+1] + \frac{m[t] - p[t] - \overline{y}}{\alpha}. \tag{76}$$

Secondly, substitute the expectations formed by technical and fundamental analyses, i.e., (6) and (9), into market expectations in (4)

$$\begin{aligned} s^e[t+1] &= \omega(\tau)(s[t] + \delta(\overline{s} - s[t])) + \\ &\quad (1 - \omega(\tau))(s[t] + \gamma(s[t] - MA[t])) \\ &= (1 - \omega(\tau)(\gamma + \delta) + \gamma) s[t] + \omega(\tau)\delta\overline{s} - \\ &\quad (1 - \omega(\tau))\gamma MA[t], \end{aligned} \tag{77}$$

and, then, substitute the long-period moving average in (7) into (77)

$$\begin{aligned} s^e[t+1] &= (1 - \omega(\tau)(\gamma + \delta) + \gamma) s[t] + \omega(\tau)\delta\overline{s} - \\ &\quad (1 - \omega(\tau))\gamma(1 - \exp(-\upsilon))\sum_{k=0}^{\infty}\exp(-k\upsilon) s[t-k] \\ &= (1 - \omega(\tau)(\gamma + \delta) + \gamma) s[t] + \omega(\tau)\delta\overline{s} - \\ &\quad (1 - \omega(\tau))\gamma(1 - \exp(-\upsilon)) s[t] - \\ &\quad (1 - \omega(\tau))\gamma(1 - \exp(-\upsilon))\sum_{k=1}^{\infty}\exp(-k\upsilon) s[t-k] \\ &= (1 - \omega(\tau)\delta + (1 - \omega(\tau))\gamma\exp(-\upsilon)) s[t] + \omega(\tau)\delta\overline{s} - \\ &\quad (1 - \omega(\tau))\gamma(1 - \exp(-\upsilon))\sum_{k=1}^{\infty}\exp(-k\upsilon) s[t-k]. \end{aligned} \tag{78}$$

Thirdly, substitute (78) into (76)

$$\begin{aligned} s[t] &= (1 - \omega(\tau)\delta + (1 - \omega(\tau))\gamma\exp(-\upsilon)) s[t] - \\ &\quad (1 - \omega(\tau))\gamma(1 - \exp(-\upsilon))\sum_{k=1}^{\infty}\exp(-k\upsilon) s[t-k] + \\ &\quad \omega(\tau)\delta\overline{s} + \frac{m[t] - p[t] - \overline{y}}{\alpha}, \end{aligned} \tag{79}$$

and, then, solve for the exchange rate

$$\begin{aligned} s\left[t\right] &= \frac{\alpha\omega\left(\tau\right)\delta\overline{s}+m\left[t\right]-p\left[t\right]-\overline{y}}{\alpha\left(\omega\left(\tau\right)\delta-\left(1-\omega\left(\tau\right)\right)\gamma\exp\left(-\upsilon\right)\right)}- \\ & \quad \frac{\left(1-\omega\left(\tau\right)\right)\gamma\left(1-\exp\left(-\upsilon\right)\right)}{\omega\left(\tau\right)\delta-\left(1-\omega\left(\tau\right)\right)\gamma\exp\left(-\upsilon\right)}\cdot\sum_{k=1}^{\infty}\exp\left(-k\upsilon\right)s\left[t-k\right]. \end{aligned} \tag{80}$$

Fourthly, the exchange rate in long-run equilibrium, according to (15) and (19), is

$$\overline{s}=m\left[t\right]-\overline{y}, \tag{81}$$

which is substituted into (80)

$$\begin{aligned} s\left[t\right] &= \frac{\left(1+\alpha\omega\left(\tau\right)\delta\right)\left(m\left[t\right]-\overline{y}\right)-p\left[t\right]}{\alpha\left(\omega\left(\tau\right)\delta-\left(1-\omega\left(\tau\right)\right)\gamma\exp\left(-\upsilon\right)\right)}- \\ & \quad \frac{\left(1-\omega\left(\tau\right)\right)\gamma\left(1-\exp\left(-\upsilon\right)\right)}{\omega\left(\tau\right)\delta-\left(1-\omega\left(\tau\right)\right)\gamma\exp\left(-\upsilon\right)}\cdot\sum_{k=1}^{\infty}\exp\left(-k\upsilon\right)s\left[t-k\right]. \end{aligned} \tag{82}$$

Finally, substitute the weight function in (5) into (82), and (73) in Section 3.5 is derived.

Chartist Trading in an Exchange Rate Target Zone Model

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Abstract

This paper develops a target zone model that takes into account that traders, to a large extent, use technical trading techniques in currency trade. Empirical surveys have revelaed that the relative importance of technical and fundamental analyses depend on the time horizon in currency trade. This finding has been incorporated in the model. The model also includes realignment expectations, which increase with the weight of chartists. The introduction of chartists may significantly reduce, what Krugman [11] called, the "honeymoon effect" of a fully credible target zone. Chartists may also reverse the "honeymoon effect". Further, in the present model chartists may cause the correlation between the exchange rate and the instantaneous interest rate differential to become either positive or negative. This conforms to previous empirical findings.

JEL codes:: E43; E52; F31; F33; F37.

Keywords: Chartists; Fundamentalists; Interest Rate Term Structure; Target Zone.

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1 Introduction

Background A target zone implies setting a central parity around which the exchange rate is allowed to fluctuate within an interval. When the exchange rate approaches the limits of the interval, the monetary authority intervenes in the domestic monetary market and prevents the exchange rate from deviating outside the interval. The ERM II (the Exchange Rate Mechanism of the European Monetary System) is an example of an existing target zone.

The seminal work of Krugman [11], which first circulated in 1988, has become the standard way of modeling target zones, and has functioned as a starting point for much of the subsequent literature. Krugman [11] assumes a perfectly credible target zone, defended by monetary authorities who use infinitesimal interventions at the edges of the zone. The model is defined over fundamentals and the exchange rate is a function of monetary fundamentals that follow a Wiener process. By using Ito calculus, Krugman [11] attains an explicit solution for the exchange rate path within the band.

Krugman's target zone model [11], causes the exchange rate curve to bend at the edges of the zone, so that it forms an S-curve. Thus, the fluctuation band for the exchange rate becomes narrower than that for fundamentals. This means that the target zone has a stabilizing effect on the exchange rate. A comparison of the S-shaped curve, representing the target zone exchange rate, with the linear 45-degree line, representing the free-float exchange rate, demonstrates what Krugman called the "honeymoon effect". The S-shaped curve is flatter than the 45-degree line, i.e., the exchange rate in a target zone is less sensitive to changes in fundamentals than a free floating exchange rate. Target zones would, in this way, have a stabilizing effect on the exchange rate without the monetary authority actually having to intervene in the foreign exchange market.

Previous literature The literature following Krugman [11] has tried, in various ways, to modify his basic model so as to make it more realistic and to fit the data better. The major extensions of the basic target zone model have focused on a few different alternatives; Bertola and Caballero [2], Bertola and Svensson [3], Neely et al. [19], Tristani [25] and Werner [26], for example, relax the assumption of a fully credible target zone and introduce expectations of realignments and, in some cases, also include actual realignments. Beetsma and Van Der Ploeg [4], Miller and Weller [17], and Neely et al. [19] introduce price rigidity. Beetsma and Van Der Ploeg [4] and Lindberg and Söderlind [13], also introduce intramarginal interventions by central banks. Finally, Krugman and Miller [12], and later Kempa and Nelles [10] introduce heterogeneity among traders into target zone models in the form of stop-loss trading. These different ways of extending the Krugman model [11] have, for varying countries and periods, improved the fit of the target zone model to empirical data, mainly from the ERM. The theoretical implications of the Krugman model [11], the empirical characteristics of the ERM exchange rates and the theoretical improvements of the Krugman model [11] are discussed more thoroughly below.

Theoretical implications and empirical data Although innovative and highly influential, Krugman's [11] model has some empirical implications that have been questioned by data from the ERM, the Bretton-Woods system and the Gold Standard.

Firstly, the Krugman model [11] implies that the target zone is fully credible. However, in the early periods of the ERM, realignments (i.e., a redefinition of central parity and the bounds of the target zone) occurred quite frequently. The ERM did, however, become more credible over time, in the sense that the periods between realignments increased (Flood et al. [7]). Empirically, realignment expectations have been found to depend on the level of the exchange rate within the zone, the variability of the exchange rate or of previous realignments (see for example Chen and Giovannini [5]). Tristani [25] and Werner [26], for example, include realignment expectations as depending on the level of the exchange rate within the zone, in their models. This improves the empirical fit of the model with regard to the much debated "honeymoon effect", which has received little support empirically (Flood et al. [7]). Including realignment expectations as depending on the level of the exchange rate within the zone makes the slope of the exchange rate steeper than the 45-degree line, and completely erases the "honeymoon effect". Bertola and Caballero [2] and Bertola and Svensson [3] for example, not only relax the assumption of a fully credible target zone but also introduce actual realignments in their models. In addition to correcting for the empirical flaws handled by Tristani [25] and Werner [26], this also increases the mean reversion in the exchange rate and creates a hump-shaped distribution for the exchange rate.

Secondly, the Krugman model [11] implies that central banks only engage in marginal interventions in order to keep the exchange rate within the target zone. However, empirical data suggests that interventions may also be intramarginal, as, for example, in the case of the Netherlands, Belgium and Sweden (Beetsma and Van Der Ploeg [4], and Lindberg and Söderlind [13]). Intramarginal interventions improve the empirical fit of the model not only with regard to when the monetary authority intervenes but also with regard to the exchange rate distribution, see below.

Thirdly, the Krugman model [11] implies via the S-shaped exchange rate curve, a very distinct non-linear relationship between the exchange rate and the fundamentals, which empirically, does not seem to be the case (Flood et al. [7]). The S-shape of the exchange rate curve is due to the assumption of marginal infinitesimal interventions used to defend the zone. There seem to exist some non-linearities in this relationship, but they are not similar to those implied by the Krugman model (Flood et al. [7]), indicating that there is no "honeymoon effect". Although, the inclusion of realignment expectations that depend on the level of the exchange rate within the band erases the "honeymoon effect", the model, nevertheless, still implies an S-shaped exchange rate distribution. The model in Bertola and Caballero [2], for example, implies an inverted S-shape for the exchange rate curve. However, this non-linear relationship has received little empirical support. Empirically, the relationship between the exchange rate and fundamentals seem, in most cases, to be approximately linear (Flood et al. [7]).

Fourthly, the S-shaped exchange rate curve of the Krugman model [11] implies a U-shaped distribution of the exchange rate within the band. This means that the exchange rate is near the edges of the band most of the time. There is weak evidence of U-shaped distributions among the currencies of some countries within the ERM (Belgium, Denmark and France, see Flood et al. [7]). However, empirical findings show that the distribution of the exchange rate within the band is often hump-shaped. This means that the exchange rate is, most of the time, around the middle of the band (Bertola and Caballero [2], Flood et al. [7] and Lindberg and Söderlind [14]). Including intramarginal interventions or realignments in the model, creates mean reversion in the exchange rate causing the distribution to become more hump-shaped.

Finally, assuming that UIP (i.e., uncovered interest rate parity) holds in the Krugman model [11] implies that there is a negative correlation between the expected rate of depreciation of the exchange rate and the current exchange rate within the zone. This in turn, implies a negative correlation between the interest rate differential and the exchange rate within the zone. The negative relationship between the exchange rate and its expected depreciation is also an effect of the assumption of a fully credible zone. The use of infinitesimal interventions at the edges of the zone results in a concave (convex) exchange rate curve at the upper (lower) edge of the zone. Hence, as the exchange rate approaches the upper limit of the zone, an appreciation of the currency is expected, and as it approaches the lower limit, a depreciation is expected. Thus, in a fully credible target zone the interest rate differential displays mean reversion towards central parity. This relationship has been found empirically to be very different between countries and over different periods. Empirical data displays a positive, zero or a negative correlation between the two (Beetsma and Van Der Ploeg [4], Bertola and Caballero [2], Flood et al. [7] and Svensson [22]).

Technical analysis The term chartism originally stems from the use of different types of time charts among traders to form expectations about the future exchange rate. The terms chartism and technical analysis are used interchangeably in the paper. Technical analysis[1], or chartism, does not rely on any underlying economic or fundamental analysis.

Today, a huge variety of different tools are used within technical analysis including both so called trend indicators and oscillating indicators. These indicators involve the use of past exchange rates, traded volumes, and the volatility of past exchange rates, which chartists extrapolate into the future. Both types of tool are used for the purpose of trying to predict trends and trend reversals in the exchange rates. Hence, technical analysis may cause the exchange

[1] For a brief description of technical analysis, the reader can turn to Neely [18].

rate to deviate from its fundamental value. There is some empirical evidence for so-called bandwagon expectations (i.e., expectations of further appreciations /depreciations of the currency of a greater magnitude than before) on the foreign exchange market (for example Ito [9]) which could very well be the result of the use of technical analysis.

The length of the time horizon in currency trade has been shown to be a decisive factor for the choice of trading techniques among foreign exchange traders. Taylor and Allen [24] conducted a questionnaire survey for the Bank of England on the foreign exchange market in London in November 1988. This survey was the first to ask, specifically, about the use of technical analysis or chartism among currency traders and the results were striking. Two percent of the respondents reported that they never used fundamental analysis in forming their exchange rate expectations, while 90 percent of the respondents reported placing some weight on technical analysis at the intraday to one week horizon. At longer time horizons, Taylor and Allen [24] found that the importance of technical analysis declined. Later surveys by, for example Cheung and Chinn [6], Lui and Mole [15], Menkhoff [16], and Oberlechner [20] have confirmed the extensive use of technical analysis in currency trade in the Asian, European and U.S. markets. A general result, found in all these surveys, is an increased reliance on technical, as opposed to fundamental, analysis for shorter time horizons.

The extensions of the Krugman model [11] discussed above, have all, except for Krugman and Miller [12], and Kempa and Nelles [10], continued to assume homogeneity, in spite of heterogeneity among traders having been well established by surveys (see referenced surveys above) of the foreign exchange market. Although Frankel and Froot, as early as in 1986 [8], offered chartists trading as a plausible explanation to the U.S. Dollar's extreme deviation from its fundamental value during the 1980's, and Taylor and Allen [24] showed already in 1992, that technical analysis is extensively used in foreign exchange trading, there has been no attempt (to our knowledge) to introduce chartists in a target zone framework.

Approach The present paper follows the Krugman [11] approach and introduces heterogeneity in the market in the form of two different types of traders, chartists and fundamentalists. The weight of chartists and fundamentalists is determined by the length of the time horizon in currency trade (i.e., the length of time on which traders intend to trade). The Krugman model [11] is augmented with realignment expectations where these depend on the weight of chartists in the market. Thus, realignment expectations are independent of the level of the exchange rate in the zone. When the time horizon in trade is infinitely long, i.e., when the weight of chartists is zero, the model is equivalent to the Krugman model. Thus, the augmented model will nest that of Krugman [11].

A specific purpose of this paper is to theoretically implement a chartist-fundamentalist set-up in a target zone framework. The qualitative results of this model will then be compared with empirical facts, and with the qualitative results of the Krugman model. In the present paper, the fundamental is a composite term that is partly a process controlled by the monetary authority, as in the Krugman model, and partly stochastic processes driven by the currency trade among chartists and fundamentalists.

The implementation of a chartist-fundamentalist set-up with realignment expectations in the spirit of Krugman's [11] target zone model is, as far as we know, a complete novelty within the target zone literature. The way in which realignment expectations, and chartist and fundamentalist traders are implemented in this model is also, as far as we know, completely novel. The paper also contains an attempt to produce a more explicit and stringent mathematical derivation of the explicit solution for the exchange rate path within a target zone than that offered by for example Krugman [11].

The main questions in focus are: how are the dynamics of the exchange rate and the interest rate differentials within the zone affected when chartism is introduced into a target zone model? How does chartism affect the correlation between the interest rate differential and the exchange rate? In summary, will including heterogeneous traders in the model create a better fit to stylized empirical findings?

Main findings The introduction of chartist and fundamentalist traders in the Krugman model [11] does create a better empirical fit with regard to the correlation between the interest rate differential and the exchange rate and with regard to exchange rate volatility. The correlation may be both positive and negative and the volatility increases when the exchange rate is further from central parity. Allowing for positive realignment expectations is also an empirical improvement as is allowing for expectations to be heterogeneous.

However, due to the use of infinitesimal marginal interventions in the model, the relationship between the exchange rate and fundamentals is still, even with the market completely dominated by chartists, somewhat S-shaped. When fundamentals are equal to central parity there is a reversed "honeymoon effect" inherent in the present model. This effect does not completely disappear when the market is completely dominated by chartists, but it is diminished for larger deviations of the fundamentals and has a reversed effect for fundamentals equal to central parity. Because of the use of infinitesimal marginal interventions in the model, the exchange rate distribution is still strongly U-shaped. Thus, it does not create a better fit with empirical data with regard to the exchange rate distribution.

The mathematical derivations, of the solution for the model, suggest another notation for the expected depreciation of the exchange rate than the one commonly used in the literature. The interpretation of this term as the expected depreciation of the exchange rate may also be a little misleading, rather it should be interpreted as a deterministic *measure* of the expected depreciation of the exchange rate. If there are no chartists, the *measure* of the expected depreciation of the exchange rate is the deterministic expected change in the composite fundamental deviation from central parity within the zone. If there are chartists, it is the deterministic expected change in the composite fundamental deviation from central parity *and* the expected change in central parity (i.e., the expected realignment).

Outline The remainder of this paper is organized as follows. The benchmark model is outlined in Section 2. The formal analysis of the model is carried out in Section 3, starting with the behavior of the exchange rate in subsection 3.1 and then continuing with the interest rate differentials in subsection 3.2. Section 4 contains some concluding remarks and the Appendix presents proofs of the propositions.

2 The model

2.1 The exchange rate

Basically, the benchmark model is a log-linear monetary model of the exchange rate in line with Krugman [11]. All variables, except the interest rate, are in natural logarithms and Greek letters denote positive structural parameters. Specifically, the spot exchange rate, $s(t)$, which is defined as units of the domestic currency per unit of the foreign currency, is

$$s(t) = x(t) + \alpha E(t), \tag{1}$$

where α is the semi-elasticity of the money demand[2] with respect to the interest rate, $E(t)$ is a deterministic measure of the expected instantaneous depreciation of the exchange rate at time t, and $x(t)$ denotes the composite monetary fundamentals in the model. Specifically, the composite monetary fundamental is

$$x(t) = m(t) + v(t). \tag{2}$$

The two parts of $x(t)$ are the domestic money supply controlled by the monetary authority, $m(t)$, and a composite currency demand shock term, $v(t)$. Henceforth, monetary policy is assumed to be passive unless needed to maintain the target zone. Thus, $m(t)$ is a process controlled by monetary authority, to be further explained below. The shock term, $v(t)$, is assumed to be an exogenous stochastic process driven by two independent Wiener processes, $W_f(t)$ and $W_c(t)$, which stem from the impact of fundamentalist and chartist currency trade in the model, again these will be explained further later.[3]

Note that Krugman [11] uses the term $E[ds]/dt$ without further ado, whereas, in the present paper, we use the notation $E(t)$ as the measure of the expected instantaneous depreciation of the exchange rate at time t. The term $E[ds]/dt$

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[2] The semi-elasticity of the money demand stems from the home and foreign money market equilibrium used to derive the benchmark model, i.e., equation (1).

[3] Equation (2) is equvalent to that in Krugman's model [11]. However, in his model the shock term, $v(t)$, represents velocity of money.

is a mathematically dubious notation[4]. However, since Krugman [11] does not produce a formal proof of his solution for the exchange rate within the band, it is hard to know the formal definition of $E[ds]/dt$, and whether or not it corresponds to the formal definition of $E(t)$, as shown in the Appendix in the present paper.[5]

2.2 Exchange rate trading

According to questionnaire surveys carried out previously, the relative importance of technical versus fundamental analysis in the foreign exchange market depends on the time horizon in currency trade. For shorter time horizons, more weight is placed on technical analysis, while more weight is placed on fundamental analysis for longer time horizons. In the present model, this observation is formulated as

$$v(t) = \omega(\tau)\, y_f(t) + (1 - \omega(\tau))\, y_c(t)\,, \tag{3}$$

where $\omega(\tau)$ is a weight function that depends on the time horizon in currency trade, τ. Specifically, the weight function [6] is

$$\omega(\tau) = 1 - e^{-\tau}. \tag{4}$$

Thus, the stochastic term, $v(t)$, reflects changes in the money supply due to currency trade and is driven by fundamentalists' and chartists' trading strategies. The trading strategies of fundamentalist and chartist traders in the foreign exchange market are represented by $y_f(t)$ and $y_c(t)$, respectively.

Fundamental analysis is based on macroeconomic fundamentals only. Specifically, fundamental analysis is driven by a Brownian motion with a zero drift and a diffusion term, $\sigma_f(t)$, according to

$$dy_f(t) = \sigma_f(t)\, dW_f(t)\,, \tag{5}$$

[4] The term $E[ds]/dt$ denoting the expected instantaneous depreciation of the exchange rate has become standard within the target zone literature.
[5] For more details, see the Appendix, proof of Proposition 1.
[6] Note that the weight function used in the present paper is inspired by the weight function used in Bask [1].

where $W_f(t)$ is a Wiener process.[7] The economic intuition behind this representation is that the Gaussian disturbance term, $\sigma_f(t)W_f(t)$, reflects the stochastic changes in liquidity needs and the zero drift reflects the assumed unchanged money supply. Thus, fundamentalists add volatility to the process, $v(t)$.

Technical analysis is based on past history that is extrapolated into the future. Specifically, technical analysis is driven by a Brownian motion with a positive drift and a diffusion term, $\sigma_c(t)$, according to

$$dy_c(t) = \gamma(y_f(t) - y_f(t-\tau))\,dt + \sigma_c(t)\,dW_c(t), \tag{6}$$

where $\gamma > 1$ is the "trend coefficient" in the drift term $\gamma[y_f(t) - y_f(t-\tau)]$, $\sigma_c(t)W_c(t)$ is the Gaussian disturbance term, and $W_c(t)$ is the Wiener process.[8] Note that $\gamma E_t[y_f(t) - y_f(t-\tau)]\,dt = 0$, and thus, the trend component in the chartist strategy actually vanishes when solving the model, and the true impact of chartists in the model is to increase volatility in the process, $v(t)$, via the Gaussian disturbance term, $\sigma_c(t)W_c(t)$. The economic intuition behind this representation is that chartists use the Wiener process, that the fundamentals follow, to extrapolate a trend from (see the first term of the right hand side of (6)), since the model is driven by fundamentals. The Gaussian disturbance term, $\sigma_c(t)W_c(t)$, reflects the fact that this type of extrapolating strategy causes bandwagon expectations, which, in turn, are volatility enhancing. Thus, chartists also add volatility to the process, $v(t)$.

Hence, $v(t)$ in (3) and, thus, $x(t)$ in (2), are driven by the two independent Wiener processes, $W_f(t)$ and $W_c(t)$. Specifically, $dv(t)$ will take the form

$$dv(t) = \omega(\tau)\,dy_f(t) + (1-\omega(\tau))\,dy_c(t). \tag{7}$$

The relative importance of the different Wiener processes is determined by the length of the market's time horizon in currency trade. If the time horizon in

[7] Or, equivalently, $y_f(t) = \int_0^t \sigma_f(s)\,dW_f(s)$, where the right hand side is an Ito integral (i.e., a stochastic integral).

[8] Or, equivalently, $y_c(t) = y_c(0) + \gamma\int_0^t (y_f(s) - y(s-\tau))\,ds + \int_0^t \sigma_c(s)\,dW_c(s)$.

currency trade is myopic, the weight of chartists is high, and thus, the impact of $dy_c(t)$ is high. On the other hand, if the time horizon is farsighted, the weight of fundamentalists is high, and thus, the impact of $dy_f(t)$ is high.

If traders' trading strategies in the foreign exchange market were homogenous (i.e., if there were only fundamentalist traders in the market), $x(t)$ would only be driven by $dy_f(t)$, and the model would be equivalent to the Krugman model [11]. However, since there are now also chartist traders in the market, the exchange rate is driven by the additional stochastic component, $dy_c(t)$, stemming from chartist trading. Chartists will, through the composite shock term, $dv(t)$, only have second order importance on the exchange rate path.

This is the first of two ways, in which, chartists affect the exchange rate in this model, the second way is through the realignment intensity, which will be further explained below. Chartists will increase volatility in the model, via the composite shock term, and chartists will push the exchange rate away from fundamentals, but only via the realignment intensity.

2.3 The target zone

The target zone is defined over fundamentals and the monetary authority sets the central parity and the upper and lower bounds for the fundamentals, which define the target zone, according to

$$c(t) = (x(t)^{upper} + x(t)^{lower})/2, \tag{8}$$

where $c(t)$ is the central parity, and $x(t)^{upper}$ and $x(t)^{lower}$ are the upper and lower bounds for the target zone. The upper and lower bounds for fundamentals correspond to an upper and a lower bound for the exchange rate, $s(t)^{\max}$ and $s(t)^{\min}$. Assuming symmetry in the model, the upper and lower bounds are of equal length from central parity. The monetary authority is assumed to intervene in the domestic monetary market to maintain the target zone for the exchange rate. In other words, the monetary authority is prepared to decrease (increase) the money supply in order to prevent the exchange rate from rising above (falling below) some upper (lower) bound. As long as the exchange rate is within the target zone, the money supply remains unchanged.

Infinitesimal interventions are used at the edges of the zone to prevent the exchange rate from moving outside the band. The regulators L (lower) and U (upper) represent these interventions and, consequently, $dL(t)$ is positive only when $x(t) = x(t)^{lower}$ and $dU(t)$ is positive only when $x(t) = x(t)^{upper}$ (i.e., when the fundamentals reach their upper or lower bounds)

$$dm(t) = dL(t) - dU(t). \tag{9}$$

Thus, the composite monetary fundamental follows the stochastic process

$$dx(t) = \omega(\tau)\, dy_f(t) + (1 - \omega(\tau))\, dy_c(t) + dL(t) - dU(t). \tag{10}$$

Inside the zone, $dm(t) = 0$, which gives the following expression for the composite monetary fundamental within the zone

$$dx(t) = \omega(\tau)\, dy_f(t) + (1 - \omega(\tau))\, dy_c(t). \tag{11}$$

2.4 Realignment expectations

In the model, there is a constant probability of a realignment of the target zone, i.e., a shift in the central parity, $c(t)$, as well as a shift in the upper and lower bounds of the fundamentals, $x(t)^{upper}$ and $x(t)^{lower}$. In the present paper, expectations of a realignment are modelled as depending on the weight of chartists in the market. Thus, the shorter the time horizon in currency trade, i.e., the more weight that is put on chartists, the higher the realignment intensity. The intuition is that because chartist trading is volatility increasing and thus destabilizing, an increase in chartist traders will also increase expectations of a realignment. Hence, this is the second impact of chartists in this model.

The expected realignment is not explicitly a function of the exchange rate level within the zone and is thus a constant. Realignment expectations will, therefore, not affect the slope of the exchange rate function.[9] Modeling realignments as also depending on the level of fundamentals within the band, does not

[9] In Neely et al. [19], for example, the probability of realignment increases when output moves away from full employment. In Werner [26] the probability of realignment increases with the distance of the fundamentals from central parity, and in Svensson [22], [23] and Bertola and Svensson [3] realignments follow a Poisson process.

change the results to any great extent. The only effect is to cause the slope of the exchange rate function to become steeper and, thus, further diminish the honeymoon effect. It does not improve the empirical fit with regard to the exchange rate distribution or the S-shaped exchange rate curve either. To isolate the impact of chartists with regard to realignment expectations, these are modeled as depending on the weight of chartists only.

In the present paper, realignment expectations are modeled as a hazard function[10] that is increasing with the weight of chartists in the market, as a percentage of the width of the zone. Hence,

$$\lambda dt = \frac{\rho(1 - \omega(\tau))}{\theta} dt, \tag{12}$$

where ρ is a constant that affects the size of the realignment, θ is the width of the band defined over fundamentals according to $\theta = x(t)^{upper} - x(t)^{lower}$, and where λ is a constant that can be interpreted as a hazard rate. The interpretation of λdt is the probability of a realignment on the interval dt, conditional on that there has been no realignment up till t.

$$\lambda dt = \frac{f(t)\, dt}{1 - F(t)}. \tag{13}$$

Defining $1 - F(t) = G(t)$, gives

$$\lambda dt = \frac{-G'(t)\, dt}{G(t)}, \tag{14}$$

and integrating (14), yields

$$-\lambda t + C = \ln G(t), \tag{15}$$

where C is a constant. Taking the antilog, gives

$$G(t) = Ce^{-\lambda t}. \tag{16}$$

Thus,

[10] The Hazard function used to model realignment probability in this paper is similar to that of Tristani [25] and Werner [26].

$$1 - F(t) = e^{-\lambda t} = e^{-\frac{\rho(1-\omega(t))}{\theta}t}, \tag{17}$$

which means that realignment expectations in this model can be interpreted as a realignment intensity.[11] The magnitude of the realignment, when it occurs, is

$$\begin{cases} \beta, & x(t) > c(t) \\ -\beta, & x(t) < c(t). \end{cases} \tag{18}$$

Thus, the expected realignment is

$$E(c(t)) = \frac{\beta\rho(1-\omega(\tau))}{\theta}. \tag{19}$$

Hence, when the exchange rate is in the upper (lower) half of the target zone, there is a positive (negative) expected realignment (i.e., an expected devaluation (revaluation)). The probability of a realignment increases the greater the weight of chartists in the market and, when a realignment takes place, both the central parity and fundamentals undergo a discrete change and the upper and lower bounds for the exchange rate are redefined.

The measure of the expected rate of depreciation of the exchange rate, $E(t)$, will, thus, depend both on the expected depreciation of the fundamentals inside the target zone, and the expected depreciation of the central parity (i.e., the realignment intensity), see equation (21) below. Note that the realignment intensity is a constant that will push the exchange rate closer to the bounds of the zone. Accordingly, expectations of a realignment are higher the closer the exchange rate is to the bounds of the target zone. This is the case even though the realignment expectations are not a function of the level of fundamentals within the band.

3 Formal analysis of the model

In the formal analysis, both the exchange rate and the term structure of the interest rate differentials within the target zone will be analyzed with respect to the implications of chartist trading in the Krugman model [11].

[11] Note that $F(t)$ follows the exponential distribution, where $E(t) = \frac{1}{\lambda}$ and $V(t) = \frac{1}{\lambda^2}$.

3.1 The exchange rate within the target zone

In the formal analysis, it is more convenient to work with the deviations of the exchange rate and the fundamentals from central parity, according to the following

$$\begin{cases} s(t) = \widetilde{s}(t) + c(t) \\ x(t) = \widetilde{x}(t) + c(t), \end{cases} \tag{20}$$

i.e., $\widetilde{s}(t)$ and $\widetilde{x}(t)$ are the deviations of the spot exchange rate and the monetary fundamentals, respectively, from central parity. In the formal analysis, it is also convenient to work with the positive part of the zone. However, as the target zone is symmetric, the qualitative results of the formal analysis hold with opposite sign for the negative part of the zone.

By assuming (20) and that the exchange rate's position within the target zone is unchanged after a realignment, the measure of the expected instantaneous depreciation of the exchange rate is a composite term of both the expected depreciation of fundamentals inside the target zone, and the expected depreciation of central parity. Hence,

$$E(t) = \widetilde{E}(t) + E(c(t)), \tag{21}$$

substituting the weight function (4) into (21), we have

$$E(t) = \widetilde{E}(t) + \frac{\beta\rho}{\theta}e^{-\tau}. \tag{22}$$

where $\frac{\beta p}{\theta}e^{-\tau}$ is the expected depreciation of the central parity (i.e., the realignment intensity), and $\widetilde{E}(t)$ is the measure of the expected instantaneous depreciation of the exchange rate inside the target zone.

Then, by combining (1, 4, 20 and 22), the exchange rate within the zone, $\widetilde{s}(t)$, is

$$\widetilde{s}(t) = \widetilde{x}(t) + \frac{\alpha\beta\rho}{\theta}e^{-\tau} + \alpha\widetilde{E}(t). \tag{23}$$

This expression for the exchange rate within the band is similar to that of Krugman [11] except for the term $\frac{\alpha\beta p}{\theta}e^{-\tau}$, which is the realignment intensity that depends on the weight of chartists in the market. Also note that the term

$\widetilde{x}(t)$ includes the impact of both chartist and fundamentalist trading. Applying Ito's lemma on (23) and solving the resulting differential equation gives equation (24) in Proposition 1.

Proposition 1 *The solution to (23) is a function of the composite fundamental*

$$\widetilde{s}(t) = \widetilde{x}(t) + \frac{\alpha\beta\rho}{\theta}e^{-\tau} + A\left(e^{r\widetilde{x}(t)} - e^{-r\widetilde{x}(t)}\right), \tag{24}$$

where

$$A = -\frac{1}{r\left(e^{r\widetilde{x}^*(t)} + e^{-r\widetilde{x}^*(t)}\right)}, \tag{25}$$

and

$$r = \sqrt{\frac{2}{\alpha}\left(\left(1 - e^{-\tau}\right)^2 \sigma_f^2 + \left(e^{-\tau}\right)^2 \sigma_c^2\right)}. \tag{26}$$

Proof. *See the Appendix for a proof.*[12]

The term, $(\alpha\beta\rho/\theta)e^{-\tau}$, is the realignment intensity, that decreases with the width of the zone, θ, and increases with, $\alpha\beta\rho e^{-\tau}$, where α is the semi-elasticity of money demand with respect to the interest rate, β is the size of the realignment when it occurs, and ρ is a constant that amplifies the size of the realignment, and finally, where, $e^{-\tau}$, is the weight of chartists in the market. The constant A depends on the root, r, of the differential equation (24), and on the bounds of the zone $\widetilde{x}^*(t)$, where, $\widetilde{x}(t)^{lower} = -\widetilde{x}(t)^{upper} = \widetilde{x}(t)^*$. The root, r, depends on α, the weight of fundamentalists and chartists in the market (i.e., of $1 - e^{-\tau}$ and $e^{-\tau}$), and their respective trading strategies (i.e., of σ_f^2 and σ_c^2).

The solution of (23) nests that of a fully credible target zone. This is because when there are no chartists in the market the second term $(\alpha\beta\rho/\theta)e^{-\tau} = 0$, and the expression for the root reduces to $r = \sqrt{2/\alpha\sigma_f^2}$. In this case, the equations in Proposition 1 reduce to those of a perfectly credible target zone, and become equal to those of the Krugman model [11]. In this case, the exchange

[12] The model has also been derived with a drift term included in the Wiener process for fundamentals to represent a positive monetary growth rate. This augmented version of the model makes it a bit more complicated since it becomes unsymmetrical, but it does not qualitatively change the results of the model to any great extent. The augmented version of the model is available on request from the author.

rate function forms the familiar S-curve, as can be seen in Figure 1, and the distribution of the exchange rate, as in the Krugman model [11], becomes U-shaped (see Figure 2).[13]

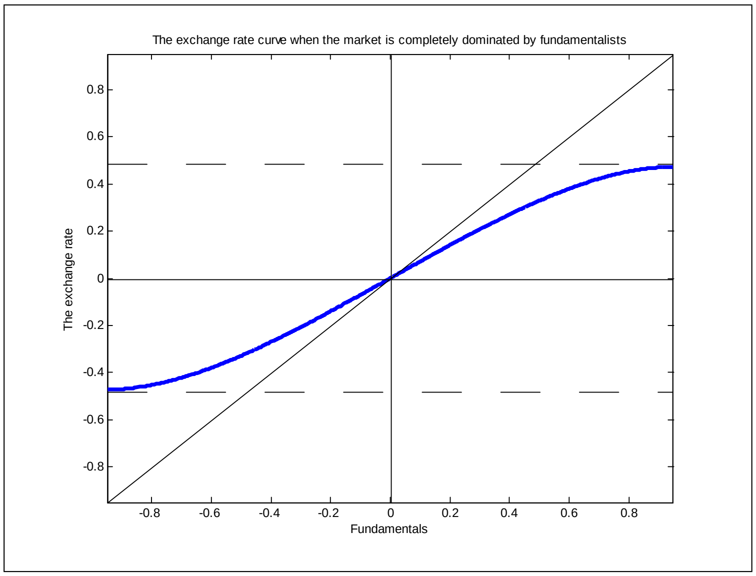

Figure 1: The exchange rate plotted against fundamentals, when there are only fundamentalists in the market (i.e., for $\tau = 1000$).

However, when there are chartists in the market, i.e., when the realignment intensity, $(\alpha\beta\rho/\theta)e^{-\tau} \neq 0$, and the root expression includes two volatility coefficients that are weighted with the time horizon in trade, τ, the exchange rate function has a double kink when fundamentals are equal to central parity (see Figure 3). In this case, the exchange rate function is always closer to the bounds of the target zone, and the root, as well as the realignment intensity, now makes

[13] The parameter values used in Figures 1-6 are chosen solely for expositional purposes and the model is simulated 2000 times for the following parameter values: $\alpha = 0.5$, $\theta = 1.9$, $\rho = 1$, $\tau = 0, 1000$, $\sigma_f = \sigma_c = 1$, $\beta = 0.95$ and $\widetilde{x}^*(t) = 0.95$. The model is sensitive to the choise of α and ρ with regard to the size of the realignment intensity, the smaller α and ρ are the lesser the impact of the realignment intensity. The parameter α also effects the slope and the linearity of the exchange rate curve. The smaller α is the more linear and the steeper the exchange rate curve becomes. Matlab routines are available on request from the author.

the model different from the Krugman model [11].

The U-shaped distribution is due to the infinitesimal interventions used at the edges of the zone. These cause the derivative of the exchange rate to vanish at the edges of the zone, i.e., they flatten the exchange rate curve. Because the curve is flattened at the edges, a large number of outcomes for the fundamentals become concentrated to a small number of outcomes for the exchange rate. In other words, the long-run density of fundamentals is positive at the edge of the zone and therefore as the derivative of the exchange rate vanishes, the distribution tends therefore to spike at these points. Accordingly, the U-shape will be evident in the model when chartists also dominate the market. The inclusion of chartists in the model actually makes the exchange rate distribution more U-shaped (see Figure 4). This is because chartists push the exchange rate function closer to the bounds of the target zone, via the increase in the realignment intensity term. The spikes of the exchange rate density at the edges of the zone increase and thus make the distribution become more strongly U-shaped.

When there are chartists in the market, i.e., when τ is finite, the term, $(\alpha\beta\rho/\theta)e^{-\tau}$, in (24) makes the exchange rate function lie partly above (below) the 45-degree line that represents the free-float (see Figure 3). This means that the "honeymoon effect", that Krugman [11] found, diminishes when the weight of chartists increases in the market. Proposition 2 presents the condition for this to be true. When the fundamentals equal central parity, chartists actually reverse the "honeymoon effect" and, for small deviations of the fundamentals from central parity, the "honeymoon effect" is still present up to a limit deviation lower than the bound for the fundamentals. In the case of deviations of the fundamentals greater than this limit, the exchange rate deviates outside of the zone.

The "honeymoon effect" diminishes in the sense that the discrepancy between the exchange rate and the fundamentals increases and the exchange rate falls closer to the bounds of the target zone. It also diminishes in the sense that it is only present for a short range of values for the fundamentals. However, the slope of the exchange rate function in this model, as in the Krugman model [11], is flatter than the 45-degree line (except when the fundamentals are equal

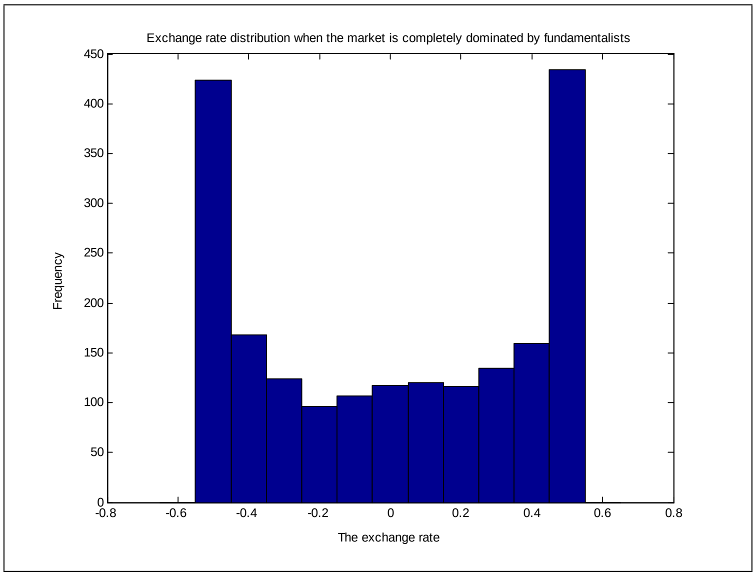

Figure 2: Exchange rate deviations from central parity when there are only fundamentalists in the market, derived from the same simulation as for Figure 1.

to central parity), and in that sense the "honeymoon effect" is still present. Hence, the "honeymoon effect" has not completely vanished in this model, and the target zone still has a stabilizing effect on the exchange rate. However, chartists cause the exchange rate to be closer to the bounds of the target zone and, thereby, diminish the "honeymoon effect".

In Proposition 2, the target zone exchange rate is compared to the free-float exchange rate (the 45-degree line) to show the impact of the model on the "honeymoon effect". Proposition 2 derives the length of the time horizon that causes the exchange rate function to lie above the 45-degree line, representing the free-float (below the 45-degree line in the negative part of the model).

Proposition 2 *The exchange rate function is above the 45-degree line, within the zone, if and only if*

$$\tau < \ln\left(\frac{\alpha\beta\rho}{\theta}\right) - \ln\left(\frac{e^{r\widetilde{x}^*(t)} - e^{-r\widetilde{x}^*(t)}}{r\left(e^{r\widetilde{x}^*(t)} + e^{-r\widetilde{x}^*(t)}\right)}\right) \tag{27}$$

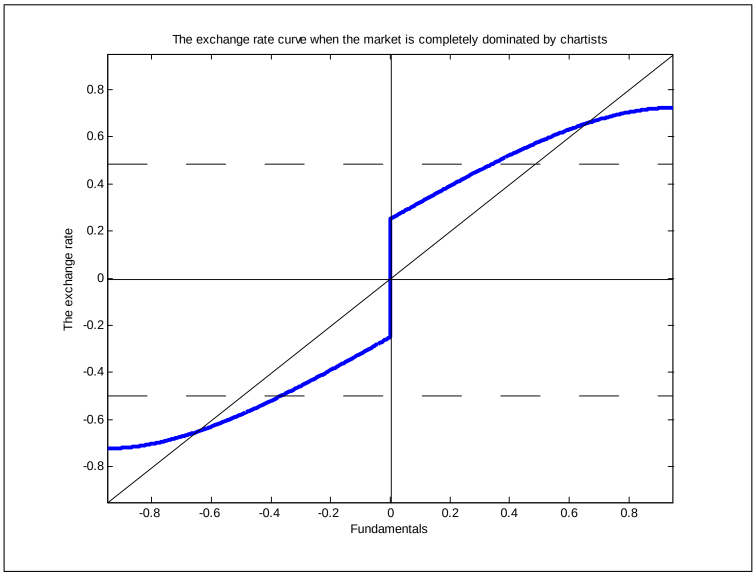

Figure 3: The exchange rate plotted against fundamentals, when there are only chartists in the market (i.e., for $\tau = 0$).

Proof. *See the Appendix for a proof.*

Hence, for the exchange rate function to be above the 45-degree line and diminish the "honeymoon effect", depends entirely on the size of the realignment intensity, which in turn, depends on the time horizon in currency trade τ and, thus, the weight of chartists. The smaller τ is, the larger is the weight of chartists, and thus, the more likely it is that the exchange rate function is above the 45-degree line (see Figures 1 and 2). Proposition 2 is derived through the comparison of $\widetilde{s}(t)$ and $\widetilde{x}(t)$ evaluated at $\widetilde{x}(t) = \widetilde{x}^*(t)$. Thus, if $\widetilde{s}(t) > \widetilde{x}^*(t)$ the exchange rate will, for all $\widetilde{x}(t)$ inside the band, be above (below) the 45-degree line. Furthermore, its maximum deviation will be greater than the width of the band.

The result of Proposition 2 is not, however, as strong as it might seem as the second term on the right hand side of (27) also contains the time horizon, τ. It is, however, necessary that τ, is small for the exchange rate function to be above

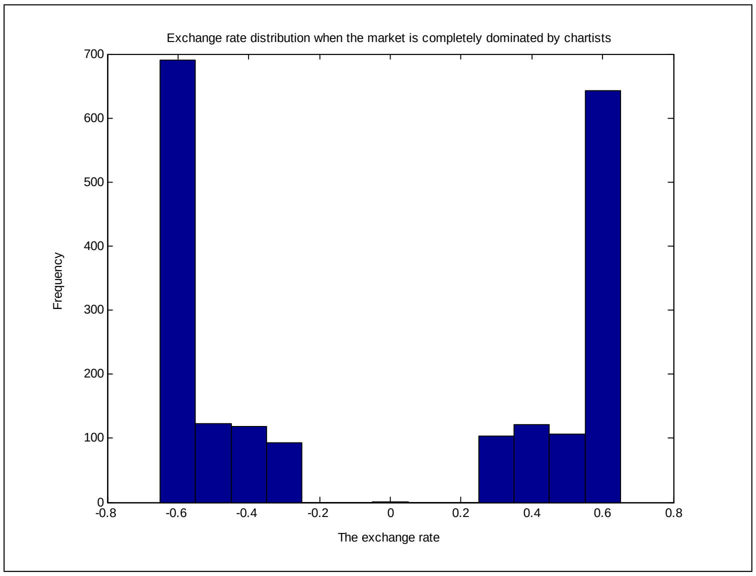

Figure 4: Exchange rate deviations from central parity when there are only chartists in the market, derived from the same simulation as for Figure 3.

(below) the 45-degree line. To see this it may be easier to look at the following inequality: $\frac{\alpha\beta\rho}{\theta}e^{-\tau} > \frac{e^{r\widetilde{x}(t)}-e^{-r\widetilde{x}(t)}}{r\left(e^{r\widetilde{x}^*(t)}+e^{-r\widetilde{x}^*(t)}\right)}$. From this inequality it is evident that the smaller τ is, the larger is the realignment intensity, and the more probable it is that the exchange rate curve is above the 45-degree line.

3.2 The interest rate differential within the target zone

This section will analyze the implications of the use of chartism in currency trade for the behavior of the interest rate differentials, in other words, it will consider the difference between the domestic and foreign interest rates for bonds of equal maturities, with regard to the level of the exchange rate inside the band. The S-shaped exchange rate curve of the Krugman model [11] implies a U-shaped distribution of the exchange rate within the band, which in turn, implies a hump-shaped distribution of the interest rate differential. Assuming that UIP holds inside the zone, implies a negative relationship between the interest rate differential and the fundamentals. This is due to the negative relationship between the expected exchange rate and the exchange rate within the zone. Hence, under the assumption of UIP in the Krugman model [11], there is a negative correlation between the interest rate differential and the exchange rate within the band. Thus, if the currency is weak, the interest rate differential is negative, (i.e., if the foreign interest rate is higher than the domestic interest rate), and the exchange rate is expected to fall, i.e., the domestic currency is expected to appreciate. Interpreted another way, when the exchange rate approaches the upper (lower) bound of the zone, expectations of interventions will predict a fall (rise) in the exchange rate.

3.2.1 The instantaneous interest rate differential

Assuming UIP in the model, the instantaneous interest rate differential is equal to the expected depreciation of the exchange rate, and it is negatively correlated with the exchange rate within the band (which is the same result as in the Krugman model [11]), if there are no chartists in the market, i.e., if the realignment intensity, $\frac{\beta\rho}{\theta}e^{-\tau}$, in (28) is zero. This is shown in Proposition 3 below.

Proposition 3 *Assume that $i(t)$ and $i^*(t)$ are the domestic and foreign instantaneous interest rates, respectively. The instantaneous interest rate differential is, then, given by*

$$i(t) - i^*(t) = \frac{\beta\rho}{\theta}e^{-\tau} - \frac{e^{r\widetilde{x}(t)} - e^{-r\widetilde{x}(t)}}{r\alpha\left(e^{r\widetilde{x}^*(t)} + e^{-r\widetilde{x}^*(t)}\right)} \tag{28}$$

Proof. *See the Appendix for a proof.*

The interest rate differential has been found empirically to have both a positive and negative correlation with the exchange rate within the band. The empirical correlation has varied with countries and periods analyzed (Bertola and Caballero [2], Flood et al. [7], and Svensson [22]). This ambiguity may possibly be explained by the type of traders dominating the market. When the time horizon in currency trade is short, i.e., when chartists dominate the market, the correlation between the interest rate differential and the exchange rate inside the band may become positive. In this case, expectations of a realignment dominate over expectations of monetary interventions in order to defend the zone, indicating market expectations of a further fall (rise) in the exchange rate (i.e., a further appreciation (depreciation) of the domestic currency).

Proposition 4 shows a bound on the time horizon for which the correlation between the instantaneous interest rate differential and the exchange rate becomes positive.

Proposition 4 *The correlation between the instantaneous interest rate differential and the exchange rate is positive, if and only if*

$$\tau < \ln\left(\frac{\beta\rho}{\theta}\right) - \ln\left(\frac{e^{r\widetilde{x}(t)} - e^{-r\widetilde{x}(t)}}{r\alpha\left(e^{r\widetilde{x}^*(t)} + e^{-r\widetilde{x}^*(t)}\right)}\right). \tag{29}$$

Proof. *See the Appendix for a proof.*

Again, this result is not as strong as it might at first seem, since the second term on the right hand side of (29) also contains the time horizon, τ, via the root, r. However, it does show the need for τ to be small for the correlation between the instantaneous interest rate differential and the exchange rate to become positive. To see this, it may be easier to look at the following inequality $\frac{\beta\rho}{\theta}e^{-\tau} > \frac{e^{r\widetilde{x}(t)} - e^{-r\widetilde{x}(t)}}{r\alpha\left(e^{r\widetilde{x}^*(t)} + e^{-r\widetilde{x}^*(t)}\right)}$, which states the necessary condition for the interest rate differential to become positive.[14]

To see how the instantaneous interest rate differential changes when the time horizon, τ, increases, it is necessary to calculate the partial derivative of the interest rate differential with regard to τ. Since τ appears not only in the expression for the realignment intensity but also in the expression for the root, the the derivative is very difficult to interpret. However, via visual inspection of Figures 5 and 6, it is evident that the instantaneous interest differential is decreasing with the time horizon in the following sense; when the time horizon increases, the correlation between the interest rate differential and the exchange rate within the band decreases and eventually becomes negative. Hence, the more fundamentalists there are in the market, the more likely it is that the correlation between the interest rate differential and the exchange rate is negative.

[14] See the Appendix for more details.

In other words, the more likely it is that the market expects the exchange rate to fall (i.e., the domestic currency to appreciate) when the interest rate differential is negative (i.e., when the foreign interest rate is greater than the domestic interest rate) and the exchange rate is near the upper bound of the zone.

Figures 5 and 6 show the instantaneous interest rate differential plotted against the fundamentals (i.e., the measure of the expected exchange rate depreciation) according to the relationship shown in (28). In Figure 5, there are only fundamentalists in the market and in Figure 6, there are only chartists in the market.

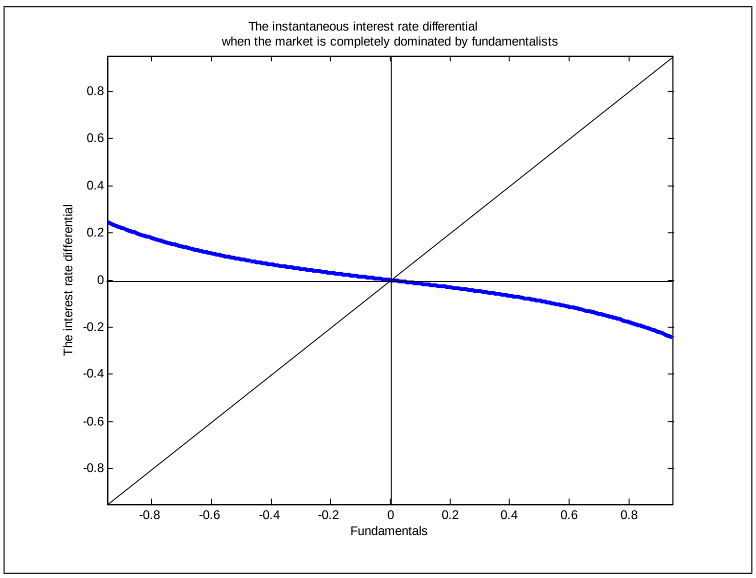

Figure 5: The interest rate differential plotted against fundamentals, when there are only fundamentalists on the market (i.e., for $\tau = 1000$).

To see how the instantaneous interest rate differential alters with changes in fundamentals, $\widetilde{x}(t)$, the partial derivative of the interest rate differential with respect to fundamentals is calculated. The instantaneous interest rate differential is decreasing in $\widetilde{x}(t)$ when $\widetilde{x}(t) < \widetilde{x}^*(t)$ according to

$$\frac{\partial (i - i^*)}{\partial \widetilde{x}(t)} = -\frac{e^{r\widetilde{x}(t)} + e^{-r\widetilde{x}(t)}}{\alpha \left(e^{r\widetilde{x}^*(t)} + e^{-r\widetilde{x}^*(t)}\right)}. \tag{30}$$

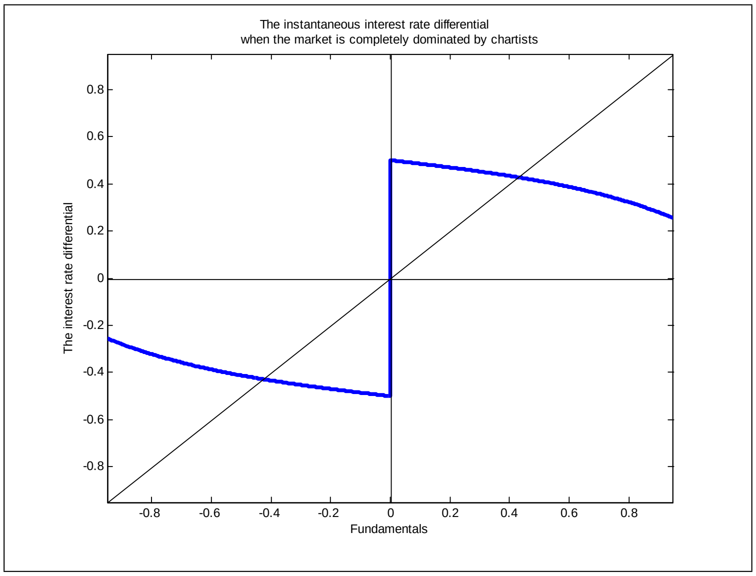

Figure 6: The intereste rate differential plotted against fundamentals, when there are only chartists on the market. (i.e., for $\tau = 0$).

When $\widetilde{x}\,(t) = \widetilde{x}^*\,(t)$ (i.e., when fundamentals are equal to its bounds), the instantaneous interest rate differential is constant and equal to $-1/\alpha$. This result corresponds to the result in Krugman [11] and is unaffected by the weight of chartists in the market. The fact that this result also holds in the model with positive realignment expectations is because the realignment intensity does not depend on fundamentals, and will, therefore, not influence the interest rate differential when fundamentals change. Thus, the instantaneous interest rate differential is decreasing in $\widetilde{x}\,(t)$ when $\widetilde{x}\,(t) < \widetilde{x}^*\,(t)$ because, when fundamentals increase, the expected rate of depreciation of the exchange rate decreases regardless of which type of traders dominate the market. The reason is that chartists have second order importance with regard to the exchange rate and only affect the realignment intensity, which is independent of the position of the exchange rate within the band. Accordingly, chartists are not able to reverse expectations of a depreciation when fundamentals increase.

3.2.2 The long-term interest rate differential

Assuming UIP in the model, the long-term interest rate differential is equal to the measure of the expected depreciation of the exchange rate divided by the time to maturity. The Krugman model [11] implies that the long term-interest rate differential equals zero within the band. However, realignment expectations that are independent of the level of fundamentals inside the band, i.e., realignment expectations reoccurring at a constant probability throughout the band, change this result. The interest rate differential for long-term maturities is derived in the Appendix[15] and presented in Proposition 5.

Proposition 5 *The interest rate differential for long maturities is a constant equal to the expected realignment caused by chartists, i.e., the realignment intensity*

$$\delta\left(\widetilde{x}, c, \infty\right) = \lim_{T \to \infty} \delta\left(\widetilde{x}, c, T\right) = \frac{\beta\rho}{\theta} e^{-\tau} \tag{31}$$

Proof*. See the Appendix for a proof.*

In a target zone model with a realignment intensity independent of the level of fundamentals or the level of the exchange rate, the interest rate for long maturities does not approach zero. Instead, it approaches the rate of expected realignment, which can either be positive, if a revaluation is expected, or negative, if a devaluation is expected. This result is in line with Svensson [22], where the realignment probability is also independent of the fundamentals but reoccurs at constant probability throughout the band. Proposition 5 also implies that if there are no chartists in the market, the interest rate differential would approach zero for long maturities and this result is in line with that of Krugman [11].

4 Concluding remarks

To summarize, the model presented in this paper implements some of the stylized empirical findings, mentioned in Section 1, that are lacking in Krugman's model

[15] The derivations of the long-term interest rate differential follow those in Svensson [22].

[11]. The empirical finding, ascertained by means of questionnaire surveys of the foreign exchange market, that the use of fundamentalism and chartism depend on the time horizon in currency trade, has been included in the model. To our knowledge, it is the first time heterogenous traders are introduced in the form of a chartist-fundamentalist setup, in an exchange rate target zone framework. The present paper also implements realignment expectations and relaxes the assumption of a fully credible target zone, another empirically realistic extension of the Krugman model [11].

Implementing chartist and fundamentalist trading in the model does not change all of the general results of Krugman's target zone model [11]. The exchange rate curve keeps its S-shape inside the band, and the distribution of the exchange rate is still U-shaped. However, when the market is completely dominated by chartists, the "honeymoon effect" highlighted by Krugman [11] diminishes, and the exchange rate stays much closer to the bounds of the target zone. The slope of the exchange rate function remains flatter than that of the 45-degree line, which still leaves some domestic monetary policy freedom. Thus, although the model includes chartist traders the "honeymoon effect" does not completely vanish.

By making these extensions to Krugman's model [11], i.e., including chartist trading and a realignment intensity depending on the weight of chartists, the model in the present paper, allows for the possibility of either a negative or a positive correlation between the instantaneous interest rate differential and the exchange rate within the band, another stylized empirical finding. This ambiguity can, in this model, be explained by the type of traders dominating the market, i.e., the length of the time horizon in currency trade. If chartists completely dominate the market, the realignment intensity may cause the correlation between the instantaneous interest rate differential and the exchange rate inside the band to become positive. The interest rate differentials for long maturities are also affected by chartists via the realignment intensity, which makes the long maturity interest rate differential different from zero.

Moreover, this way of modeling heterogeneity still allows for an explicit solution of the model and results in, from a number of respects, an empirically more realistic model compared to Krugman [11]. The Appendix is an attempt to produce a more explicit and stringent mathematical derivation of the solution of the exchange rate path within a target zone framework. It shows the formal definition of the measure of the expected exchange rate depreciation, as it must be, for a proper mathematical derivation of the solution for the exchange rate path.

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Appendix

Proof of Proposition 1. Assume that the general solution for the differential equation for the exchange rate is given by

$$\widetilde{s}\,(t) = h(\widetilde{x}\,(t)), \tag{A 1}$$

i.e., a function of fundamentals. Assuming that $h\,(\widetilde{x}\,(t))$ is well-behaved and twice differentiable, a Taylor expansion of $h(\widetilde{x}\,(t))$ including second order terms, gives

$$dh(\widetilde{x}\,(t)) = h_{\widetilde{x}} d\widetilde{x}\,(t) + \frac{1}{2} h_{\widetilde{x}\widetilde{x}} d(\widetilde{x}\,(t))^2, \tag{A 2}$$

Using Ito's formula, gives

$$\begin{aligned} d\widetilde{s}\,(t) &= h_{\widetilde{x}}\,(\widetilde{x}\,(t))\,[\omega\,(\tau)\,dy_f\,(t) + (1-\omega\,(\tau))\,dy_c\,(t)] \\ &\quad + \frac{1}{2} h_{\widetilde{x}\widetilde{x}}\,(\widetilde{x}\,(t)) \left[\begin{array}{c} \omega\,(\tau)^2\,(dy_f\,(t))^2 + 2\omega\,(\tau)\,(1-\omega\,(\tau))\,dy_f\,(t)\,dy_c\,(t) \\ + (1-\omega\,(\tau))^2\,(dy_c\,(t))^2 \end{array} \right] \end{aligned} \tag{A 3}$$

where

$$dy_f\,(t) = \sigma_f\,(t)\,dW_f\,(t)\,, \tag{A 4}$$

and

$$dy_c\,(t) = (\gamma\,(y_f\,(t) - y_f\,(t-\tau))\,dt + \sigma_c\,(t)\,dW_c\,(t))\,, \tag{A 5}$$

where

$$(dy_f\,(t))^2 = (\sigma_f\,(t))^2\,dt, \tag{A 6}$$

and

$$(dy_c\,(t))^2 = (\sigma_c\,(t))^2\,dt, \tag{A 7}$$

Finally, assuming that $W_f\,(t)$ and $W_c\,(t)$ are independent, we have that

$$dy_f\,(t)\,dy_c\,(t) = \sigma_f\,(t)\,dW_f\,(t)\,[(\gamma\,(y_f\,(t) - y_f\,(t-\tau))\,dt + \sigma_c\,(t)\,dW_c\,(t))] = 0. \tag{A 8}$$

Hence,

$$\begin{aligned} d\widetilde{s}\left(t\right) &= h_{\widetilde{x}}\left(\widetilde{x}\left(t\right)\right)\left[\omega\left(\tau\right)dy_f\left(t\right)+\left(1-\omega\left(\tau\right)\right)dy_c\left(t\right)\right] \\ &\quad +\frac{1}{2}h_{\widetilde{x}\widetilde{x}}\left(\widetilde{x}\left(t\right)\right)\left[\omega\left(\tau\right)^2\sigma_f^2\left(t\right)+\left(1-\omega\left(\tau\right)\right)^2\sigma_c^2\left(t\right)\right]dt. \end{aligned} \quad \text{(A 9)}$$

As $\widetilde{s}\left(t\right)=h(\widetilde{x}\left(t\right))$ we have

$$\begin{aligned} h(\widetilde{x}\left(t\right))-h(\widetilde{x}\left(0\right)) &= \int_0^t h_{\widetilde{x}}\left(\widetilde{x}\left(s\right)\right)\left[\omega\left(\tau\right)dy_f\left(s\right)+\left(1-\omega\left(\tau\right)\right)dy_c\left(s\right)\right] \\ &\quad +\int_0^t \frac{1}{2}\left[h_{\widetilde{x}\widetilde{x}}\left(\widetilde{x}\left(s\right)\right)\right]\left[\omega\left(\tau\right)^2\sigma_f^2\left(s\right)+\left(1-\omega\left(\tau\right)\right)^2\sigma_c^2\left(s\right)\right]ds. \end{aligned} \quad \text{(A 10)}$$

Assuming that $\sigma_f^2\left(t\right)$ and $\sigma_c^2\left(t\right)$ are deterministic and taking expectations of (A 10), we get, by construction and the fact that the expectation of an Ito integral is zero, that

$$E\left[h(\widetilde{x}\left(t\right))\right]-E\left[h(\widetilde{x}\left(0\right))\right]=\int_0^t \frac{1}{2}E\left[h_{\widetilde{x}\widetilde{x}}\left(\widetilde{x}\left(s\right)\right)\right]\left[\omega\left(\tau\right)^2\sigma_f^2\left(s\right)+\left(1-\omega\left(\tau\right)\right)^2\sigma_c^2\left(s\right)\right]ds. \quad \text{(A 11)}$$

Differentiating (A 11) with regard to t gives

$$\frac{d}{dt}E\left[h(\widetilde{x}\left(t\right))\right]=\frac{1}{2}E\left[h_{\widetilde{x}\widetilde{x}}\left(\widetilde{x}\left(t\right)\right)\left(\omega\left(\tau\right)^2\sigma_f^2\left(t\right)+\left(1-\omega\left(\tau\right)\right)^2\sigma_c^2\left(t\right)\right)\right]. \quad \text{(A 12)}$$

According to equation (23) in the main text, we have

$$h(\widetilde{x}\left(t\right))=\widetilde{x}\left(t\right)+\frac{\alpha\beta\rho}{\theta}e^{-\tau}+\alpha\widetilde{E}\left(t\right) \quad \text{(A 13)}$$

Now, defining $\widetilde{E}\left(t\right)$ formally as[16]

$$\widetilde{E}\left(t\right)=\frac{d}{dt}E\left[h(\widetilde{x}\left(t\right))\right] \quad \text{(A 14)}$$

we have

$$h(\widetilde{x}\left(t\right))=\widetilde{x}\left(t\right)+\frac{\alpha\beta\rho}{\theta}e^{-\tau}+\alpha\frac{d}{dt}E\left[h(\widetilde{x}\left(t\right))\right]. \quad \text{(A 15)}$$

[16] The proper definition of $\widetilde{E}\left(t\right)$ for this argumentation is $\widetilde{E}\left(t\right)=\frac{d}{dt}E\left[h(\widetilde{x}\left(t\right))\right]$. Hence, $E\left(t\right)=\widetilde{E}\left(t\right)+\frac{\beta\rho}{\theta}e^{-\tau}=\frac{d}{dt}E\left[h(\widetilde{x}\left(t\right))\right]+\frac{\beta\rho}{\theta}e^{-\tau}=\frac{d}{dt}E\left[\left(x\left(t\right)\right)\right]-\frac{d}{dt}c\left(t\right)+\frac{\beta\rho}{\theta}e^{-\tau}$, inside the band.

Combining (A 12) with (A 15) gives

$$\begin{aligned} h(\widetilde{x}(t)) &= \widetilde{x}(t) + \frac{\alpha\beta\rho}{\theta}e^{-\tau} \\ &+\frac{\alpha}{2}E\left[h_{\widetilde{x}\widetilde{x}}(\widetilde{x}(t))\right]\left[\omega(\tau)^2\sigma_f^2(t) + (1-\omega(\tau))^2\sigma_c^2(t)\right]. \end{aligned} \tag{A 16}$$

Taking expectations of (A 16), gives

$$E\left[\begin{array}{c} h(\widetilde{x}(t)) - \widetilde{x}(t) - \frac{\alpha\beta\rho}{\theta}e^{-\tau} \\ -\frac{\alpha}{2}\left[h_{\widetilde{x}\widetilde{x}}(\widetilde{x}(t))\right]\left[\omega(\tau)^2\sigma_f^2(t) + (1-\omega(\tau))^2\sigma_c^2(t)\right] \end{array}\right] = 0 \tag{A 17}$$

Equation A(17) implies a restriction on $h(\widetilde{x}(t))$, and (A 16) holds if the integrand is zero partwise, i.e., if

$$h(\widetilde{x}(t)) = \widetilde{x}(t) + \frac{\alpha\beta\rho}{\theta}e^{-\tau} + \frac{\alpha}{2}E\left[h_{\widetilde{x}\widetilde{x}}(\widetilde{x}(t))\left(\omega(\tau)^2\sigma_f^2(t) + (1-\omega(\tau))^2\sigma_c^2(t)\right)\right] \tag{A 18}$$

is true, for fixed t at every $\widetilde{x} \in \mathbb{R}$. This implies that for fixed t,

$$h(\widetilde{x}(t)) = \widetilde{x}(t) + \frac{\alpha\beta\rho}{\theta}e^{-\tau} + \frac{\alpha}{2}h_{\widetilde{x}\widetilde{x}}(\widetilde{x}(t))\left(\omega(\tau)^2\sigma_f^2(t) + (1-\omega(\tau))^2\sigma_c^2(t)\right). \tag{A 19}$$

Thus, by the assumption of symmetry in the model, i.e., that the bounds of the target zone are of equal length from central parity, i.e., $\widetilde{s}(t)^{\min} = -\widetilde{s}(t)^{\max}$, and $\widetilde{x}(t)^{lower} = -\widetilde{x}(t)^{upper} = \widetilde{x}(t)^{*}$ the general solution to the second order differential equation (A 19) is

$$\widetilde{s}(t) = \widetilde{x}(t) + \frac{\alpha\beta\rho}{\theta}e^{-\tau} + A\left(e^{r\widetilde{x}(t)} - e^{-r\widetilde{x}(t)}\right), \tag{A 20}$$

where r_1 and r_2 are the characteristic roots of the characteristic equation

$$\frac{\alpha}{2}\left(\left(1-e^{-\tau}\right)^2\sigma_f^2 + \left(e^{-\tau}\right)^2\sigma_c^2\right)r^2 - 1 = 0, \tag{A 21}$$

and

$$r_1, r_2 = \pm\sqrt{\frac{2}{\alpha}\left((1-e^{-\tau})^2\sigma_f^2 + (e^{-\tau})^2\sigma_c^2\right)}. \tag{A 22}$$

To solve for the constant A, we use the smooth pasting condition

$$h_{\widetilde{x}}\left(\widetilde{x}^{*}\left(t\right)\right)=0, \tag{A 23}$$

which gives

$$A=-\frac{1}{r\left(e^{r\widetilde{x}^{*}(t)}+e^{-r\widetilde{x}^{*}(t)}\right)}. \tag{A 24}$$

Substituting this result into the general solution (A 18), we have that

$$\widetilde{s}\left(t\right)=\widetilde{x}\left(t\right)+\frac{\alpha\beta\rho}{\theta}e^{-\tau}-\frac{\left(e^{r\widetilde{x}(t)}-e^{-r\widetilde{x}(t)}\right)}{r\left(e^{r\widetilde{x}^{*}(t)}+e^{-r\widetilde{x}^{*}(t)}\right)}, \tag{A 25}$$

where

$$r=\sqrt{\frac{2}{\alpha}\left(\left(1-e^{-\tau}\right)^{2}\sigma_{f}^{2}+\left(e^{-\tau}\right)^{2}\sigma_{c}^{2}\right)}, \tag{A 26}$$

and the proof is completed. ■

Proof of Proposition 2. To establish whether or not the S-curve of the exchange rate is above (below) the 45-degree line, i.e., to see whether $\widetilde{s}\left(t\right)>\widetilde{x}\left(t\right)$, we need to evaluate the exchange rate function using this inequality. However, since $\widetilde{s}\left(t\right)$ is increasing $\widetilde{x}\left(t\right)$ we can evaluate the exchange rate function at $\widetilde{x}\left(t\right)=\widetilde{x}^{*}\left(t\right)$, and thus if $\widetilde{s}\left(t\right)>\widetilde{x}^{*}\left(t\right)$ the exchange rate will, for all $\widetilde{x}\left(t\right)$ inside the band, be above (below) the 45-degree line. Furthermore, its maximum deviation will be greater than the width of the band. Thus,

$$\widetilde{s}\left(t\right)=\widetilde{x}^{*}\left(t\right)+\frac{\alpha\beta\rho}{\theta}e^{-\tau}-\frac{e^{r\widetilde{x}^{*}(t)}-e^{-r\widetilde{x}^{*}(t)}}{r\left(e^{r\widetilde{x}^{*}(t)}+e^{-r\widetilde{x}^{*}(t)}\right)}>\widetilde{x}^{*}\left(t\right), \tag{A 26}$$

which implies

$$\frac{\alpha\beta\rho}{\theta}e^{-\tau}>\frac{\left(e^{r\widetilde{x}^{*}(t)}-e^{-r\widetilde{x}^{*}(t)}\right)}{r\left(e^{r\widetilde{x}^{*}(t)}+e^{-r\widetilde{x}^{*}(t)}\right)}, \tag{A 27}$$

and

$$e^{-\tau}>\frac{\frac{\left(e^{r\widetilde{x}^{*}(t)}-e^{-r\widetilde{x}^{*}(t)}\right)}{r\left(e^{r\widetilde{x}^{*}(t)}+e^{-r\widetilde{x}^{*}(t)}\right)}}{\frac{\alpha\beta p}{\theta}}. \tag{A 28}$$

Taking logarithms of (A 28), gives

$$\tau<\ln\left(\frac{\alpha\beta\rho}{\theta}\right)-\ln\left(\frac{\left(e^{r\widetilde{x}^{*}(t)}-e^{-r\widetilde{x}^{*}(t)}\right)}{r\left(e^{r\widetilde{x}^{*}(t)}+e^{-r\widetilde{x}^{*}(t)}\right)}\right), \tag{A 29}$$

and the proof is completed. Note that $r = \sqrt{\frac{2}{\alpha}\left(\left(1-e^{-\tau}\right)^2\sigma_f^2 + \left(e^{-\tau}\right)^2\sigma_c^2\right)}$ and thus the inequality (A 29) is not as strong as it may seem. However, trying to solve completely for τ is not fruitful since the resulting expression is too complex to interpret. ■

Proof of Proposition 3. Assuming UIP in the model, i.e., that the instantaneous interest rate differential is equal to the expected depreciation of the exchange rate, then from equation (1) in the main text, where $E(t)$ is the measure of the instantaneous expected depreciation of the exchange rate, we have that

$$E(t) = \frac{s(t) - x(t)}{\alpha}, \tag{A 30}$$

and thus, inside the band

$$\begin{aligned} i(t) - i^*(t) &= \frac{\widetilde{s}(t) - \widetilde{x}(t)}{\alpha} \qquad \text{(A 31)} \\ &= \frac{\widetilde{x}(t)}{\alpha} + \frac{\alpha\beta\rho}{\alpha\theta}e^{-\tau} - \frac{\left(e^{r\widetilde{x}(t)} - e^{-r\widetilde{x}(t)}\right)}{\alpha r\left(e^{r\widetilde{x}^*(t)} + e^{-r\widetilde{x}^*(t)}\right)} - \frac{\widetilde{x}(t)}{\alpha} \end{aligned}$$

Assuming (20), and substituting the solution for the exchange rate (24) into (A 31), we get

$$i(t) - i^*(t) = \frac{\beta\rho}{\theta}e^{-\tau} - \frac{e^{r\widetilde{x}(t)} - e^{-r\widetilde{x}(t)}}{r\alpha\left(e^{r\widetilde{x}^*(t)} + e^{-r\widetilde{x}^*(t)}\right)} \lesseqgtr 0, \tag{A 32}$$

which completes the proof. ■

Proof of Proposition 4. From the instantaneous interest differential derived in the proof of Proposition 3, it is easy to see that for the interest rate differential to have a positive correlation with the expected exchange rate depreciation, the first term on the right hand side of (A 32) must be greater than the second term on the right hand side of (A 32)

$$\frac{\beta\rho}{\theta}e^{-\tau} > \frac{\left(e^{r\widetilde{x}(t)} - e^{-r\widetilde{x}(t)}\right)}{r\alpha\left(e^{r\widetilde{x}^*(t)} + e^{-r\widetilde{x}^*(t)}\right)}, \tag{A 33}$$

which gives

$$e^{-\tau} > \frac{\frac{\left(e^{r\widetilde{x}(t)} - e^{-r\widetilde{x}(t)}\right)}{r\alpha\left(e^{r\widetilde{x}^*(t)} + e^{-r\widetilde{x}^*(t)}\right)}}{\frac{\beta\rho}{\theta}}. \tag{A 34}$$

Taking logarithms of (A 34), gives

$$\tau < \ln\left(\frac{\beta\rho}{\theta}\right) - \ln\left(\frac{\left(e^{r\widetilde{x}(t)} - e^{-r\widetilde{x}(t)}\right)}{r\alpha\left(e^{r\widetilde{x}^*(t)} + e^{-r\widetilde{x}^*(t)}\right)}\right), \tag{A 35}$$

and this completes the proof. Note that $r = \sqrt{\frac{2}{\alpha}\left(\left(1 - e^{-\tau}\right)^2 \sigma_f^2 + \left(e^{-\tau}\right)^2 \sigma_c^2\right)}$ and thus the inequality (A 35) is not as strong as it may seem. However, trying to solve completely for τ is not fruitful since the resulting expression is too complex to interpret. ■

Proof of Proposition 5. Assume uncovered interest parity and let $i^*(T, t)$ be the foreign nominal interest rate of a pure discount currency bond, purchased at time t for term-T. That is, the bond is maturing at time $t + T$ with $T > 0$. Let i be the domestic nominal interest rate when fundamentals are equal to x. Then, the term-T interest rate differential, denoted $\delta(x, c, T)$, given the current level of fundamentals and expected realignment during term-T, is given by

$$i(x, t, T) - i^*(t, T) = \delta(x, c, T) = \frac{E_t[E(t+T) \mid x(t) = x] - E(t)}{T}, \tag{A 36}$$

i.e., the measure of the expected depreciation of the exchange rate divided by the time to maturity.

Since the exchange rate and the fundamentals are Markov processes, the interest rate differential will only depend on the level of the fundamentals at the time of purchase, $x(t)$, and the term-T. Hence, the interest rate differential will not depend on the time of the purchase of the bond, i.e., of t. This implies that we can set the purchase time equal to zero and the exchange rate as a function of x and T only. Now, the measure of the expected exchange rate is, in this model, a sum of two components; the expected depreciation of the exchange rate within the band and the expected realignment of the band, i.e., the realignment intensity, which implies that (A 36) can be rewritten as

$$\delta\left(x,c,T\right)=\frac{E_0\left[\widetilde{E}\left(T\right)\mid\widetilde{x}\left(0\right)=\widetilde{x},c\left(0\right)=c\right]-\widetilde{E}\left(0\right)}{T}+\frac{E_0\left[c\left(T\right)\mid c\left(0\right)=c\right]}{T}. \tag{A 37}$$

The second term in (A 37) is the expected change in central parity. Since the parameters β, θ and ρ are assumed to be constant, and unchanged by realignment, and since $e^{-\tau}$ is determined exogenously, the second term in (A 37) is simply

$$\frac{E_0\left[c\left(T\right)\mid c\left(0\right)=c\right]}{T}=\frac{\beta\rho}{\theta}e^{-\tau}. \tag{A 38}$$

Let $f\left(\widetilde{x},T\right)$ be the expected change of the exchange rate inside the band, (i.e., $f\left(\widetilde{x},T\right)=E_0\left[\widetilde{E}\left(T\right)\mid\widetilde{x}\left(0\right)=\widetilde{x},c\left(0\right)=c\right]-\widetilde{E}\left(0\right)$), (A 37) can be written

$$\delta\left(\widetilde{x},c,T\right)=\frac{f\left(\widetilde{x},T\right)}{T}+\frac{\beta\rho}{\theta}e^{-\tau}, \tag{A 39}$$

since the numerator of the first term in (A 39) is bounded, $\frac{f(\widetilde{x},T)}{T}$ approaches zero for long maturities, i.e., when $T\to\infty$. Thus,

$$\delta\left(\widetilde{x},c,\infty\right)=\lim_{T\to\infty}\delta\left(\widetilde{x},c,T\right)=\frac{\beta\rho}{\theta}e^{-\tau}. \tag{A 40}$$

This completes the proof. ■

Sterilized Intervention - A Chartist Channel?

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Abstract

Using a chartist-fundamentalist set-up, the present paper derives the effects on the current exchange rate of central bank intervention. Fundamentalists have rational expectations and chartists use a technical trading strategy which involves the use of so called support and resistance levels for the exchange rate. Chartists' trading technique results in two different types of expectations for chartists; bandwagon expectations and regressive expectations. These expectations give rise to a variety of effects on the current exchange rate when the central bank intervenes in the foreign exchange market. Chartists may enhance or suppress the effect of intervention depending on their expectations. The results of this paper indicate that a chartist channel exists that central banks may be able to exploit when intervening in the foreign exchange market.

JEL codes: E52; F31; F33.

Keywords: Foreign exchange; Intervention; Technical trading; Chartist channel

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1 Introduction

Background The breakdown of the Bretton Woods system forced central banks to seek new ways of managing the exchange rate since then, central bank foreign exchange intervention has been in focus. In particular, attention has been paid to the much disputed effects on the exchange rate of sterilized intervention. Sterilized intervention is most easily described as central banks buying (selling) domestic currency on the foreign exchange market at the same time as they make an offsetting sell (buy) of domestic currency for domestic bonds on the domestic monetary market. In this way money supply remains unchanged and only the stock of worldwide private holdings of domestic bonds changes. Sterilized intervention is, however, a controversial policy tool. According to traditional monetary models, it will be ineffective because it does not change monetary fundamentals. Never the less, just because sterilized intervention does not affect monetary fundamentals, it does not mean that it is not an appealing tool to use in exchange rate policy.[1] It may, for example, be effective in managing the exchange rate. If this is the case, it may be possible for the central bank to use sterilized intervention to manage the exchange rate without affecting domestic policy objectives. Indeed sterilized interventions are frequently used by a number of central banks in Asia. The Bank of Japan, for example, openly admits to using interventions when conducting exchange rate policy (Dooley et al. [13] and Dooley et al. [14]).[2]

The effectiveness of sterilized intervention is connected to its capacity to influence the level or volatility of the exchange rate. Central banks may for instance, use sterilized intervention to correct a perceived exchange rate misalignment (such as when the exchange rate has deviated from fundamentals or from an explicit or implicit target for the exchange rate), or to decrease the volatility of the exchange rate. When the central bank is attempting to pursue a target rate or correct a misalignment, it may intervene in the foreign exchange market in order to reinforce or reverse an existing trend in the exchange rate. If

[1] Given that the redistribution of wealth, arising from the sterilized intervention, does not affect the demand for goods or the output, so that inflation changes.

[2] See also the official website for the Bank of Japan.

the central bank acts to reinforce an existing trend, it adopts a "leaning with the wind" strategy (i.e., sell (buy) when the exchange rate is depreciating (appreciating)). If, instead, the central bank acts to reverse an existing trend, it adopts a "leaning against the wind" strategy (i.e., to sell (buy) if the currency is appreciating (depreciating). Note that, a "leaning with the wind strategy" increases exchange rate volatility whereas a "leaning against the wind" decreases it. The effectiveness of using a "leaning against the wind" strategy in order to correct a misalignment has, however, been questioned both empirically and theoretically. The main criticism has been on the basis of the smallness of the central banks' trading volumes in comparison with the daily turnover in the foreign exchange market (Humpage [19]).

There are two dominating explanations as to why sterilized intervention may have an effect on the exchange rate level, even though monetary fundamentals remain unchanged. These are firstly, through the signalling channel and secondly, through the portfolio balance channel. A third explanation has also been suggested, a chartist channel. The chartist channel has, however, received much less attention, despite some empirical support for its existence. The empirical evidence supporting the two dominating channels is mixed and the controversies surrounding sterilized intervention have, not yet, been solved satisfactory and much work remains.

The signalling channel The signalling channel was first proposed by Mussa [26] and the hypothesis is fairly straightforward; the central bank conveys, i.e., signals, its intentions about future monetary policy through sterilized foreign exchange interventions. Assuming that expectations about the future spot exchange rate affect the current spot exchange rate and that the market believes in the central bank's intervention signal, a sterilized intervention has an effect on the spot exchange rate because it alters markets expectations about the future spot exchange rate.

The idea of central banks being able to signal future monetary policy through interventions implies that they have private information about future fundamentals, and that interventions are followed by changes in monetary policy. Kaminsky and Lewis [22] empirically examined the presumption that interventions are followed by changes in monetary policy. They found that interventions were informative about future monetary policy, but most often in the opposite direction from the signalling hypothesis (i.e., the selling (buying) of domestic currency was followed by a decrease (increase) in money supply). Humpage [19] tested for the existence of private information within U.S. monetary authorities, via testing the superiority of their forecasting skills. He found no evidence of private information, suggesting that interventions could not be used as a means of signalling new information to the market. Furthermore, central banks are known to often use covert sterilized interventions, which is in strict contrast to the operating of the signalling channel. Dominguez [11] and Hung [20] show that covert sterilized interventions do actually affect both the volatility and the level of the exchange rate, which speaks against the signalling channel.

The portfolio balance channel The portfolio balance channel suggests that, if foreign and domestic bonds are considered imperfect substitutes (Ricardian equivalence does not hold), and portfolio managers are risk averse and choose their optimal portfolio according to a mean-variance behavior, sterilized interventions will affect the exchange rate when portfolio managers readjust their portfolios according to changes in the relative supply of foreign and domestic bonds. The strongest support for the portfolio balance channel was presented by Dominguez and Frankel [12],who found empirical evidence of this hypothesis during the mid 1980's. On the other hand, controlling for the possibility of signalling, Gosh [18] finds no supportive evidence for the portfolio balance channel for a sample covering roughly the same sample period. A major issue regarding the effectiveness of this channel is that it, theoretically, requires the size of interventions to be rather large in order to have an impact on the relative supply and demand for domestic and foreign bonds. This is because daily turnover in trade volume is much larger in comparison with intervention volume.

Technical trading According to questionnaire surveys, technical analysis is widely used among traders in the foreign exchange market when forming expectations about future exchange rates (Cheung and Chinn [9], Lui and Mole [24], Menkhoff [25], Oberlechner [29] and Taylor and Allen [32]). Technical analysis, or chartism, does not rely on any underlying economic or fundamental analysis. Technical analysis involves the use of past exchange rates, traded volumes, and the volatility of past exchange rates. Chartists extrapolate these into the future and use them to try and predict trends and trend reversals within the exchange rates.

The term chartism originally stems from traders' use of different types of time charts when applying technical analysis to form expectations about the future exchange rate. Today, a huge variety of different tools are used within technical analysis including both trend indicators and oscillating indicators based on either the exchange rate itself, its variance, volatility or the traded volume of the currency. All these different tools are used to detect trends and predict trend reversals in the exchange rate.[3]

The chartist channel The chartist channel has received little attention so far in the literature. However, Hung [20] concludes that the presence of non-fundamentalist traders, what she calls noise traders (i.e., chartists), whose behavior is more or less predictable, constitutes a channel through which sterilized intervention can be effective. Furthermore, Hung [20] argues that central banks can, through covert interventions and a "leaning with the wind strategy", cause noise traders to believe in a trend reversal and make them act accordingly. If noise traders dominate the market, or the market is thin enough, this strategy may result in central banks effectively managing the exchange rate through covert sterilized interventions. Hung [20] also tests whether or not sterilized intervention has resulted in increased volatility in the exchange rate, indicating the use of a "leaning with the wind strategy" and finds supportive empirical evidence for this hypothesis during parts of the sample period. Dominguez [11], although not testing the same hypothesis, also found that covert central bank intervention increased volatility whereas open intervention was found to decrease volatility. Interpreting Dominguez's [11] results in the spirit of Hung [20], would provide support for the chartist channel and the hypothesis that central banks may, under some circumstances, use chartists in their exchange rate policy.

[3] For a more thorough, but still brief, description of technical analysis the reader can turn to Neely [28].

However, it would appear that the chartist channel is not an independent channel as it is necessary for central bank's sterilized intervention to move the exchange rate, at least initially, so that chartist traders have something to react on. The chartist channel explains way sterilized intervention may have a greater effect on the exchange rate than in a situation where there are no chartists in the market, and how it possibly has a little more persistent effect than otherwise. Through the chartist channel, chartists may act to reinforce a trend or to cause a trendreversal in the exchange rate, arising from central bank sterilized intervention.

Previous research There are a few papers, within this are of literature, which adopt a chartist-fundamentalist set-up in modelling central bank intervention. However, these papers rely more on a type of fundamentalist channel than on the chartist channel proposed by Hung [20], when analyzing the effectiveness of sterilized intervention.

De Grauwe and Grimaldi [10] use a chartist-fundamentalist model where the sterilized central bank intervention strategy is known to the market and implemented by fundamentalists in their trading strategy. The central bank's target rate for the exchange rate is equal to the fundamental value and the central bank adopts a "leaning against the wind" strategy in interventions to correct misalignments in the exchange rate. Fundamentalists implement the central bank's interventions in their trading strategy and increase mean reversion in the exchange rate. In this way fundamentalists cause chartist trading strategies to become less profitable and, since the weight between chartists and fundamentalists depends on the profitability of their respective trading strategies, the amount of chartists will decrease with intervention. In this way, intervention becomes effective in reducing misalignments, as it decreases the amount of chartists in the market and increases the mean reversion of the exchange rate. Thus, the effectiveness of sterilized interventions rely on the fact that the central bank has a target for the exchange rate that is equal to its fundamental value, and that fundamentalists incorporate the central bank's intervention strategy into their trading strategy and thereby act to reinforce the effect of intervention.

Kubelec [23] uses a similar model to that of De Grauwe and Grimaldi [10], but also incorporates the evolutionary fitness measure, developed in a series of papers by Brock and Hommes ([5], [6], [7], [8]), in determining the weights between chartists and fundamentalists. This evolutionary fitness measure is based on the profitability of the different trading strategies, which in turn depends on the deviation of the exchange rate from its fundamental value. The weight of the fundamentalists increases when the deviation of the exchange rate from the fundamental value of the exchange rate increases. This is because the expected loss of a chartist strategy, in case of a busting bubble, increases when the deviation of the exchange rate from its fundamental rate increases. In Kubelec [23] central bank interventions are also sterilized and follow a "leaning against the wind" strategy, making chartist trading rules less profitable and, hence, decrease the amount of chartists in the market. The decrease in chartist traders has the effect of driving the exchange rate closer to fundamentals and of bursting bubbles caused by chartists. Thus, once again the effectiveness of sterilized interventions depends on the behavior of the fundamentalists rather than on the behavior of the chartists.

The intervention strategies used in both of these papers are focused on exploiting fundamentalists. In De Grauwe and Grimaldi [10] they are used to drive the exchange rate towards fundamentals, and in Kubelec [23] to correct misalignments. However, if the central bank has a target for the exchange rate that is not equal to fundamentals, it would not be able to exploit fundamentalist behavior, since fundamentalists would then act opposite to central bank intervention.

Schmidt and Wollmershäuser [30] also model sterilized interventions in a chartist and fundamentalist set-up. In their model, the weights between chartists and fundamentalists are fixed and equal. Schmidt and Wollmershäuser [30] assume that the central bank, through covert intervention, tries to exploit chartists. They use simulations to test different strategies that the central bank can use for their interventions. These are: a targeting strategy, a trend reverting strategy, and a smoothing strategy. The targeting strategy implies intervening when the exchange rate deviates from its fundamental value, the trend reverting strategy uses a moving average rule which implies to sell (buy) when the exchange rate is above (below) the moving average, and the smoothing strategy implies going against the wind on an everyday basis. The strategies are evaluated in terms of an intertemporal loss function in which the loss increases the greater the exchange rate deviation from target and the higher the cost of intervention. The trend reverting strategy is most effective in dampening misalignments followed by the targeting strategy. Smoothing the day-to-day movements in the exchange rate is less effective in correcting misalignments and, with a more aggressive intervention rule, also increases the variance and thereby the loss. Schmidt and Wollmershäuser [30] argues that their results support the existence of a chartist channel.

Approach The present paper takes a portfolio balance framework approach and incorporates a chartist channel into a monetary model for the foreign exchange market. In particular, the chartist channel consists solely of the behavior of chartists in the market. The portfolio balance approach is, in this context, necessary to theoretically derive any effects of sterilized intervention through a chartist channel. Although the portfolio balance channel has not received much empirical support it is the most reasonable channel to combine with the chartist channel. This is because it does not require interventions to be open, in contrast to the signaling channel. Further, even though weak, the portfolio balance channel has received more empirical support than the signaling channel.

The purpose of this paper is to develop a theoretical model for central bank intervention in which it is possible to explicitly derive the effects of different types of intervention on the current exchange rate in the case of both homogeneous and heterogeneous expectations (i.e., without and with chartists), and to compare the effects of intervention on the current exchange rate in the different cases. More specifically, the aim is to analyze whether there is a chartist channel through which chartists may enhance, suppress or even reverse the effect of intervention, i.e., to examine the theoretical support for the existence of a chartist channel. Accordingly, it is not necessary to model an explicit policy objective or exchange rate target for the central bank in order to be able to analyze the questions in focus for the present paper. However, a benchmark model is required with which the effects of intervention can be compared when a chartist channel is incorporated. Chartists must also be allowed to have a trading strategy that results in a change in expectations concerning the future exchange rate, depending on how chartists value the current exchange rate. For the purpose of the paper and for reasons of mathematical tractability, it is unnecessary to derive central bank intervention through optimization behavior. The same is true for the monetary model used in the paper, which is based on a few equilibrium conditions and is not derived from an explicit optimizing micro foundation. The same monetary approach is used in Kubelec [23] to model the foreign exchange market.

The heterogeneous expectations model will offer a chartist channel through which sterilized central bank interventions are able to affect the level of the current exchange rate. Traders in the benchmark model have rational expectations, and in the heterogeneous expectations model, expectations are based on fundamental and technical analysis. Short-term expectations are based on technical analysis and long-term expectations on fundamental analysis. Technical analysis is implemented in such a way, that short-term expectations can vary depending on how the current exchange rate is valued by chartists and fundamental analysis is implemented as rational expectations.

Main findings The present paper finds support for the existence of a chartist channel. However, the stability restrictions on the model in the heterogeneous expectations case dampen the effect of chartists. Although they may indeed enhance or suppress the effect of sterilized intervention on the current exchange rate, they can not, in this model, reverse the effect of intervention. This is, again, solely because of the stability conditions that restrict the weight of chartists and the strength of their expectations. There is also a trade-off between the weight of chartists and the strength of chartist expectations in the stability condition for the model. In other words, if the weight of the chartists is low enough, their expectations may be explosive, whereas if their expectations are non-explosive, they may dominate the market.

The effect of sterilized intervention on the current exchange rate may be greater in the heterogeneous expectations model than in the benchmark model when chartists act to reinforce the effect of intervention, i.e., when chartists have bandwagon expectations. The effect of intervention on the current exchange rate may also be less in the heterogeneous expectations model than in the benchmark model when chartists act to oppose intervention, i.e., when chartists have regressive expectations. Thus, the results indicate the existence of a chartists channel through which central banks may be able to effectively manage the exchange rate. However, in this model, this postulates a portfolio balance behavior among traders, i.e., that traders are risk averse, and that foreign and domestic bonds are imperfect substitutes.

Outline The rest of the paper is as follows, the benchmark model is outlined in Section 2.The effects of different types of intervention in the benchmark

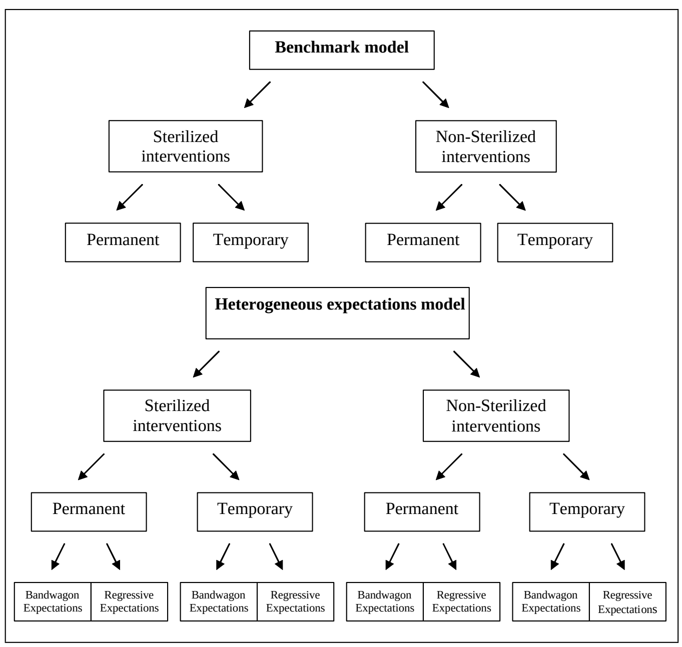

Figure 1: The different types of intervention in the benchmark and heterogeneous expectations models. Table 1 in subsection 2.1 presents the size of the effects of the different types of intervention in the benchmark model, shown in Figure 1, third row. Tables 2A and 2B in subsection 3.3 present the size of the effects of the different types of intervention in the heterogeneous expectations model, shown in Figure 1, last row.

model are presented in subsection 2.1. Subsection 2.2 compares the different effects of intervention in the benchmark model. The heterogeneous expectations model is outlined in Section 3, where expectation formations are presented in subsection 3.1 and the heterogeneous expectations model is formally presented in subsection 3.2. The effects of different types of intervention in the heterogeneous expectations model are presented in subsection 3.3 and subsection 3.4 compares the different effects of intervention in the heterogeneous expectations model. As can be seen from Figure 1, a huge variety of possible comparisons can be made. However, not all of these are of interest for this paper. Accordingly, the comparisons will be limited to the following: comparisons of the effects of intervention when the type and character of intervention are held constant (i.e., sterilized or non-sterilized, temporary or permanent) and chartist expectations are of different types (i.e., bandwagon or regressive), and comparisons of the effects of the different types of intervention (i.e., sterilized or non-sterilized) when chartist expectations and the character of the intervention are held constant. Finally, Section 4 compares the different effects of interventions in the different models and Section 5 summarizes and draws conclusions. The Appendix derives the benchmark and heterogeneous expectations models and the effects of the different types of intervention in the two models.

2 The benchmark model

Basically, the benchmark model is a two-country monetary model (where the foreign economy is regarded as the rest of the world) of the exchange rate in which interventions are introduced via a portfolio balance view. The model is based on an interest rate parity condition, a purchasing power parity condition and money market equilibrium conditions.

The formal structure of the model is presented below, all variables, except the interest rate, are in natural logarithms and Greek letters denote structural parameters.

The interest rate parity condition is formulated as

$$i_t - i_t^* = s_{t+1}^e - s_t + \theta \left(c + b_t - b_t^* - s_t \right), \tag{1}$$

where $i_t - i_t^*$ is the interest rate difference between the domestic and foreign market and $s_{t+1}^e - s_t$ is the expected change of the exchange rate, where s is defined as the domestic price of foreign currency and the superscript e denotes expectations. The term $\theta\left(c + b_t - b_t^* - s_t\right)$ is the linearized risk premium on foreign government bonds derived by Flood and Marion [15], and $\theta = \alpha\sigma^2$, where α is a measure of risk aversion and σ^2 is the variance of the future exchange rate (where exchange rate variance is assumed constant). Hence, θ shows how investors desired bond holdings are influenced by risk. The worldwide private holdings of domestic and foreign bonds, expressed in domestic currency, are b_t and $b_t^* + s_t$, respectively, and c is a positive constant that ensures that the risk premium increases, when θ increases. Thus, the term $\theta\left(c + b_t - b_t^* - s_t\right)$ summarizes traders attitude to risk.[4]

The portfolio balance interest rate parity condition is necessary to derive the effects of sterilized interventions. Note that without the assumption of different risks for home and foreign bonds the effect of non-sterilized intervention on the current exchange rate would equal one, (i.e., the change in the monetary fundamentals would equal the change in the current exchange rate), put another way the elasticity would equal one. Of course, in this case, there would be no effect at all of sterilized intervention.

The purchasing power parity condition states that the exchange rate is equal to the difference between domestic and foreign price levels according to the following equation

$$s_t = p_t - p_t^*, \tag{2}$$

where p_t and p_t^* are the domestic and foreign nominal price levels.

The domestic and foreign money markets are in equilibrium when their respective real money supply is equal to real money demand

$$m_t - p_t = -\beta i_t, \tag{3}$$

and

[4] The risk premium derived by Flood and Marion [15] is based on a model where a foreign investor maximizes his welfare, that depends positively on future real wealth and negatively on the variance of future real wealth relative to current real wealth.

$$m_t^* - p_t^* = -\beta i_t^*, \tag{4}$$

where m, p and i and are the domestic money supply, the domestic price level, and the nominal interest rate, respectively, and y is the natural level of output that has been normalized to 1 before taking logartihms. m^*, $p^*$and i^* represent the foreign counterparts. β is the semi-elasticity of money demand with respect to the interest rate and is assumed to be equal between countries. For reasons of stability, it is necessary to restrict β according to $\beta \leqslant 1$. Assuming β to be smaller than one does not necessarily have to be so restrictive since empirical estimates of β in monetary models are most often smaller than one (Gosh [18], Najand and Bond [27], Schröder and Dornau [31]).

Combining (1)-(4), the exchange rate is given by

$$s_t = m_t - m_t^* + \beta \left(s_{t+1}^e - s_t + \theta \left(c + b_t - b_t^* - s_t \right) \right). \tag{5}$$

In the benchmark model we assume that traders are homogeneous and have rational expectations about the future exchange rate, thus

$$s_{t+1}^e = E_t \left[s_{t+1} | \, I_t \right], \tag{6}$$

where E_t denotes the mathematical expectations and I_t is the information set available at time t, which includes information on the complete model and the values of the different variables up till time t. Substitution of (6) into (5), gives

$$s_t = m_t - m_t^* + \beta \left(E_t \left(s_{t+1} \right) - s_t + \theta \left(c + b_t - b_t^* - s_t \right) \right). \tag{7}$$

After solving the model forward, the current exchange rate in the benchmark model is given by (8) below.[5]

$$s_t = x_{1,bm} \sum_{k=0}^{\infty} x_{4,bm}^k E \left(f_{t+k}^m \right) + x_{3,bm} \sum_{k=0}^{\infty} x_{4,bm}^k E \left(f_{t+k}^b \right), \tag{8}$$

where the difference in monetary fundamentals is given by

$$f_t^m \equiv m_t - m_t^*, \tag{9}$$

[5] For a more detailed derivation of the benchmark model see Appendix A.

and the difference in outstanding bonds, plus a constant, is given by

$$f_t^b \equiv c + b_t - b_t^*, \tag{10}$$

and finally where

$$\begin{cases} x_{1,bm} \equiv \frac{1}{1+\beta(1+\theta)} \\ x_{3,bm} \equiv \frac{\beta\theta}{1+\beta(1+\theta)} \\ x_{4,bm} \equiv \frac{\beta}{1+\beta(1+\theta)}. \end{cases} \tag{11}$$

The parameters in the quotient $x_{1,bm}$ summarize the effect on the exchange rate due to a change in monetary fundamentals and, the parameters in the quotient $x_{3,bm}$ summarize the effect on the exchange rate resulting from a change in outstanding bonds. This can be shown by differentiating (8) with regard to f_t^m, f_t^b respectively, when $k = 0$. These effects are also shown in the Appendix where the effects of non-sterilized and sterilized interventions of a temporary character are derived. The treatment of $x_{4,bm}$ as a convergent infinite geometric series means that the parameters in the quotient $x_{4,bm}$ summarize the enhanced effect on the exchange rate if the intervention is permanent and, thus, changes market expectations. The total effect in such a case is multiplied by either $x_{1,bm}$ or $x_{3,bm}$ depending on the type of fundamentals that change. These effects are derived for the case of non-sterilized and sterilized interventions of a temporary and a permanent character in the Appendix. Finally, the subscript bm denotes the benchmark model. Thus, the current exchange rate depends solely on expectations of future fundamentals (see the two terms on the right hand side of (8) in the benchmark model). Since expectations in the benchmark model are rational and forward looking, the effects of intervention on the current exchange rate will differ depending on whether intervention is perceived by the market as being of a temporary or a permanent character.

The distinction between interventions of a temporary and a permanent character, is made because, even if the central bank does not announce its intentions, fundamentalists may perceive the intervention to be either of a temporary or a permanent character. The effect on the current exchange rate of intervention will, of course, be different if intervention is of a permanent character than if it is of a temporary character. In the former case, the fundamentalist expectations on future monetary fundamentals will change, (see equation (8) for the current exchange rate), and, if the intervention is of temporary character, the fundamentalist expectations on future fundamentals will, of course, not change. Henceforth, the type of intervention referred to will always be in italics for easier reading. We are now ready to analyze the effects of intervention on the current exchange rate in the benchmark model.

2.1 The effects of intervention in the benchmark model

The effects on the current exchange rate of the different types of intervention in the benchmark model are derived formally in the Appendix and presented in Table 1 below.

Table 1: The effects of intervention in the benchmark model.

	Sterilized	Non-Sterilized
Temporary	$0 < \frac{\beta\theta}{1+\beta(1+\theta)} < 1$	$0 < \frac{1}{1+\beta(1+\theta)} < 1$
Permanent	$0 < \frac{\beta\theta}{\beta\theta+1} < 1$	$0 < \frac{1}{\beta\theta+1} < 1$

Note that the effect on the current exchange rate of a *temporary* intervention may be interpreted as the short-term elasticity of the exchange rate with regard to fundamentals, and the effect on the current exchange rate of a *permanent* intervention may be interpreted as the long-term elasticity of the exchange rate with regard to fundamentals.

2.1.1 Sterilized intervention

Sterilized intervention is characterized by the central bank buying (selling) domestic currency for foreign currency on the foreign exchange market and offsetting the effect on the money supply by completing the opposite type of transaction on the domestic monetary market, selling (buying) domestic currency for domestic bonds.

In the benchmark model, when the foreign exchange market is completely dominated by fundamentalists with rational expectations, the effect on the current exchange rate of a *sterilized* intervention of a *temporary* character, is less than one. The effect of a *sterilized* intervention of a *permanent* character is greater than the effect of a *temporary* change, but is also less than one (See Table 1). Formally we, can express this difference in effects according to

$$0 < \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|_{temp,ster} < \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|_{perm,ster} < 1.$$

2.1.2 Non-sterilized intervention

Non-sterilized intervention is equivalent to a change in money supply, but instead of completing the transaction in the domestic monetary market, the central bank completes its transactions on the foreign exchange market, where the central bank buys (sells) domestic currency for foreign currency. The effect on the current exchange rate of a *non-sterilized* intervention of a *temporary* character is less than one. The effect of a *non-sterilized* intervention of a *permanent* character has a greater effect than an intervention of a *temporary* character but is still less than one (See Table 1).

$$0 < \left.\frac{ds_{t_0}}{df^m_{t_0}}\right|_{temp,non-ster} < \left.\frac{ds_{t_0}}{df^m_{t_0}}\right|_{perm,non-ster} < 1$$

The fact that the effect is less than one, when the change in the money supply is *permanent* is due to the inclusion of risk aversion in the model. If traders are risk neutral, (i.e., $\theta = 0$), the effect of an intervention of a *permanent* character will have a one-to-one effect on the current exchange rate (see the effect of an intervention of a *permanent* character in Table 1).[6]

[6] Note that, the effects of intervention derived in this model are derived when output is held constant (at its natural level), and thus, there are no real effects of intervention. If output is affected by intervention the effects may be different.

2.2 Comparing the different effects of intervention in the benchmark model

The size of the effect of intervention on the current exchange rate in the *non-sterilized* and *sterilized* cases will differ only with regard to parameters $\beta\theta$ which appear in the numerator of the terms representing the effects of *sterilized* intervention (see Table 1). The comparison of *non-sterilized* and *sterilized* interventions of a *permanent* character is qualitatively the same as the comparison of *non-sterilized* and *sterilized* interventions of a *temporary* character (see Table 1).

Thus, comparing the effects of intervention when intervention is *non-sterilized* and *sterilized* gives the following ambiguous result: if $\beta\theta > 1$, *sterilized* intervention has a greater effect on the exchange rate than *non-sterilized* intervention; if $\beta\theta < 1$, *non-sterilized* intervention has a greater effect on the exchange rate than *sterilized* intervention; and finally if $\beta\theta = 1$, the effect of *non-sterilized* intervention equals that of *sterilized* intervention (see Table 1).

$$\left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{\beta\theta>1}_{non-ster} < \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{\beta\theta>1}_{ster}$$

$$\left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{\beta\theta=1}_{non-ster} = \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{\beta\theta=1}_{ster}$$

$$\left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{\beta\theta<1}_{non-ster} > \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{\beta\theta<1}_{ster}$$

Hence, under certain circumstances, *sterilized* intervention may have a greater effect on the current exchange rate than *non-sterilized* intervention. This ambiguity depends on the size of the risk premium, i.e., on θ and on how sensitive money demand is to the interest rate, i.e., on β. The more risk averse investors are, and the more sensitive money demand is to the interest rate, the more likely it is that *sterilized* intervention has a greater effect on the exchange rate than *non-sterilized* intervention. However, note that a stability condition on the model is that $\beta < 1$. Thus, for $\beta\theta$ to be greater than one it is necessary that θ is greater than one, i.e., it is necessary that traders are rather risk averse. Hence, it seems reasonable to assume that $\beta\theta \leqslant 1$, and that, the effect on the current exchange rate of *sterilized* intervention is, at most, equal to the effect of *non-sterilized* intervention.

Note also that if traders are risk neutral, (i.e., $\theta = 0$), there is no effect on the current exchange rate of *sterilized* intervention.

3 Heterogeneous expectations

In accordance with De Grauwe and Grimaldi [10], Kubelec [23] and Schmidt and Wollmershäuser [30], the heterogeneous expectations model in this paper has a chartist-fundamentalist set-up. In the mentioned papers, the weight function between chartists and fundamentalists is a function of the profitability of the respective chartist and fundamentalist trading strategy. In the present paper, the weight function in the heterogeneous expectations model is, instead, a function of the time horizon in currency trade which, according to questionnaire surveys on the foreign exchange market, is the major determining factor for the choice of trading strategy (see for example Cheung and Chinn [9], Lui and Mole [24], Menkhoff [25], Oberlechner [29] and Taylor and Allen [32]). Furthermore, in the present model, the central bank does not, unlike the above mentioned papers, have an explicit (or implicit) target rate for the exchange rate equal to fundamentals, which is crucial for the ability of the central bank to exploit fundamentalists when intervening. Moreover, within the present model, chartist expectations are also allowed to change with the level of the exchange rate which according to the different trading tools within technical analysis is characteristic for chartist behavior.

3.1 Expectations formation

In the heterogeneous expectations model, we assume that traders have heterogeneous expectations in the form of chartist and fundamentalist expectations. According to questionnaire surveys (see references in Section 3), the relative importance of technical versus fundamental analysis in the foreign exchange market depends on the time horizon in currency trade. For shorter time horizons, more weight is placed on technical analysis, whereas more weight is placed on fundamental analysis for longer time horizons. In this paper, this empirical observation is formulated as

$$s^e_{t+1} = \omega(\tau)\, s^e_{f,t+1} + (1 - \omega(\tau))\, s^e_{c,t+1}, \tag{12}$$

where s^e_{t+1} denotes the market expectations of the future spot exchange rate, and $s^e_{f,t+1}$ and $s^e_{c,t+1}$ are the expectations formed by fundamentalists and chartists, respectively. In (12) chartist and fundamentalist expectations are weighted according to a weight function that depends on the time horizon in the currency trade, τ. More specifically, the weight function is

$$\omega(\tau) = 1 - e^{-\tau}, \tag{13}$$

where the time horizon τ is exogenously given in the model[7]

3.1.1 Technical analysis

According to questionnaire surveys and empirical studies, short term trading is mainly based on technical analysis, and often results in expectations of static, extrapolative, regressive or bandwagon type (Allen and Taylor [1], Frankel and Froot [16], Frankel and Froot [17], Ito [21]). Technical analysis, or chartism, is based on past exchange rates in order to detect patterns that are extrapolated into the future. Typically, trend indicators and oscillating indicators are used conjunctively to help predict trends and trend reversals in the exchange rates.

Oscillating indicators are constructed to oscillate within a specific interval restricted by so called "oversold" and "overbought" limits. These limits are constructed for the purpose of detecting when the market is oversold or overbought in a currency (indicating that the currency is undervalued or overvalued). When the indicator oscillates outside of the interval it is interpreted as a signal of a trend reversal.

Trend indicators are usually a smoothed version of the exchange rate (most commonly a type of moving average). These are often used in conjunction with, so called support and resistance levels (alternatively two differently smoothed

[7] Note that the weight function used in the present paper is inspired by the weight function used in Bask [2].

versions of the exchange rate are used to identify crossovers), to help detect trendreversals.

In the present paper, technical analysis is based on the use of such an interval, delimited by a type of support and resistance levels. This means that if the exchange rate moves outside the interval a trend reversal is expected. More specifically, if the exchange rate is above the support level, the currency is considered to be "undervalued" and is expected to appreciate and, if the exchange rate is below the resistance level, the currency is considered to be "overvalued" and is expected to depreciate. Thus, the support and resistance levels are set with the purpose of identifying trend reversals in the exchange rate path, and in the present paper, the support and resistance levels are assumed to be fixed.

Hence, chartists expectations will become regressive, when the exchange rate is outside the interval delimited by the support and resistance levels. This means that, if the exchange rate is depreciating, it is expected to start appreciating, and vice versa. When the exchange rate is in between the support and resistance levels, expectations are of bandwagon type, meaning that, if the exchange rate is depreciating (appreciating) it is expected to keep depreciating (appreciating). This is an extremely trivial type of trading rule and is constructed for simplicity and tractability. However, it mimics a simple version of the chartist trading rules most commonly used by the traders included in the survey carried out by Taylor and Allen [32], i.e., moving averages, oscillators and momentums.

Let S and R be the support and resistance levels for the exchange rate and assume for simplicity that they are set exogenously. Specifically, chartist expectations are formulated as

$$\begin{cases} \Delta s^e_{c,t+1} = \gamma \Delta s_t, & if\ R < s_t < S, \qquad \text{where } 0 < \gamma \\ \Delta s^e_{c,t+1} = -\gamma \Delta s_t, & if\ s_t \leqslant R,\ or\ if\ s_t \geqslant S, \qquad \text{where } 0 < \gamma. \end{cases} \tag{14}$$

Hence, if the exchange rate is in between the interval delimited by the support and resistance levels, (i.e., if $R < s_t < S$), chartist expectations are of bandwagon type. This means that if $\Delta s_t < 0$, then $\Delta s^e_{c,t+1} < 0$ and the exchange rate is expected to further depreciate, and if $\Delta s_t > 0$, then $\Delta s^e_{c,t+1} > 0$ and the exchange rate is expected to further appreciate. If the exchange rate is outside this interval, (i.e., if $s_t < R$, or $s_t > S$), chartist expectations shift and become regressive. Thus, if $\Delta s_t < 0$, then the exchange rate is expected to start appreciating, and if $\Delta s_t > 0$, then the exchange rate is expected to start depreciating[8]. Hence, chartists act to reinforce or reverse trends in the exchange rate.

Note that there is a stability condition in this model that restricts γ according to $\gamma e^{-\tau} \leqslant 1 - e^{-\tau}$. Accordingly, there is a trade-off between the weight of chartists and the size of γ in the model.[9] Put another way, the stability condition restricts the time horizon in currency trade according to; $\tau > \ln(1+\gamma)$. Thus, the time horizon in currency trade may not become too myopic, i.e., the weight of chartists may not become too high. However, how high depends on the size of γ.

According to the conventional definition of bandwagon expectations, the parameter γ in (14) above is defined as $\gamma > 1$. Thus, chartists expectations are, in this model, of bandwagon type. However, they do not necessarily have to be bandwagon expectations, since, in this model $0 < \gamma$. Nevertheless, they will, henceforth, be referred to as bandwagon expectations. Further, even though chartist expectations are referred to as regressive in this model, it does not mean, as it generally does, that the exchange rate is expected to regress to its fundamental value. Instead it means that the exchange rate is expected to regress back between the chartist support and resistance levels. Nevertheless, this type of expectations will, henceforth, be referred to as being regressive.

3.1.2 Fundamental analysis

According to questionnaire surveys, medium to long term trading is mainly based on fundamental analysis. Two empirical papers providing support for the use of monetary fundamentals (monetary models) in forecasting exchange rates in the medium to long run[10] are Najand and Bond [27] and Schröder and Dornau [31]. In the present paper, fundamental analysis is based on rational expectations, i.e., the fundamentalists know the complete monetary model for exchange

[8] Hence, we can view the switch from bandwagon expectations to regressive expectations as a multiplication of γ with -1.

[9] See Appendix A for more details.

[10] Six months in Schröder and Dornau [31] and 12-36 months in Najand and Bond [27].

rate determination. Specifically, fundamentalist expectations are formulated as

$$s^e_{f,t+1} = E_t \left[s_{t+1} \right| I_t \right], \tag{15}$$

where $E_t(s_{t+1})$ is the mathematical expectation of s_{t+1} based on the information set available at time t, which includes information on the complete model and the values of the different variables up till time t. Thus, the fundamentalists take chartist trading into account when forming their expectations about the future exchange rate.

3.2 The heterogeneous expectations model

The heterogeneous expectations model, with both chartists and fundamentalists in the market, becomes rather complicated. The Appendix presents a derivation of the solution of the heterogeneous expectations model presented in (16) below. However, in simple terms, the model is solved forward by the use of the undetermined coefficient method and by combining equations (5) and (12)-(15). The solution below is given for when chartists have bandwagon expectations (i.e., when the exchange rate is in between the chartist support and resistance levels). When chartist expectations change and become regressive (i.e., when the exchange rate is outside of the chartist interval restricted by the support and resistance levels) the root expression in (17) changes, but otherwise (16) remains unchanged. In the heterogeneous expectations model, the current exchange rate is given by

$$\begin{aligned} s_t &= -\lambda_1 s_{t-1} + \frac{x_{1,hm}}{(1+x_{4,hm}\lambda_1)} \sum_{k=0}^{\infty} \left(\frac{x_{4,hm}}{(1+x_{4,hm}\lambda_1)} \right)^k E_t\left(f^m_{t+k}\right) \\ &\quad + \frac{x_{3,hm}}{(1+x_{4,hm}\lambda_1)} \sum_{k=0}^{\infty} \left(\frac{x_{4,hm}}{(1+x_{4,hm}\lambda_1)} \right)^k E_t\left(f^b_{t+k}\right), \end{aligned} \tag{16}$$

where the parameter λ_1, the stable root, is

$$\lambda_1 = \frac{1}{2x_{4,hm}} - \sqrt{\left(\frac{-1}{2x_{4,hm}} \right)^2 + \frac{x_{2,hm}}{x_{4,hm}}}, \tag{17}$$

where $|\lambda_1| < 1$. The characteristic equation, and thus, the root expression changes slightly depending on chartists' type of expectations. Given that chartists have bandwagon expectations, the stable root, λ_1^{bw}, is positive and <1 and, given that chartists have regressive expectations, the stable root, λ_1^{reg}, is negative and >-1.[11] The parameters in the $x_{1,hm}$, $x_{2,hm}$, $x_{3,hm}$ and $x_{4,hm}$ quotients are

$$\begin{cases} x_{1,hm} \equiv \frac{1}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})} \\ x_{2,hm} \equiv \frac{\beta e^{-\tau}\gamma}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})} \\ x_{3,hm} \equiv \frac{\beta\theta}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})} \\ x_{4,hm} \equiv \frac{\beta\left(1-e^{-\tau}\right)}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})}, \end{cases} \tag{18}$$

and f_t^m and f_t^b in (16) are given by (9) and (10), respectively. The subscript hm denotes the heterogeneous expectations model.

The x_{hm}-parameters in the heterogeneous expectations model are similar to the x_{bm}-parameters in the benchmark model but differ with regard to both the weight of chartists and fundamentalists, and the chartist expectations, which are included in the x_{hm}-parameters but not in the x_{bm}-parameters. In the heterogeneous expectations model, the x_{hm}-parameters do not appear single-handedly but are combined in the quotients in equation (16). However, the interpretation of the quotients including the x_{hm}-parameters in (16) are equivalent to the x_{bm}-parameters in equation (8) for the current exchange rate in the benchmark model. The quotients also include the stable root λ_1, which results in a saddle path solution for the model. We are now ready to analyze the effects of intervention on the current exchange rate in the heterogeneous expectations model.

3.3 The effects of intervention in the heterogeneous expectations model

Tables 2 and 3 present the effects of *non-sterilized* and *sterilized* intervention on the current exchange rate in the heterogeneous expectations model. The effects of intervention are derived formally in the Appendix.

[11] For more detailed expressions see Appendix A.

Table 2: The effects of intervention in the heterogeneous expectations model when chartists have bandwagon expectations.

	Sterilized	Non-sterilized
Temp	$\frac{\theta\beta}{1+\beta(\theta-\gamma e^{-\tau})+\beta(1-e^{-\tau})(1+\lambda_1^{bw})}$	$\frac{1}{1+\beta(\theta-\gamma e^{-\tau})+\beta(1-e^{-\tau})\left(1+\lambda_1^{bw}\right)}$
Perm	$\frac{\theta\beta}{1+\beta(\theta-\gamma e^{-\tau})+\beta(1-e^{-\tau})\lambda_1^{bw}}$	$\frac{1}{1+\beta(\theta-\gamma e^{-\tau})+\beta(1-e^{-\tau})\lambda_1^{bw}}$

Table 3: The effects of intervention in the heterogeneous expectations model when chartists have regressive expectations.

	Sterilized	Non-sterilized
Temp	$\frac{\theta\beta}{1+\beta(\theta+\gamma e^{-\tau})+\beta(1-e^{-\tau})(1+\lambda_1^{reg})}$	$\frac{1}{1+\beta(\theta+\gamma e^{-\tau})+\beta(1-e^{-\tau})\left(1+\lambda_1^{reg}\right)}$
Perm	$\frac{\theta\beta}{1+\beta(\theta+\gamma e^{-\tau})+\beta(1-e^{-\tau})\lambda_1^{reg}}$	$\frac{1}{1+\beta(\theta+\gamma e^{-\tau})+\beta(1-e^{-\tau})\lambda_1^{reg}}$

3.3.1 Sterilized intervention

The effect of a *temporary sterilized* intervention on the current exchange rate is positive and less than one regardless of the type of expectations held by chartists. The effect of a *permanent sterilized* intervention on the current exchange rate may also be less than one (see Tables 2 and 3) but larger than the effect when intervention is of a *temporary* character, given that expectations are held constant.

$$0 < \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{bandwagon\ exp}_{temp,ster} < \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{bandwagon\ exp}_{perm,ster} \lesseqgtr 1$$

$$0 < \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{regressive\ exp}_{temp,ster} < \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{regressive\ exp}_{perm,ster} \lesseqgtr 1$$

Note that when chartists have regressive expectations, the effect on the current exchange rate of both types of intervention becomes smaller the greater the weight put on chartists, i.e., the smaller τ, the time horizon in trade, becomes. The opposite is true for when chartists have bandwagon expectations, that is, the greater the weight put on chartists, the larger the effect of intervention. Hence, when chartists have bandwagon expectations, they act to increase the effect of intervention on the current exchange rate and, when chartists have regressive expectations, they act to decrease the effect of intervention. The stronger the chartists' expectations are, i.e., the larger γ is, the more emphasized these effects become. This result is in line with the workings of the chartist channel as it was described in the introduction.

3.3.2 Non-sterilized intervention

The analysis of the effects of *non-sterilized* interventions, is similar to the analysis of the *sterilized* interventions both in the case of a *temporary* and a *permanent* change (see Tables 2 and 3), given that expectations are held constant.

$$0 < \left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{bandwagon\ exp}_{temp,non-ster} < \left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{bandwagon\ exp}_{perm,non-ster} \lesseqgtr 1$$

$$0 < \left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{regressive\ exp}_{temp,non-ster} < \left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{regressive\ exp}_{perm,non-ster} \lesseqgtr 1$$

Note that it is also possible that the effects on the current exchange rate of a *permanent* intervention, in both the *sterilized* and *non-sterilized* cases, may be larger than one (i.e., a long-term elasticity of the exchange rate with regard to fundamentals greater than one). The greater the weight put on chartists, when they have bandwagon expectations, the more likely it is that the effect will be larger than one (see the expressions for *permanent sterilized* and *non-sterilized* intervention in Tables 2 and 3). Again, γ works to emphasize these effects.

3.4 Comparing the different effects of intervention in the heterogeneous expectations model

3.4.1 Comparing the effects of non-sterilized and sterilized intervention

Given that chartist expectations are held constant and that interventions are of the same character (*temporary* or *permanent*), this comparison is qualitatively equivalent to the comparison of *non-sterilized* and *sterilized* intervention in the benchmark model. Thus, whether the effect on the current exchange rate is greater in the case of *non-sterilized* intervention or not, depends on whether $\beta\theta \lesseqgtr 1$. If $\beta\theta < 1$, the effect of intervention on the current exchange rate is greater in the *non-sterilized* case and vice versa if $\beta\theta > 1$. The effects are of the same size if $\beta\theta = 1$ (see Tables 2 and 3).

$$\left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{\beta\theta>1}_{non-ster} < \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{\beta\theta>1}_{ster}$$

$$\left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{\beta\theta=1}_{non-ster} = \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{\beta\theta=1}_{ster}$$

$$\left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{\beta\theta<1}_{non-ster} > \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{\beta\theta<1}_{ster}$$

Again, this result is due to the portfolio balance approach used in the model and is independent of chartist expectations. Accordingly, for a more detailed interpretation of these results the reader can turn to the comparison of the effects of *non-sterilized* and *sterilized* intervention in the benchmark model.

3.4.2 Comparing the effects of intervention when chartist expectations are different

Given that the type and character of the intervention (*permanent* or *temporary* and *sterilized* or *non-sterilized*) are qualitatively identical in both cases, it is possible to compare the effects of intervention on the current exchange rate when chartist expectations are of different type. Whether or not the effect of an intervention on the current exchange rate is greater when chartist expectations are of bandwagon type than when they are of regressive type, hinges, in all cases, on the sign of following expression; $-2\gamma e^{-\tau}+(1-e^{-\tau})\left(\lambda_1^{bw}-\lambda_1^{reg}\right)$. This is derived from the difference between the denominators of the expressions in Tables 2 and 3 for bandwagon and regressive expectations, where the first term is negative and the second term is positive as $0>\lambda_1^{reg}>-1$. Thus in the case of *sterilized* intervention, if: $-2\gamma e^{-\tau}+(1-e^{-\tau})\left(\lambda_1^{bw}-\lambda_1^{reg}\right)<0$, the effect on the current exchange rate of *sterilized* intervention is greater when chartists have bandwagon expectations than if they have regressive expectations; if $-2\gamma e^{-\tau}+(1-e^{-\tau})\left(\lambda_1^{bw}-\lambda_1^{reg}\right)>0$, the effect on the current exchange rate of *sterilized* intervention is greater when chartists have regressive expectations than if they have bandwagon expectations: and finally if $-2\gamma e^{-\tau}+(1-e^{-\tau})\left(\lambda_1^{bw}-\lambda_1^{reg}\right)=0$ the effect on the current exchange rate of *sterilized* intervention is the same in the case of both regressive and bandwagon expectations (see Tables 2A and 2B).

$$\left.\frac{ds_{t_0}}{df_{t_0}^b}\right|_{bandwagon}^{-2\gamma e^{-\tau}+\left(1-e^{-\tau}\right)(\lambda_1^{bw}-\lambda_1^{reg})<0} > \left.\frac{ds_{t_0}}{df_{t_0}^b}\right|_{regressive}^{-2\gamma e^{-\tau}+\left(1-e^{-\tau}\right)(\lambda_1^{bw}-\lambda_1^{reg})<0}$$

$$\left.\frac{ds_{t_0}}{df_{t_0}^b}\right|_{bandwagon}^{-2\gamma e^{-\tau}+\left(1-e^{-\tau}\right)(\lambda_1^{bw}-\lambda_1^{reg})=0} = \left.\frac{ds_{t_0}}{df_{t_0}^b}\right|_{regressive}^{-2\gamma e^{-\tau}+\left(1-e^{-\tau}\right)(\lambda_1^{bw}-\lambda_1^{reg})=0}$$

$$\left.\frac{ds_{t_0}}{df_{t_0}^b}\right|_{banwagon}^{-2\gamma e^{-\tau}+\left(1-e^{-\tau}\right)(\lambda_1^{bw}-\lambda_1^{reg})>0} < \left.\frac{ds_{t_0}}{df_{t_0}^b}\right|_{regressive}^{-2\gamma e^{-\tau}+\left(1-e^{-\tau}\right)(\lambda_1^{bw}-\lambda_1^{reg})>0}$$

It is clear that the effect of intervention on the current exchange rate is larger in the case of bandwagon expectations when these are strong (i.e., if γ is large) and the weight on chartists is high (i.e.,if τ is small). The size of $\gamma e^{-\tau}$ need not exceed the sufficient stability condition for the inequality $-2\gamma e^{-\tau}+(1-e^{-\tau})\left(\lambda_1^{bw}-\lambda_1^{reg}\right)<0$ to hold.[12] This result truly lends support to the existence of a chartist channel. Thus, depending on their expectations, chartists may act to increase or decrease the effect of intervention on the current exchange rate, but they can not reverse the effect of intervention. Given that the

[12] For simplicity lets assume that $\gamma e^{-\tau}=\left(1-e^{-\tau}\right)$, i.e., that the stability condition is at its bound, then we have $-2\left(1-e^{-\tau}\right)+\left(1-e^{-\tau}\right)\left(\lambda_1^{bw}-\lambda_1^{reg}\right)<0$, because the roots are both in absolute values smaller than one. Simulation show that the inequality remains robust for most values of $\gamma e^{-\tau}$ under the stability condition.

exchange rate is in between chartist support and resistance levels, chartists will act to increase the effect of intervention and, given that the exchange rate is outside this interval, they will act to decrease the effect of intervention.

Note that the stability condition used in the model is a sufficient condition, and given that the weight of chartists is low enough, it is not necessary for γ to be smaller than one. On the other hand, if the weight of chartists is higher than the weight of fundamentalists, it is necessary for γ to be smaller than one. Thus, when the model is stable, it is possible to include chartists with explosive bandwagon expectations given that the weight of chartists is low enough, or alternatively, it is possible to have chartists dominating the market given that their expectations are not explosive.

Now, the reader may wonder what will happen if the intervention causes the exchange rate to fall outside the chartist interval (i.e., to cross either the support or the resistance level). We have not been able to derive this effect as it would require a two period model. However, a reasonable guess would be that, in the fist period, chartists would enhance the intervention and push the exchange rate rather far outside the interval and in the next period they would (given that the weight of chartists is high enough for them to affect the level of the current exchange rate) cause a trend reversal in the exchange rate path. A comprehensive theoretical model is required, however, in order to be able to further investigate this issue.

4 Comparing the benchmark model to the heterogeneous expectations model

Comparing the results of the different models gives an indication of how chartists may influence the effects of intervention on the exchange rate. We have already noted that the effects of *sterilized* and *non-sterilized* intervention are qualitatively the same in the different models (again see Tables 1, 2 and 3). The results of the comparisons show that the effect of intervention on the current exchange rate is more likely to be greater in the heterogeneous expectations case when chartist expectations are of bandwagon type and, in the homogenous expectations case, when chartist expectations are regressive.

We start by looking at *temporary* intervention. In this case, it is clear that the effect of intervention on the current exchange rate is greater in the heterogeneous expectations model if chartists have bandwagon expectations. If chartist expectations are regressive the effect of intervention on the current exchange rate is likely to be smaller in the heterogeneous expectations model (see Table 1, 2 and 3).

$$\left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{bm}_{temp} < \left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{hm,BW}_{temp}$$

$$\left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{bm}_{temp} \lesseqgtr \left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{hm,REG}_{temp}$$

Looking at interventions of a *permanent* character, the result is more ambiguous. However, if bandwagon expectations are strong in the heterogeneous expectations model, i.e., γ is large, or if the weight of chartists is high, i.e., τ is small, (although within the assumed limit of the model), the effect of a *permanent* intervention is larger in the heterogeneous expectations model. On the other hand, if regressive expectations are strong in the heterogeneous expectations model or if the weight of chartists is high, the effect is greater in the benchmark model.

$$\left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{bm}_{perm} \lesseqgtr \left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{hm,BW}_{perm}$$

$$\left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{bm}_{perm} \lesseqgtr \left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{hm,REG}_{perm}$$

The model's stability conditions which restrict the size of γ and the weight of chartists $e^{-\tau}$ mean that chartists are not able to reverse the effect of intervention on the current exchange rate, i.e., cause a so called perverse response to intervention, when their expectations are regressive. A perverse response to intervention is when traders act to cause the domestic currency to depreciate (appreciate) when the central bank intervenes to decrease (increase) the money supply. Note that, the stability condition used in the model is a sufficient condition and, depending on other parameter values, we could have a stable solution even when $\ln(1+\gamma) > \tau$. In this case a perverse response is highly likely, especially if, as Hung [20] argues, the market is thin enough.

5 Concluding remarks

The main purpose of the present paper was to establish whether or not there exists a chartist channel through which *sterilized* intervention may be effective. The results of the benchmark and heterogeneous expectations models clearly show that *sterilized* intervention may have a greater effect on the current exchange rate than *non-sterilized* intervention. In the benchmark model, this result is due to the portfolio balance framework, whereas, in the heterogeneous expectations model, it also depends on how the market perceives the central bank's intentions with intervention and on market expectations about the future exchange rate. The results of the heterogeneous expectations model lend support to the existence of a chartist channel which central banks may be able to exploit. Earlier theoretical papers within this area have not, to our knowledge, been able to show any clear indication of the existence of a chartist channel. However, as is clearly shown in the present paper, the chartist channel does not appear to be an independent channel. Traders have to be risk averse in order for a *sterilized* intervention to have an effect on the current exchange rate, i.e., a "portfolio balance behavior" among traders is required.

Within the heterogeneous expectations model, technical and fundamental analysis, as well as the weight function between chartists and fundamentalists are modelled with the aim of mimicking real behavior among traders such as that described in the results of questionnaire surveys on the foreign exchange market (see previously mentioned surveys). To be able to support a chartist channel it appears to be necessary to allow chartists to act on the foreign exchange market either to reinforce or reverse trends in the exchange rate. As *sterilized* central bank interventions are often small in volume compared with the daily market turnover and, given that they have an effect, they are only likely to change the current exchange rate slightly. Nevertheless this may be sufficient to cause chartists to act on a trend or a trend reversal in the exchange rate. In this way, the effect of *sterilized* intervention on the current exchange rate may become significant.

In the present paper, chartist expectations are formulated with sufficient flexibly to allow for variations in behavior among chartists and they may both enhance and suppress the effects of intervention depending on their type of expectations. If chartist have bandwagon expectations, they act to increase the effect of *sterilized* intervention on the current exchange rate whereas, if their expectations are regressive, they act to decrease its effect. Thus, the effect of *sterilized* intervention may indeed be larger (smaller) in the case of heterogeneous expectations, when chartist expectations are of bandwagon (regressive) type than in the case of homogeneous expectations. The greater the weight on chartists, the more likely it is that the effect of *sterilized* intervention on the current exchange rate will be larger in the heterogeneous case when chartists have bandwagon expectations.

However, the model's stability condition means that chartists cannot cause a perverse response to intervention. An interesting path to pursue in future research would be to explore whether, and under what circumstances, chartists are able to reverse the effect of intervention. Empirically perverse responses to intervention have been documented, by for example, Kaminsky and Lewis [22]. Perverse responses to intervention also coincide well with how Hung [20] explains the operation of a chartist channel. Another interesting extension to this paper would be to include a reason for the central bank to intervene in the foreign exchange market. This could, for example, be a policy objective for the central bank which is implemented through a loss function. The gains and losses occurring to the central bank as a consequence of its intervention could also be considered. If, for example, chartists cause a perverse response to intervention and the central bank tries to use this fact, it will result in losses in wealth for the central bank. However, implementing the type of chartist behavior used in this model is rather complicated and requires a fairly simple model to be (mathematically) computational. Including policy objectives, a loss function, and maybe some type of budget restriction on central bank interventions, tend to complicate matters when chartists are incorporated in the model in this way.

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Appendix

Derivation of the benchmark model

Combining (7 and 6) we get

$$s_t = m_t - m_t^* + \beta \left(E_t \left(s_{t+1} \right) - s_t + \theta \left(c + b_t - b_t^* - s_t \right) \right). \tag{A 1}$$

Hence, the current exchange rate is now given by

$$s_t = x_{1,bm} f_t^m + x_{3,bm} f_t^b + x_{4,bm} E \left(s_{t+1} \right). \tag{A 2}$$

where

$$\begin{cases} x_{1,bm} = \frac{1}{1+\beta(1+\theta)} \\ x_{3,bm} = \frac{\beta\theta}{1+\beta(1+\theta)} \\ x_{4,bm} = \frac{\beta}{1+\beta(1+\theta)}, \end{cases} \tag{A 3}$$

and where the difference in monetary fundamentals are given by

$$f_t^m \equiv m_t - m_t^*, \tag{A 4}$$

and the difference in outstanding bonds is given by

$$f_t^b \equiv c + b_t - b_t^*. \tag{A 5}$$

Iterating (A 2) one period ahead and substituting back into the last term on the right hand side of (A 2), gives

$$s_t = x_{1,bm} f_t^m + x_{3,bm} f_t^b + x_{4,bm} E \left(x_{1,bm} f_{t+1}^m + x_{3,bm} f_{t+1}^b + x_{4,bm} E \left(s_{t+2} \right) \right). \tag{A 6}$$

Repeating the procedure for k periods ahead, gives

$$s_t = x_{1,bm} \sum_{k=0}^{k_{\max}} x_{4,bm}^k E \left(f_{t+k}^m \right) + x_{3,bm} \sum_{k=0}^{k_{\max}} x_{4,bm}^k E \left(f_{t+k}^b \right) + x_{4,bm}^{k_{\max}+1} E \left(s_{t+k_{\max}+1} \right). \tag{A 7}$$

Assuming no bubbles, i.e., that the following transversality condition holds[13]

$$\lim_{k_{\max}\to\infty} x_4^{k_{\max}+1} E\left(s_{t+k_{\max}+1}\right) = 0. \tag{A 8}$$

Thus, the current exchange rate is given by

$$s_t = x_{1,bm} \sum_{k=0}^{\infty} x_{4,bm}^k E\left(f_{t+k}^m\right) + x_{3,bm} \sum_{k=0}^{\infty} x_{4,bm}^k E\left(f_{t+k}^b\right), \tag{A 9}$$

which completes the derivation of the benchmark model given in (8)-(11) in the main text.

The effects of intervention in the benchmark model

The following computations derive the effects of different types intervention on the current exchange rate in the benchmark model. The effects are derived under the assumption that the current exchange rate is given by (8)-(11) in the main text and that central bank intervention that takes place at time $t = t_0$. All the effects derived below are presented in Table 1, subsection 2.1.

The effect of a *sterilized* intervention of a *temporary* character, is

$$ds_{t_0} = x_{3,bm} E(df_{t_0}^b), \tag{A 10}$$

which gives

$$\frac{ds_{t_0}}{df_{t_0}^b} = x_{3,bm} = \frac{\beta\theta}{1+\beta\left(1+\theta\right)} \tag{A 11}$$

and, thus

$$\left.\frac{ds_{t_0}}{df_{t_0}^b}\right|_{temp,ster}^{bm} = \frac{\beta\theta}{1+\beta\left(1+\theta\right)} < 1. \tag{A 12}$$

The effect of a *sterilized* intervention of a *permanent* character, is

$$\begin{aligned} ds_{t_0} &= x_{3,bm} \sum_{k=0}^{\infty} x_{4,bm}^k E\left(df_{t_0+k}^b\right) \\ &= x_{3,bm} df_{t_0}^b \sum_{k=0}^{\infty} x_{4,bm}^k \end{aligned} \tag{A 13}$$

[13] If the transversality condition does not hold, there exists a rational bubble. However, in this paper, we concentrate on the stable solution.

which, since $x_{4,bm} = \frac{\beta}{1+\beta(1+\theta)} < 1$ and $\frac{ds_{t_0}}{df^m_{t_0}}$ may be calculated as a convergent infinite geometric series, gives

$$\frac{ds_{t_0}}{df^b_{t_0}} = \frac{x_{3,bm}}{1 - x_{4,bm}} = \frac{\frac{\beta\theta}{1+\beta(1+\theta)}}{1 - \frac{\beta}{1+\beta(1+\theta)}}. \tag{A 14}$$

Hence,

$$\left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{bm}_{perm,ster} = \frac{\beta\theta}{\beta\theta + 1} < 1. \tag{A 15}$$

The effect on the current exchange rate of a *non-sterilized* intervention of a *temporary* character, is

$$ds_{t_0} = x_{1,bm} df^m_{t_0}, \tag{A 16}$$

which gives

$$\frac{ds_{t_0}}{df^m_{t_0}} = x_{1,bm} = \frac{1}{1 + \beta(1+\theta)} \tag{A 17}$$

and, thus

$$\left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{bm}_{temp,non-ster} = \frac{1}{1 + \beta(1+\theta)} < 1. \tag{A 18}$$

The effect of a *non-sterilized* intervention of a *permanent* character is

$$\begin{aligned} ds_{t_0} &= x_{1,bm} \sum_{k=0}^{\infty} x^k_{4,bm} E\left(df^m_{t_0+k}\right) \qquad \text{(A 19)} \\ &= x_{1,bm} df^m_{t_0} \sum_{k=0}^{\infty} x^k_{4,bm} \end{aligned}$$

which, since $x_{4,bm} = \frac{\beta}{1+\beta(1+\theta)} < 1$ and $\frac{ds_{t_0}}{df^m_{t_0}}$ may be calculated as a convergent infinite geometric series, gives

$$\frac{ds_{t_0}}{df^m_{t_0}} = \frac{x_{1,bm}}{1 - x_{4,bm}} = \frac{\frac{1}{1+\beta(1+\theta)}}{1 - \frac{\beta}{1+\beta(1+\theta)}}. \tag{A 20}$$

Hence,

$$\left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{bm}_{perm,non-ster} = \frac{1}{\beta\theta + 1} < 1. \tag{A 21}$$

Derivation of the heterogeneous expectations model

From (7) in the main text, we have

$$s_t = m_t - m_t^* + \beta\left(s_{t+1}^e - s_t + \theta\left(c + b_t - b_t^* - s_t\right)\right). \tag{A 22}$$

Combining (A 22) with market expectations (12) in the main text gives

$$s_t = x_{1,hm} f_t^m - x_{2,hm} s_{t-1} + x_{3,hm} f_t^b + x_{4,hm} E\left(s_{t+1}\right), \tag{A 23}$$

where

$$\begin{cases} x_{1,hm} = \frac{1}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})} \\ x_{2,hm} = \frac{\beta e^{-\tau}\gamma}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})} \\ x_{3,hm} = \frac{\beta\theta}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})} \\ x_{4,hm} = \frac{\beta\left(1-e^{-\tau}\right)}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})}, \end{cases} \tag{A 24}$$

and f_t^m and f_t^b are given by (A 4) and (A 5), respectively. Following the method of undetermined coefficients (the UC method), (Blanchard and Khan [4], Blanchard and Fisher [3]), the form of the general solution for the model, based upon repeated substitution, is

$$s_t = -\lambda s_{t-1} + \sum_{k=0}^{\infty} c_k E_t\left(f_{t+k}^m\right) + \sum_{k=0}^{\infty} d_k E_t\left(f_{t+k}^b\right). \tag{A 25}$$

The UC method requires finding values of λ, c_k, d_k that make (A 25) a solution to (A 23). We start by iterating (A 25) one step ahead, to get an expression for $E_t\left(s_{t+1}\right)$

$$E_t(s_{t+1}) = -\lambda s_t + \sum_{k=0}^{\infty} c_k E_t\left(f_{t+k+1}^m\right) + \sum_{k=0}^{\infty} d_k E_t\left(f_{t+k+1}^b\right). \tag{A 26}$$

Substituting (A 26) into (A 23), gives

$$\begin{aligned} s_t \quad = \quad & x_{4,hm}\left[-\lambda s_t + \sum_{k=0}^{\infty} c_k E_t\left(f_{t+k+1}^m\right) + \sum_{k=0}^{\infty} d_k E_t\left(f_{t+k+1}^b\right)\right] \qquad \text{(A 27)} \\ & -x_{2,hm} s_{t-1} + x_{1,hm} f_t^m + x_{3,hm} f_t^b. \end{aligned}$$

Finally, solving (A 27) for s_t, gives

$$s_t = \frac{1}{(1+x_{4,hm}\lambda)} \left(\begin{array}{c} x_{4,hm} \left[\sum_{k=0}^{\infty} c_k E_t \left(f^m_{t+k+1} \right) + \sum_{k=0}^{\infty} d_k E_t \left(f^b_{t+k+1} \right) \right] \\ -x_{2,hm} s_{t-1} + x_{1,hm} f^m_t + x_{3,hm} f^b_t \end{array} \right). \tag{A 28}$$

For (A 25) to be a solution to (A 23), then (A 28) must be equal to (A 25), i.e., the coefficients before the variables s, f^b, f^m must be equal in both equations. Thus, setting (A 25) equal to (A 28) and solving for the undetermined coefficients, gives for s_{t-1}:

$$-\lambda = -\frac{x_{2,hm}}{(1+x_{4,hm}\lambda)}, \tag{A 29}$$

which gives the characteristic equation

$$\lambda^2 + \frac{1}{x_{4,hm}}\lambda - \frac{x_{2,hm}}{x_{4,hm}} = 0, \tag{A 30}$$

where the roots are

$$\lambda_1, \lambda_2 = -\frac{1}{2x_{4,hm}} \pm \sqrt{\frac{(\frac{1}{x_{4,hm}})^2}{4} - \frac{-x_{2,hm}}{x_{4,hm}}}. \tag{A 31}$$

Depending on the type of chartist expectations, the roots change slightly. Specifically, the roots are

$$\begin{aligned} \lambda_1^{BW} &= -\frac{1+\beta(1-e^{-\tau}+\theta-\gamma e^{-\tau})}{2\beta\,(1-e^{-\tau})} \qquad \text{(A 32a)} \\ &\quad +\sqrt{\left(\frac{1+\beta(1-e^{-\tau}+\theta-\gamma e^{-\tau})}{2\beta\,(1-e^{-\tau})}\right)^2 + \frac{\gamma e^{-\tau}}{1-e^{-\tau}}}, \end{aligned}$$

$$\begin{aligned} \lambda_2^{BW} &= -\frac{1+\beta(1-e^{-\tau}+\theta-\gamma e^{-\tau})}{2\beta\,(1-e^{-\tau})} \qquad \text{(A 32b)} \\ &\quad -\sqrt{\left(\frac{1+\beta(1-e^{-\tau}+\theta-\gamma e^{-\tau})}{2\beta\,(1-e^{-\tau})}\right)^2 + \frac{\gamma e^{-\tau}}{1-e^{-\tau}}}, \end{aligned}$$

where the superscript BW denotes bandwagon expectations, and

$$\begin{aligned}\lambda_1^{REG} &= -\frac{1+\beta(1-e^{-\tau}+\theta+\gamma e^{-\tau})}{2\beta\,(1-e^{-\tau})} \\ &+\sqrt{\left(\frac{1+\beta(1-e^{-\tau}+\theta+\gamma e^{-\tau})}{2\beta\,(1-e^{-\tau})}\right)^2-\frac{\gamma e^{-\tau}}{1-e^{-\tau}}},\end{aligned} \tag{A 32c}$$

$$\begin{aligned}\lambda_2^{REG} &= -\frac{1+\beta(1-e^{-\tau}+\theta+\gamma e^{-\tau})}{2\beta\,(1-e^{-\tau})} \\ &-\sqrt{\left(\frac{1+\beta(1-e^{-\tau}+\theta+\gamma e^{-\tau})}{2\beta\,(1-e^{-\tau})}\right)^2-\frac{\gamma e^{-\tau}}{1-e^{-\tau}}},\end{aligned} \tag{A 32d}$$

where the superscript REG denotes regressive expectations. Given that $\gamma e^{-\tau} \leqslant 1 - e^{-\tau}$, which gives $\tau \geqslant \ln(1+\gamma)$, and that $\beta \leqslant 1$, the stable roots are λ_1^{BW} and λ_1^{REG}, see the following proofs by contradiction.

In shorthand notation

$$\lambda_1^{BW} = -A + \sqrt{A^2 + \frac{\gamma e^{-\tau}}{1-e^{-\tau}}} \geqslant 1 \tag{A 33a}$$

which gives

$$\begin{aligned}\sqrt{A^2 + \frac{\gamma e^{-\tau}}{1-e^{-\tau}}} &\geqslant 1 + A \\ A^2 + \frac{\gamma e^{-\tau}}{1-e^{-\tau}} &\geqslant 1 + A^2 + 2A \\ \frac{\gamma e^{-\tau}}{1-e^{-\tau}} &\geqslant 1 + 2A,\end{aligned} \tag{A 33b}$$

which is a contradiction given that $\gamma e^{-\tau} \leqslant 1 - e^{-\tau}$. Thus, the root λ_1^{BW} is stable and

$$0 < \lambda_1^{BW} < 1. \tag{A 33c}$$

In shorthand notation

$$\lambda_1^{REG} = -B + \sqrt{B^2 - \frac{\gamma e^{-\tau}}{1-e^{-\tau}}} \leqslant -1 \tag{A 34a}$$

which gives

$$\begin{aligned}\sqrt{B^2 - \frac{\gamma e^{-\tau}}{1-e^{-\tau}}} &\leqslant -(1+B) \\ B^2 - \frac{\gamma e^{-\tau}}{1-e^{-\tau}} &\leqslant 1 + B^2 - 2B \\ -\frac{\gamma e^{-\tau}}{1-e^{-\tau}} &\leqslant 1 - 2B \\ 1 + \frac{\gamma e^{-\tau}}{1-e^{-\tau}} &\geqslant 2B\end{aligned} \tag{A 34b}$$

which is a contradiction given that $\gamma e^{-\tau} \leqslant 1 - e^{-\tau}$, and that $\beta \leqslant 1$, since

$$B = \frac{1 + \beta(1 - e^{-\tau} + \theta + \gamma e^{-\tau})}{2\beta\,(1 - e^{-\tau})} > 1, \qquad \text{if } \beta \leqslant 1,$$

$$\begin{aligned}\frac{1 + \beta(1 - e^{-\tau} + \theta + \gamma e^{-\tau})}{2\beta\,(1 - e^{-\tau})} - 1 &> 0 \\ \frac{1 + \beta(1 - e^{-\tau} + \theta + \gamma e^{-\tau})}{2\beta\,(1 - e^{-\tau})} - \frac{2\beta\,(1 - e^{-\tau})}{2\beta\,(1 - e^{-\tau})} &> 0 \\ \frac{1 + \beta - \beta e^{-\tau} + \beta\theta + \beta\gamma e^{-\tau} - 2\beta + 2\beta e^{-\tau}}{2\beta - 2\beta e^{-\tau}} &> 0 \\ \frac{1 - \beta + \beta e^{-\tau} + \beta\theta + \beta\gamma e^{-\tau}}{2\beta - 2\beta e^{-\tau}} &> 0.\end{aligned}$$

Assuming $\theta = 0$, and disregarding the denominator as it is positive, we have

$$1 + \beta(e^{-\tau} + \gamma e^{-\tau} - 1) > 0$$

thus, if $\beta \leqslant 1$, this inequality holds.

Accordingly, the root λ_1^{REG} is stable and

$$-1 < \lambda_1^{REG} < 0. \tag{A 34c}$$

The unstable roots are λ_2, which follows from the proofs above, and thus,

$$\lambda_2^{BW},\, \lambda_2^{REG} < -1. \tag{A 35}$$

To continue solving the undetermined coefficients, the rest are as follows:

- For f_t^m:

$$c_0 = \frac{x_{1,hm}}{(1 + x_{4,hm}\lambda_1)}, \tag{A 36a}$$

and for $E(f^m_{t+1}\,|t)$:

$$c_1 = \frac{x_{4,hm}}{(1 + x_{4,hm}\lambda_1)} c_0, \tag{A 36b}$$

and finally for $E(f^m_{t+1+k}\,|t)$:

$$c_k = \frac{x_{4,hm}}{(1 + x_{4,hm}\lambda_1)} c_{k-1}. \tag{A 36c}$$

- For f^b_t:

$$d_0 = \frac{x_{3,hm}}{(1 + x_{4,hm}\lambda_1)}, \tag{A 37a}$$

and for $E(f^b_{t+1}\,|t)$:

$$d_1 = \frac{x_{4,hm}}{(1 + x_{4,hm}\lambda_1)} d_0, \tag{A 37b}$$

and finally for $E(f^b_{t+1+k}\,|t)$:

$$d_k = \frac{x_{4,hm}}{(1 + x_{4,hm}\lambda_1)} d_{k-1}. \tag{A 37c}$$

Substituting the expressions for the new coefficients into the solution for (A 23), i.e., into (A 25), and using the stable root λ_1, gives

$$\begin{aligned} s_t \quad = \quad & \lambda_1 s_{t-1} + \frac{x_{1,hm}}{(1 + x_{4,hm}\lambda_1)} \sum_{k=0}^{\infty} \left(\frac{x_{4,hm}}{(1 + x_{4,hm}\lambda_1)} \right)^k E_t\left(f^m_{t+k}\right) \quad \text{(A 38)} \\ & + \frac{x_{3,hm}}{(1 + x_{4,hm}\lambda_1)} \sum_{k=0}^{\infty} \left(\frac{x_{4,hm}}{(1 + x_{4,hm}\lambda_1)} \right)^k E_t\left(f^b_{t+k}\right). \end{aligned}$$

Equation (A 37) is the solution of the model when there are both chartists and fundamentalists in the market, i.e., the heterogeneous expectations model. Thus, the derivation of the heterogeneous expectations model given in (16)-(18) in the main text is completed.

The effects of intervention in the heterogeneous expectations model

The following computations derive the effects of different types of intervention on the current exchange rate in the heterogeneous expectations model. It is assumed that the current exchange rate is given by (16)-(18), and that the central bank intervention takes place at time $t = t_0$. In the heterogeneous expectations model, the effects of intervention will differ depending on which type of expectations chartists have when the intervention takes place. Thus, the effects of intervention are presented separately in each case for different type of expectations. Henceforth, BW denotes bandwagon expectations and REG denotes regressive expectations among chartists. All the effects derived below are presented in Tables 2A and 2B, subsection 3.3.

The effect on the current exchange rate of a *sterilized* intervention of a *temporary* character, is

$$ds_{t_0} = \frac{x_3}{(1+x_4\lambda_1)} E(df^b_{t_0}), \tag{A 39}$$

which gives

$$\begin{aligned} \frac{ds_{t_0}}{df^b_{t_0}} &= \frac{x_3}{(1+x_4\lambda_1)} \\ &= \frac{\frac{\beta\theta}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})}}{1+\frac{\beta(1-e^{-\tau})}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})}\lambda_1} \end{aligned} \tag{A 40}$$

and thus, depending on the type of chartist expectations

$$\begin{aligned} BW: \quad & \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{hm}_{temp,ster} = \frac{\theta\beta}{1+\beta(\theta-\gamma e^{-\tau})+\beta(1-e^{-\tau})(1+\lambda_1^{bw})} < 1 \\ REG: \quad & \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{hm}_{temp,ster} = \frac{\theta\beta}{1+\beta(\theta+\gamma e^{-\tau})+\beta(1-e^{-\tau})(1+\lambda_1^{reg})} < 1 \end{aligned} \tag{A 41}$$

The effect on the current exchange rate of a *sterilized* intervention of a *permanent* character, is

$$ds_{t_0} = \frac{x_3}{(1+x_4\lambda_1)} \sum_{k=0}^{\infty} \left(\frac{x_4}{(1+x_4\lambda_1)}\right)^k E\left(df^b_{t_0}\right), \tag{A 42}$$

since $\frac{x_4}{(1+x_4\lambda_1)} = -\frac{1}{\lambda_2}$ and $|\lambda_2| > 1$, $\frac{x_4}{(1+x_4\lambda_1)} < 1$, thus $\sum_{k=0}^{\infty}\left(\frac{x_4}{(1+x_4\lambda_1)}\right)^k$ may be calculated as a convergent infinite geometric series.[14] This gives

$$ds_{t_0} = \frac{x_3}{(1+x_4\lambda_1)}\frac{1}{1-\frac{x_4}{(1+x_4\lambda_1)}}E(df^b_{t_0}), \tag{A 43}$$

and thus,

$$\begin{aligned}\frac{ds_{t_0}}{df^b_{t_0}} &= \frac{\frac{x_3}{(1-x_4\lambda_1)}}{1-\frac{x_4}{(1-x_4\lambda_1)}} \qquad \text{(A 44)}\\ &= \frac{x_3}{1+x_4\lambda_1 - x_4}\\ &= \frac{\frac{\beta\theta}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})}}{1+\frac{\beta(1-e^{-\tau})}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})}\lambda_1 - \frac{\beta(1-e^{-\tau})}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})}}.\end{aligned}$$

Depending on the type of chartist expectations

$$\begin{aligned} BW: \quad & \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{hm}_{perm,ster} = \frac{\theta\beta}{1+\beta(\theta-\gamma e^{-\tau})+\beta(1-e^{-\tau})\lambda_1^{bw}} \gtrless 1\\ REG: \quad & \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{hm}_{perm,ster} = \frac{\theta\beta}{1+\beta(\theta+\gamma e^{-\tau})+\beta(1-e^{-\tau})\lambda_1^{reg}} \gtrless 1 \end{aligned} \tag{A 45}$$

The effect of a *non-sterilized* intervention of a *temporary* character, is

$$ds_{t_0} = \frac{x_{1,hm}}{(1+x_{4,hm}\lambda_1)}E(df^m_{t_0}), \tag{A 46}$$

which gives

$$\begin{aligned}\frac{ds_{t_0}}{df^m_{t_0}} &= \frac{x_{1,hm}}{(1+x_{4,hm}\lambda_1)} \qquad \text{(A 47)}\\ &= \frac{\frac{1}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})}}{1+\frac{\beta(1-e^{-\tau})}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})}\lambda_1}\end{aligned}$$

and thus, depending on the type of chartist expectations

[14] Since $\lambda_1 + \lambda_2 = -\frac{1}{x_4}$, which gives $\lambda_2 = -\frac{1}{x_4} - \lambda_1$, and $-\lambda_2 = \frac{1+x_4\lambda_1}{x_4}$ and thus, $-\frac{1}{\lambda_2} = \frac{x_4}{1+x_4\lambda_1}$.

$$\begin{aligned} BW: \quad & \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{hm}_{temp,non-ster} = \frac{1}{1+\beta(\theta-\gamma e^{-\tau})+\beta(1-e^{-\tau})\left(1+\lambda_1^{bw}\right)} < 1 \\ REG: \quad & \left.\frac{ds_{t_0}}{df^b_{t_0}}\right|^{hm}_{temp,non-ster} = \frac{1}{1+\beta(\theta+\gamma e^{-\tau})+\beta(1-e^{-\tau})\left(1+\lambda_1^{reg}\right)} < 1 \end{aligned} \tag{A 48}$$

The effect on the current exchange rate of a *non-sterilized* intervention of a *permanent* character, is

$$ds_{t_0} = \frac{x_1}{(1+x_4\lambda_1)}\sum_{k=0}^{\infty}\left(\frac{x_4}{(1+x_4\lambda_1)}\right)^k E\left(df^m_{t_0}\right), \tag{A 49}$$

and again, since $\frac{x_4}{(1+x_4\lambda_1)} = -\frac{1}{\lambda_2}$ and $|\lambda_2| > 1$, $\frac{x_4}{(1+x_4\lambda_1)} < 1$, thus $\sum_{k=0}^{\infty}\left(\frac{x_4}{(1-x_4\lambda_1)}\right)^k$ may be calculated as a convergent infinite geometric series. This gives

$$ds_{t_0} = \frac{x_1}{(1+x_4\lambda_1)}\frac{1}{1-\frac{x_4}{(1+x_4\lambda_1)}}df^m_{t_0}, \tag{A 50}$$

and thus,

$$\begin{aligned} \frac{ds_{t_0}}{df^m_{t_0}} &= \frac{\frac{x_1}{(1+x_4\lambda_1)}}{1-\frac{x_4}{(1+x_4\lambda_1)}} \\ &= \frac{x_1}{1+x_4\lambda_1-x_4} \\ &= \frac{\frac{1}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})}}{1+\frac{\beta(1-e^{-\tau})}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})}\lambda_1-\frac{\beta(1-e^{-\tau})}{1+\beta(1+\theta-e^{-\tau}-\gamma e^{-\tau})}}. \end{aligned} \tag{A 51}$$

Depending on the type of chartist expectations,

$$\begin{aligned} BW: \quad & \left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{hm}_{perm,non-ster} = \frac{1}{1+\beta(\theta-\gamma e^{-\tau})+\beta(1-e^{-\tau})\lambda_1^{bw}} \lesseqgtr 1 \\ REG: \quad & \left.\frac{ds_{t_0}}{df^m_{t_0}}\right|^{hm}_{perm,non-ster} = \frac{1}{1+\beta(\theta+\gamma e^{-\tau})+\beta(1-e^{-\tau})\lambda_1^{reg}} \lesseqgtr 1 \end{aligned} \tag{A 52}$$

This completes the Appendix.

Robust Taylor Rules in an Open Economy with Heterogeneous Expectations*

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Abstract

The aim of this paper is threefold: (i) to investigate if there is a unique REE in the small open economy in Galí and Monacelli [10] that is augmented with technical trading in the foreign exchange market; (ii) to investigate if the unique REE is adaptively learnable in recursive least squares sense; and (iii) to investigate if the unique and adaptively learnable REE is desirable in an inflation rate targeting regime in the sense that a low and not too variable CPI inflation rate in equilibrium is achieved. The monetary authority is using a Taylor rule when setting the nominal interest rate, and we investigate the properties of the model developed numerically. A main conclusion is that the monetary authority should increase the interest rate when the CPI inflation rate increases and when the currency gets stronger to have a desirable rule that is robust with respect to the degree of technical trading in the foreign exchange market. Thus, the value of the currency is a better response variable than the output gap in the most desirable parametrizations of the interest rate rule.

JEL codes: E52; F31.

Keywords: Determinacy; Foreign Exchange; Inflation Rate Targeting Regime; Taylor Rule; Robust Monetary Policy; Technical Trading.

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1 Introduction

Background During the last two decades, a new paradigm in monetary policy has evolved. This paradigm concerns independent central banks, openness and inflation rate targeting. In other words, monetary policy is conducted by the central bank without political influence, with the purpose of creating price-stability and credibility to evade the time-inconsistency problem. Further on, monetary policy is conducted through interest rate managing with an explicit target for the inflation rate. This new paradigm has been developed almost without any guidance from the academic literature (see p. 3 in Woodford [21]). However, since this practise nowadays is established among central banks of the industrialized countries, the literature within this area is flourishing.

In 1993, John B. Taylor [16] demonstrated that the monetary policy of the Federal Reserve could be described by the following interest rate rule

$$r_t = 0.04 + 1.5\,(\pi_t - 0.02) + 0.5\,(y_t - \overline{y})\,, \tag{1}$$

where r_t is the Federal Reserve's operating target for the funds rate, π_t is the inflation rate according to the GDP deflator, y_t is the logarithm of real GDP, and $\overline{y}$ is the logarithm of potential real GDP. This kind of rule has been the center of attention within the monetary policy literature since it was presented and is often referred to as a Taylor rule. In particular, the Taylor rule in (1) prescribes setting an operating interest rate target in response to the inflation rate and the (output) gap between the logarithm of real GDP and the logarithm of potential real GDP.

The key question in the literature is whether this type of interest rate rule, which does not incorporate a target path for the monetary aggregates, can control the price level and create price-stability. In other words, the success of this type of monetary policy rule hinges on the central bank's ability to shape market expectations of future interest rates, inflation rates and income levels. It is, therefore, important for the central bank to commit to the rule, be as transparent as possible in its decision making, and make the correct policy-decisions as often as possible. Taylor [17] also argues that since the interest rate rule in (1) describes the Federal Reserve's policy during a successful period, one should adopt a rule like this in policy-making in which the interest rate is set in response to the inflation rate and the output gap.

However, since most countries trade extensively with other countries, and, therefore, should be considered as open economies, one might ask whether some exchange rate index also should be included in the monetary policy rule. Taylor [18] does not think so, and the reason is that

> "... rules that react directly to the exchange rate ... sometimes work *worse* than policy rules that do not react directly to the exchange rate" (p. 267, italics added).

Instead, Taylor [18] argues that the indirect effect that exchange rates have on monetary policy, via its effect on the inflation rate and the output gap, is to prefer since it results in fewer and less erratic changes in the interest rate.

Our model In this paper, two types of Taylor rules are embedded in a theoretical framework consisting of a dynamic IS-type equation, a new Keynesian Phillips curve, and a parity condition at the international asset market. The first rule is a contemporaneous data specification of the output gap, the inflation rate and the change in an exchange rate index (that, in the analysis below, consists of a single exchange rate), whereas the second rule is a contemporaneous expectations specification of the same variables. Further on, technical trading is incorporated into the foreign exchange market in the form of extrapolation of trends in the exchange rate index, and the reason is that several questionnaire surveys made at currency markets around the world confirm that technical trading, or chartism, is extensively used in currency trade.

Examples of questionnaire surveys include Cheung and Chinn [7], who conducted a survey at the U.S. market; Lui and Mole [11], who conducted a survey at the Hong Kong market; Menkhoff [13], who conducted a survey at the German market; Oberlechner [14], who conducted surveys at the markets in Frankfurt, London, Vienna and Zurich; and Taylor and Allen [19], who conducted a survey at the London market. An extensive exploration of the trading behavior at the foreign exchange market is also found in Oberlechner [15] that is based on surveys conducted at the European and the North American markets.

Thus, we include the change in an exchange rate index into the monetary authority's interest rate rule, even though Taylor [18] claims that this kind of rule might worsen the outcome of monetary policy. However, as also is argued in Taylor [18], more research is needed to investigate whether this claim holds in all types of models, and our contribution to the literature is to examine to what extent monetary policy *is* and *should be* affected when currency trade is partly driven by chartism.

Our approach It is well-known that models in economics and finance, in which agents have rational expectations regarding some of the variables in the model, may exhibit a multiplicity of rational expectations equilibria (REE). This is problematic. For instance, without imposing additional restrictions into such a model, it is not known in advance which of the REE that the agents will coordinate on, if there will be any coordination at all. To give an example, the effects of monetary policy is not known beforehand: is it the case that the agents will coordinate on an equilibrium that has undesirable properties, like a too high inflation rate, or an equilibrium with a low inflation rate?

Therefore, after augmenting the small open economy in Galí and Monacelli [10] with technical trading in the foreign exchange market, we explore for which parameter values we have Taylor rules that give rise to determinacy, i.e., a unique REE. Further on, which is a self-evident fact, but often neglected in the literature, is that a unique REE is not the same as a desirable REE. For this reason, we check whether the REE is desirable in an inflation rate targeting regime. In other words, is the unique inflation rate low enough and not too variable in equilibrium?

In between the questions on determinacy and the desirability of the inflation rate in equilibrium, we investigate if the REE is adaptively learnable in recursive least squares sense. The reason is that rational expectations is a rather strong assumption since it assumes that agents often have an outstanding capacity when it comes to deriving equilibrium outcomes of the variables in a model. This assumption has, therefore, in the more recent literature, been complemented by an analysis of the possible convergence to the REE (see Evans and Honkapohja [9] for an introduction to this literature).

To be more precise, it is assumed that expectations are formed by a correctly specified model, i.e., a model that nests the REE, but without having perfect knowledge about the parameter values in the model. However, using past and current values of all variables in the model, the parameter values are learned over time since the beliefs are revised as new information is gained. Thus, the question in focus is this: will the agents learn the parameter values in the model that corresponds to the unique REE?

Even though questionnaire surveys made at foreign exchange markets around the world demonstrate that technical trading techniques are used extensively in currency trade, it is not obvious to what extent these techniques are used at each moment in time. Clearly, the aforementioned surveys reveal an inverse relationship between the extent of chartism and the time horizon in currency trade, but the exact proportion of technical trading is still not known when conducting monetary policy. Therefore, to find robust parametrizations of the Taylor rules, the desirable properties of a rule should be relatively unaffected by the degree of technical trading in the foreign exchange market.

Finally, since the model developed is too large for theoretical analysis, we have to illustrate our findings numerically.[1] Specifically, we use calibrated values of the structural parameters in our model that are found in other papers within this research area (see Bullard and Mitra [4] and references therein).

Relation to the literature To slightly simplify the picture, there are two strands of literature that explore the effects of monetary policy in the new Keynesian framework. In the first strand of literature, an optimal policy rule for the monetary authority is derived via optimization of a welfare function, but the conditions for determinacy and adaptive learnability of the REE are often neglected (see, e.g., Galí and Monacelli [10]). In the second strand of literature, the focus is on finding parametrizations of Taylor rules that give rise to a unique REE that also is adaptively learnable in recursive least squares sense. However, the interest rate rules that satisfy these criteria are not evaluated using a welfare function as the metric (see, e.g., Bullard and Mitra [4]).

Our paper fills the gap between the two aforementioned papers since we, like

[1] MATLAB routines for this purpose are available on request from the authors.

Bullard and Mitra [4], search for Taylor rules that are associated with a unique and an adaptively learnable inflation rate in equilibrium, but also, like Galí and Monacelli [10], evaluate this equilibrium using a loss-function. However, a discrepancy between our paper and papers in which optimal monetary policy rules are derived is that we restrict the search for the most desirable rules among those rules that give rise to determinacy and adaptive learnability of the REE. The loss-function that we make use of in the analysis concerns the expected inflation rate and the conditional volatility of the inflation rate in equilibrium.

Our main finding Contrary to what Taylor [18] claims, we find parametrizations of interest rate rules with robust and desirable properties that include the change in an exchange rate index. Further on, these rules do not include the output gap, which might be an advantage since it comes closer to the reality of central banking. To be more specific, due to data revisions, it is often the case that policy-makers do not have the correct information on a variable such as real GDP when needed. This is even more true when it comes to a variable such as potential real GDP.

Of course, one should not take our finding that the Taylor rule should include the change in an exchange rate index to be robust and desirable too literally since this result relies on calibrated values of the structural parameters in the model. Instead, our message is this, if we travesty the quote by Taylor [18]:

> Monetary policy rules that react directly to the exchange rate, or an exchange rate index, sometimes work *better* than policy rules that do not react directly to such quantities.

It is self-evident that future research should explore the robustness of our finding.

A caveat There are not too many papers that incorporate chartism in a foreign exchange model, and we believe there are two reasons for this. The first reason is that many researchers do not believe that currency traders using technical analysis can survive in the market, and the second reason is that even if some of these researchers are aware of the use of chartism in currency trade, most of them argue that it is of uttermost importance to explain *why* these traders survive in the market.

We are sympathetic to this standpoint, but we also believe that this may be a hindrance to a better understanding of the *effects* of technical trading in the foreign exchange market since it is not easy to develop theoretical models that satisfactorily explain human behavior at the currency market or at any financial asset market.

Outline of the paper The theoretical framework is outlined in Section 2, whereas the search for robust Taylor rules with desirable properties is in focus in Section 3. The paper is concluded in Section 4, and the Appendix contains technical details.

2 Theoretical framework

Our theoretical framework consists of three parts: (i) the small open economy in Galí and Monacelli [10], which is our baseline model; (ii) equations that describe the trading behavior at the foreign exchange market; and (iii) a Taylor rule for the monetary authority. Due to the findings in Bullard and Mitra [4], two types of Taylor rules are investigated: (i) a contemporaneous data specification; and (ii) a contemporaneous expectations specification. Specifically, these two types of Taylor rules have appealing properties in a closed economy, and our aim is to investigate if these rules still have appealing properties in an open economy. The three parts are outlined in Sections 2.1-2.3, respectively.

2.1 Baseline model

Basically, the Galí and Monacelli [10] model is a dynamic stochastic general equilibrium model with imperfect competition and nominal rigidities. In their model, the world economy is represented by a continuum of infinitely small economies, meaning that since each economy is of measure zero, its policy decisions do not have any impact on the rest of the world. Consequently, there is no room for strategic behavior in monetary policy-making. It is also assumed that the economies share identical household preferences, firm technology and market structure, while different economies are subject to correlated productivity shocks. Finally, firms set prices in a staggered fashion as in Calvo [6].

After extensive derivations, the Galí and Monacelli [10] model can be reduced to a dynamic IS-type equation and a new Keynesian Phillips curve

$$\begin{cases} x_t = x^e_{t+1} - \alpha\left(r_t - \pi^e_{H,t+1} - \overline{rr}_t\right) \\ \pi_{H,t} = \beta\pi^e_{H,t+1} + \gamma x_t, \end{cases} \tag{2}$$

where x_t is the output gap, r_t is the nominal interest rate, $\pi_{H,t}$ is the domestic inflation rate, and $\overline{rr}_t$ is the natural rate of interest. To be more specific, the output gap is the deviation of output from its natural level, where the latter is output in the absence of nominal rigidities. The domestic inflation rate is the rate of change in the index of domestic goods prices, and the natural rate of interest is the real interest rate that is consistent with output's natural level. Finally, the superscript e denotes expectations. (In Section 2.2, we will discuss how expectations are formed in the model.)

For our purpose, (2) is not in an appropriate form since there are no expected exchange rate terms in the equations. These terms are necessary when modeling the trading behavior at the foreign exchange market. It is, however, possible to use the following equations, which are derived in Galí and Monacelli [10], to rewrite (2) into a suitable form

$$\begin{cases} \pi_t = \pi_{H,t} + \delta\Delta s_t \\ s_t = e_t + p^*_t - p_{H,t}, \end{cases} \tag{3}$$

where π_t is the CPI inflation rate, s_t is the terms of trade, e_t is the nominal exchange rate (or, more broadly, an exchange rate index), p^*_t is the index of foreign goods prices, and $p_{H,t}$ is the index of domestic goods prices. Specifically, the terms of trade is the relative price of the home country's import goods in terms of its domestically produced goods, and the CPI inflation rate is the rate of change in the index of goods prices. Thus, the difference between the two measures of the inflation rate, $\pi_{H,t}$ and π_t, is that the former measure is based on all prices for domestically produced goods, whereas the latter measure is based on all prices within the home country, imported goods included. CPI is also an abbreviation for consumer price index. Finally, the asterisk denotes a foreign quantity.

Now, if we rewrite the equations in (2) with help of those in (3), we get two of the equations that form our baseline model[2]

$$\begin{cases} x_t = x_{t+1}^e - \alpha\left(r_t - \frac{1}{1-\delta}\cdot\left(\pi_{t+1}^e - \delta\left(\Delta e_{t+1}^{e,m} + \pi_{t+1}^{e,*}\right)\right) - \overline{rr}_t\right) \\ \pi_t = \beta\pi_{t+1}^e + \gamma\left(1-\delta\right)x_t + \delta\left(\Delta e_t - \beta\Delta e_{t+1}^{e,m} + \pi_t^* - \beta\pi_{t+1}^{e,*}\right), \end{cases} \tag{4}$$

where the superscript e, m denotes (aggregated) expectations at the foreign exchange market. The third equation in the baseline model, which also is derived in Galí and Monacelli [10], is the condition for uncovered interest rate parity (UIP)

$$r_t - r_t^* = \Delta e_{t+1}^{e,m}. \tag{5}$$

Thus, (5) is a parity condition at the international asset market. Finally, we assume that the natural rate of interest is governed by the following stochastic process

$$\overline{rr}_t = \rho\overline{rr}_{t-1} + \varepsilon_t, \tag{6}$$

where $0 \leq \rho < 1$ is the serial correlation in the process, and $\varepsilon_t \in IID\left(0, \sigma_\varepsilon^2\right)$. To sum up, (4)-(6) is the complete baseline model that will be augmented with equations that describe the trading behavior at the foreign exchange market as well as a Taylor rule for the monetary authority. Note that the stochastic process in (6) also is assumed to hold in Bullard and Mitra [4].

At this stage, let us say a few words about the structural parameters in our baseline model. $\beta > 0$ is the discount factor that is used when the representative household in the home country maximizes a discounted sum of instantaneous utilities derived from consumption and leisure. $\delta \in [0, 1]$ is the share of consumption in the home country allocated to imported goods, meaning that δ is an index of openness of the economy. For example, the equations in (4) reduces to those in Bullard and Mitra [4] when $\delta = 0$ since the home country is a closed economy in this case.

[2] See the Appendix for the derivation of (4).

The other two parameters in the model, α and γ, are not that easy to interpret since they are functions of structural parameters in the Galí and Monacelli [10] model. Shortly, α depends on four parameters: (i) the openness index, δ; (ii) the intertemporal elasticity of substitution in consumption; (iii) the elasticity of substitution between domestic and foreign goods in consumption; and (iv) the elasticity of substitution between foreign goods in consumption. Moreover, γ depends on α as well as three other parameters: (i) the discount factor, β; (ii) the intertemporal elasticity of substitution in labor supply; and (iii) the share of firms that set (new) prices in each time period (see Calvo [6]).

Since we investigate the properties of the model developed numerically, we do not need to emphasize the exact relationships between the structural parameters in our baseline model and the structural parameters in the Galí and Monacelli [10] model. Of course, to fully grasp the micro-foundations in the baseline model and their relationships with the dynamic IS-type equation and the new Keynesian Phillips curve in (4) as well as the UIP condition in (5), it is necessary to consult Galí and Monacelli [10].

2.2 Trading behavior at the foreign exchange market

There are two types of traders in the foreign exchange market; (i) agents who use chartism, or technical analysis, in their trade, meaning that they utilize past exchange rates to detect patterns that are extrapolated into the future; and (ii) agents who use fundamental analysis in their trade, meaning that they have rational expectations regarding the next time period's exchange rate, or, as in our model, the next time period's change in the exchange rate. Thus, these agents know that there are agents who use technical trading techniques in currency trade, and they take this into account when forming their exchange rate expectations.

In this paper, we assume that the chartists use a simple technical trading technique

$$\Delta e_{t+1}^{e,c} = \Delta e_t, \tag{7}$$

i.e., the chartists expect that the nominal exchange rate will continue to increase (decrease) in the next time period, if it has increased (decreased) in the current time period. To be more specific, if the exchange rate increased (decreased) between time periods $t-1$ and t, the chartists believe that the exchange rate also will increase (decrease) between time periods t and $t+1$. Moreover, to keep the structural parameters in the model developed as few as possible, it is assumed that these two consecutive increases (decreases) in the exchange rate are of the same size. Finally, the superscript e, c denotes chartist expectations.

Then, if we move on to the fundamentalists, it is assumed that they have rational expectations regarding the change in the nominal exchange rate

$$\Delta e_{t+1}^{e,f} = \Delta e_{t+1}^{e}, \tag{8}$$

which means that the expected change in the exchange rate is equal to the mathematically expected change in the exchange rate, conditioned on all information available to this type of currency trader. This information includes the structure of the complete model as well as past and current values of all variables in the model, meaning that the dating of expectations is time period t. (However, as will be discussed in Section 3.2.2, when a contemporaneous expectations specification of the Taylor rule is used by the monetary authority, we assume that the dating of expectations is time period $t-1$.) Finally, the superscript e, f denotes fundamentalist expectations.

The expected exchange rate terms that appear in (4)-(5) are aggregated expectations at the foreign exchange market. Specifically, these expectations are a weighted average of chartist and fundamentalist expectations

$$\begin{aligned} \Delta e_{t+1}^{e,m} &= \omega \Delta e_{t+1}^{e,c} + (1-\omega)\, \Delta e_{t+1}^{e,f} \\ &= \omega \Delta e_{t} + (1-\omega)\, \Delta e_{t+1}^{e}, \end{aligned} \tag{9}$$

where $\omega \in [0,1]$ is the proportion of chartists in currency trade. Thus, aggregated expectations are a weighted average of the current change in the exchange rate and the next time period's mathematically expected change in the exchange rate. Consequently, as long as there are chartists present in the foreign exchange market, aggregated expectations do not coincide with rational expectations.

2.3 Taylor rules

We will investigate the properties of the complete model using two specifications of the Taylor rule: (i) a contemporaneous data specification of the rule; and (ii) a contemporaneous expectations specification of the rule. Moreover, since the nominal exchange rate or the change in this exchange rate may affect the economy's outcome in equilibrium, a term including the latter variable is included in both types of rules

$$r_t = \zeta_c + \zeta_x x_t + \zeta_\pi \pi_t + \zeta_e \Delta e_t, \tag{10}$$

and

$$r_t = \zeta_c + \zeta_x x_t^e + \zeta_\pi \pi_t^e + \zeta_e \Delta e_t^{e,f}, \tag{11}$$

where also a constant has been added. (In Section 3.3.1, it will be shown that this constant is equal to the foreign nominal interest rate.) Thus, in the Taylor rule in (11), the monetary authority has rational expectations regarding the change in the exchange rate since it is assumed that they behave as the fundamentalists.

3 A unique and desirable REE that is learnable?

Now, after having completed the description of our theoretical framework, we will investigate the properties of the model developed: (i) is there a unique REE in the model?; (ii) is the unique REE characterized by recursive least squares learnability?; and (iii) is the unique and adaptively learnable REE desirable in an inflation rate targeting regime in the sense that the inflation rate is low enough and not too variable in equilibrium? All three questions will be answered, for both specifications of the Taylor rule in (10)-(11), in Sections 3.1-3.3, respectively.

3.1 Determinacy

Let us begin with the question if there are any parametrizations of the Taylor rules in (10)-(11) that give rise to a unique CPI inflation rate in equilibrium.

3.1.1 Contemporaneous data in the Taylor rule

If the Taylor rule in (10) is used when the monetary authority is setting the nominal interest rate, meaning that they respond to current data of the output gap, the CPI inflation rate and change in the nominal exchange rate, the complete model in (4)-(6) and (9)-(10) can be written in matrix form as follows[3]

$$
\begin{aligned}
&\begin{bmatrix} 1+\alpha\zeta_x & \alpha\zeta_\pi & \alpha\left(\frac{\delta\omega}{1-\delta}+\zeta_e\right) \\ \gamma\left(\delta-1\right) & 1 & \delta\left(\beta\omega-1\right) \\ \zeta_x & \zeta_\pi & \zeta_e-\omega \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ \Delta e_t \end{bmatrix} \qquad (12) \\
= &\begin{bmatrix} -\alpha\zeta_c - \frac{\alpha\delta}{1-\delta}\cdot\pi^{e,*}_{t+1} \\ \delta\left(\pi^*_t - \beta\pi^{e,*}_{t+1}\right) \\ r^*_t - \zeta_c \end{bmatrix} + \\
&\begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & \frac{\alpha\delta(\omega-1)}{1-\delta} \\ 0 & \beta & \beta\delta\left(\omega-1\right) \\ 0 & 0 & 1-\omega \end{bmatrix} \begin{bmatrix} x^e_{t+1} \\ \pi^e_{t+1} \\ \Delta e^e_{t+1} \end{bmatrix} + \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix} \cdot \overline{rr}_t.
\end{aligned}
$$

Thus, to have a unique and stable REE, all three eigenvalues of the following coefficient matrix must be inside the unit circle since x_t, π_t and Δe_t are free (see, e.g., Blanchard and Kahn [2])

$$
\begin{aligned}
\mathbf{\Gamma} = &\begin{bmatrix} 1+\alpha\zeta_x & \alpha\zeta_\pi & \alpha\left(\frac{\delta\omega}{1-\delta}+\zeta_e\right) \\ \gamma\left(\delta-1\right) & 1 & \delta\left(\beta\omega-1\right) \\ \zeta_x & \zeta_\pi & \zeta_e-\omega \end{bmatrix}^{-1} \times \qquad (13) \\
&\begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & \frac{\alpha\delta(\omega-1)}{1-\delta} \\ 0 & \beta & \beta\delta\left(\omega-1\right) \\ 0 & 0 & 1-\omega \end{bmatrix}.
\end{aligned}
$$

However, deriving necessary and sufficient conditions for determinacy is not meaningful for practical reasons since these expressions would be too large and cumbersome to interpret. Consequently, we adopt the strategy in Bullard and Mitra [4], and illustrate our findings for determinacy using calibrated values of the structural parameters.

[3] See the Appendix for the derivation of (12).

To be more specific, the following parameter values, or range of values, are used in the analysis that are the same values as in Bullard and Mitra [4]

$$\begin{cases} \alpha = \frac{1}{0.157}, & \beta = 0.99, \qquad \gamma = 0.024, \qquad \delta = 0.2, \\ \rho = 0.35 & 0 \leq \zeta_x \leq 4, \quad 0 \leq \zeta_\pi \leq 10, \quad -5 \leq \zeta_e \leq 5. \end{cases} \tag{14}$$

Of course, the parameters δ, ω and ζ_e do not appear in Bullard and Mitra [4] since their model is for a closed economy. The index of openness of the economy is $\delta = 0.2$. However, to perform a sensitivity analysis of the numerical findings, we will also investigate the case when this index is $\delta = 0.4$.[4] In the former case, the index is slightly larger than the import/GDP ratio in the U.S., and in the latter case, which is the parameter setting in Galí and Monacelli [10], the index corresponds roughly to the import/GDP ratio in Canada and Sweden.

In all figures below[5], the regions in the parameter space of $(\omega, \zeta_x, \zeta_\pi, \zeta_e)$ for which we have a unique REE are shown. Specifically, since ω and ζ_x are given, it is the combinations of ζ_π and ζ_e that are in the light areas in the figures that give rise to determinacy. In Figure 1, there is no technical trading in the foreign exchange market (i.e., $\omega = 0$), meaning that all currency trade is guided by fundamental analysis, and the monetary authority does not take into account the output gap when setting the interest rate (i.e., $\zeta_x = 0$).[6] In Figures 2 a-b, the proportion of chartists in currency trade has increased to 25 percent, meaning that 75 percent of the trade is guided by fundamental analysis, and the parameter value in the Taylor rule that describes the output gap reaction is $\zeta_x = 0$ and $\zeta_x = 2$, respectively. A visible result in Figures 2 a-b is that the region for a unique REE decreases when the monetary authority reacts stronger to the output gap. This is also true when there is no chartism in currency trade, even though we do not show this result explicitly (since there are no determinacy regions for $\zeta_x \geq 2$, which also is true for $\zeta_x = 4$ when $\omega = 0.25$). Moreover,

[4] Detailed results are available on request from the authors.

[5] Figures and Tables are shown after the Appendix.

[6] To keep the number of figures in the paper at a minimum, the regions in the figures are not only the regions for determinacy, but also the regions for adaptive learnability. Thus, as also will be clear in Section 3.2.1, when there is unique REE, the agents that use fundamental analysis, which also includes the monetary authority, will learn this REE.

if we compare Figure 1 and Figure 2 a, the determinacy region is larger when there is technical trading in the foreign exchange market.

In Figures 3 a-c, half of the trade in the foreign exchange market is driven by technical analysis, whereas in Figures 4 a-c, chartism is used in 75 percent of the trade. Finally, in Figures 5 a-c, all trade in foreign exchange is based on technical analysis, meaning that no trade is guided by fundamental analysis. In all these figures, the parameter value that describes the output gap reaction is $\zeta_x = 0$, $\zeta_x = 2$ and $\zeta_x = 4$, respectively. When at least half of the trade in the foreign exchange market is driven by technical analysis, the region for a unique REE increases when the monetary authority reacts stronger to the output gap, which is in contrast with the result in Figures 2 a-b. Moreover, given the output gap reaction in the Taylor rule, the determinacy region gets larger when the proportion of chartists in currency trade increases (even if it seems that the size of the regions are the same in Figure 4 c and Figure 5 c).

What is the intuition behind the result that an increase in technical trading induces a larger determinacy region? There is a similar result in Bullard and Mitra [5] who investigate the conditions for determinacy (and learnability that we will discuss in Section 3.2.1) in a closed economy like the one in Bullard and Mitra [4], where the monetary authority uses a Taylor rule that is augmented with a term that includes the previous time period's nominal interest rate to have policy inertia. Bullard and Mitra [5] conclude that policy inertia helps to alleviate the problem with a multiplicity of REE, and the similarity with our model is that an increase in chartism is a form of increased inertia since there is a larger emphasize on the current exchange rate change instead of the next time period's (mathematically) expected exchange rate change.

We will restrict our discussion about the findings in the figures to the results previously mentioned, and the reason is that we save the conclusions till after we have investigated the robustness and desirability of a specific REE in the perspective of an inflation rate targeting regime. It might, for example, be tempting to conclude that the monetary authority should not react to exchange rate changes, if the reaction to the output gap is strong enough and at least half of the trade in foreign exchange is based on chartism (that is a reliable assumption according to questionnaire surveys). However, as will be clear in

Section 3.3.1, it is not a favorable approach to restrict the parameter ζ_e in the Taylor rule to 0 since there are several parametrizations of the rule that give rise to a better outcome in equilibrium in terms of the expected inflation rate and the conditional volatility of the inflation rate when $\zeta_e < 0$.

Sensitivity analysis When the index of openness of the economy increases from $\delta = 0.2$ to $\delta = 0.4$, none of the findings are affected. That is, when at least half of the trade in the currency market is driven by technical analysis, the determinacy region increases when the monetary authority reacts stronger to the output gap, whereas the opposite is true when less than half of the trade is driven by chartism. Moreover, given the output gap reaction in the Taylor rule, the determinacy region gets larger when the proportion of chartists in currency trade increases. Finally, since we will learn in Section 3.3.1 that robust and desirable Taylor rules do not include a reaction to the output gap, we observe that the determinacy region is smaller when the economy is more open when $\zeta_x = 0$.

3.1.2 Contemporaneous expectations in the Taylor rule

If the Taylor rule in (11) is used when the monetary authority is setting the nominal interest rate, meaning that they respond to current expectations of the output gap, the CPI inflation rate and change in the nominal exchange rate, the complete model in (4)-(6), (9) and (11) can be written in matrix form as follows[7]

$$\begin{bmatrix} 1 & 0 & \frac{\alpha\delta\omega}{1-\delta} \\ \gamma(\delta-1) & 1 & \delta(\beta\omega-1) \\ 0 & 0 & -\omega \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ \Delta e_t \end{bmatrix} \tag{15}$$

[7] See the Appendix for the derivation of (15).

$$= \begin{bmatrix} -\alpha\zeta_c - \frac{\alpha\delta}{1-\delta}\cdot\pi^{e,*}_{t+1} \\ \delta\left(\pi^*_t - \beta\pi^{e,*}_{t+1}\right) \\ r^*_t - \zeta_c \end{bmatrix} + \begin{bmatrix} -\alpha\zeta_x & -\alpha\zeta_\pi & -\alpha\zeta_e \\ 0 & 0 & 0 \\ -\zeta_x & -\zeta_\pi & -\zeta_e \end{bmatrix} \begin{bmatrix} x^e_t \\ \pi^e_t \\ \Delta e^{e,f}_t \end{bmatrix}$$
$$\begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & \frac{\alpha\delta(\omega-1)}{1-\delta} \\ 0 & \beta & \beta\delta(\omega-1) \\ 0 & 0 & 1-\omega \end{bmatrix} \begin{bmatrix} x^e_{t+1} \\ \pi^e_{t+1} \\ \Delta e^e_{t+1} \end{bmatrix} + \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix} \cdot \overline{rr}_t.$$

The dating of current expectations in (15) is time period $t-1$, meaning that the monetary authority has rational expectations regarding the variables in the interest rate rule, conditioned on all information available in the previous time period.

When deriving conditions for determinacy, we do the following substitution

$$\left[x^e_t, \pi^e_t, \Delta e^{e,f}_t\right]' = [x_t, \pi_t, \Delta e_t]' + \boldsymbol{\epsilon}_t. \tag{16}$$

where $\boldsymbol{\epsilon}_t$ is a vector with error terms. Consequently, to have a unique and stable REE, all three eigenvalues of the same coefficient matrix as in Section 3.1.1 must be inside the unit circle since x_t, π_t and Δe_t are free (see (13)).[8] Therefore, we refer to Figures 1-5 and the discussion around them for the regions in the parameter space of $(\omega, \zeta_x, \zeta_\pi, \zeta_e)$ for which we have a unique inflation rate in equilibrium.[9]

Sensitivity analysis Obviously, we get exactly the same results when increasing the index of openness of the economy from $\delta = 0.2$ to $\delta = 0.4$ as when increasing the same index when a contemporaneous data specification of the Taylor rule is used by the monetary authority. This is because the same coefficient matrix determines the conditions for determinacy for both types of interest rate rules (see (13)).

[8] See the Appendix for the derivation of this result.

[9] That is, the regions in Figures 1-5 are the determinacy regions when a contemporaneous expectations specification of the Taylor rule is used, even if it is written "contemporaneous data specification of the Taylor rule" in some of the figures (see Figure 1, Figure 2 a, Figure 3 a, Figure 4 a and Figure 5 a). In fact, it will turn out in Section 3.2.2 that there are parametrizations of the contemporaneous expectations specification of the interest rate rule that give rise to a unique REE that is not adaptively learnable.

3.2 Least squares learning

The assumption in (8) is that when fundamental analysis is used in currency trade, the agents have rational expectations in the sense that the expected change in the exchange rate is equal to the mathematically expected change in the exchange rate, conditioned on all information available to the currency trader. Thus, since this information not only includes past and current values of all variables in the model, but also a perfect knowledge about the structure of the model, rational expectations is a rather strong assumption. This assumption has, therefore, in the more recent literature, been complemented by an analysis of the possible convergence to the REE.

It is assumed that expectations are formed by a correctly specified model, i.e., a model that nests the REE, but without having perfect knowledge about the parameter values in the model. However, using past and (depending on the dating of expectations) current values of all variables in the model, the parameter values are learned over time since the beliefs are revised as new information is gained. To be more precise, we will examine if the unique REE is characterized by recursive least squares learnability. But since expectational stability, or E-stability, implies learnability (see, e.g., Evans and Honkapohja [9]), the focus in the analysis will be on E-stability. This is because the latter concept is easier to handle mathematically.

When there is a unique REE in the model, we make use of the minimal state variable (MSV) solution, which is the solution of a linear difference equation that depends linearly on a set of variables such that there does not exist a solution that depends linearly on a smaller set of variables (see McCallum [12]). This is also the approach taken in Bullard and Mitra [4].

Finally, recall that the agents that use technical analysis do not learn anything since they use a mechanical rule in their trade in foreign exchange.

3.2.1 Contemporaneous data in the Taylor rule

Let us start with the contemporaneous data specification of the interest rate rule as it is presented in (10).

First, using matrices and vectors, the model in (12) can be written as follows

$$\boldsymbol{\Xi} \cdot \mathbf{y}_t = \boldsymbol{\Pi} + \boldsymbol{\Sigma} \cdot \mathbf{y}_{t+1}^e + \boldsymbol{\Upsilon} \cdot \overline{rr}_t, \tag{17}$$

where $\mathbf{y}_t = [x_t, \pi_t, \Delta e_t]'$ is the state of the economy. A suggested MSV solution of the model in (17) is, therefore,

$$\mathbf{y}_t = \widehat{\boldsymbol{\Theta}} + \widehat{\boldsymbol{\Lambda}} \cdot \overline{rr}_t, \tag{18}$$

where $\widehat{\boldsymbol{\Theta}}$ and $\widehat{\boldsymbol{\Lambda}}$ are parameter vectors to be determined with the method of undetermined coefficients. Hence, calculate the mathematically expected state of the economy in the next time period

$$\begin{aligned} \mathbf{y}_{t+1}^e &= \widehat{\boldsymbol{\Theta}} + \widehat{\boldsymbol{\Lambda}} \cdot \overline{rr}_{t+1}^e \\ &= \widehat{\boldsymbol{\Theta}} + \widehat{\boldsymbol{\Lambda}} \cdot \rho \overline{rr}_t, \end{aligned} \tag{19}$$

where (6) is used in the second step in (19), and the dating of expectations is time period t. Thereafter, substitute (19) into the model in (17)

$$\boldsymbol{\Xi} \cdot \mathbf{y}_t = \boldsymbol{\Pi} + \boldsymbol{\Sigma} \cdot \left(\widehat{\boldsymbol{\Theta}} + \widehat{\boldsymbol{\Lambda}} \cdot \rho \overline{rr}_t \right) + \boldsymbol{\Upsilon} \cdot \overline{rr}_t, \tag{20}$$

or, if solved for the contemporaneous values of the model's variables,

$$\begin{aligned} \mathbf{y}_t &= \boldsymbol{\Xi}^{-1} \cdot \boldsymbol{\Pi} + \boldsymbol{\Xi}^{-1} \cdot \boldsymbol{\Sigma} \cdot \left(\widehat{\boldsymbol{\Theta}} + \widehat{\boldsymbol{\Lambda}} \cdot \rho \overline{rr}_t \right) + \boldsymbol{\Xi}^{-1} \cdot \boldsymbol{\Upsilon} \cdot \overline{rr}_t \\ &= \boldsymbol{\Xi}^{-1} \cdot \boldsymbol{\Pi} + \boldsymbol{\Gamma} \cdot \left(\widehat{\boldsymbol{\Theta}} + \widehat{\boldsymbol{\Lambda}} \cdot \rho \overline{rr}_t \right) + \boldsymbol{\Xi}^{-1} \cdot \boldsymbol{\Upsilon} \cdot \overline{rr}_t \\ &= \boldsymbol{\Xi}^{-1} \cdot \boldsymbol{\Pi} + \boldsymbol{\Gamma} \cdot \widehat{\boldsymbol{\Theta}} + \left(\boldsymbol{\Gamma} \cdot \widehat{\boldsymbol{\Lambda}} \cdot \rho + \boldsymbol{\Xi}^{-1} \cdot \boldsymbol{\Upsilon} \right) \cdot \overline{rr}_t, \end{aligned} \tag{21}$$

where $\boldsymbol{\Gamma} = \boldsymbol{\Xi}^{-1} \cdot \boldsymbol{\Sigma}$. Finally, by comparing the parameters in (18) and (21), we can solve for the MSV solution

$$\mathbf{y}_t = (\mathbf{I} - \boldsymbol{\Gamma})^{-1} \cdot \boldsymbol{\Xi}^{-1} \cdot \boldsymbol{\Pi} + (\mathbf{I} - \boldsymbol{\Gamma} \cdot \rho)^{-1} \cdot \boldsymbol{\Xi}^{-1} \cdot \boldsymbol{\Upsilon} \cdot \overline{rr}_t, \tag{22}$$

since

$$\left\{ \begin{array}{l} \widehat{\boldsymbol{\Theta}} = \boldsymbol{\Xi}^{-1} \cdot \boldsymbol{\Pi} + \boldsymbol{\Gamma} \cdot \widehat{\boldsymbol{\Theta}} \\ \widehat{\boldsymbol{\Lambda}} = \boldsymbol{\Gamma} \cdot \widehat{\boldsymbol{\Lambda}} \cdot \rho + \boldsymbol{\Xi}^{-1} \cdot \boldsymbol{\Upsilon}, \end{array} \right. \tag{23}$$

where $\mathbf{I}$ is the identity matrix.

Now, is the MSV solution in (22) characterized by recursive least squares learnability? To have a REE that is learnable, the parameter values in the perceived law of motion (PLM) of the economy have to converge to the parameter values in the economy's actual law of motion (ALM) (see, e.g., Evans and Honkapohja [9]). In fact, the suggested MSV solution in (18) is also the PLM of the economy (which is emphasized by the "hat"-symbol since $\mathbf{\Theta}$ and $\mathbf{\Lambda}$ are parameter vectors that are estimated), and the solution in (21) is the ALM of the economy.

To be more precise, to have the ALM of the economy, a possibly non-rational forecast of the next time period's state of the economy should be substituted into the model in (17) allowing for non-rational expectations. However, since the mathematical expression in (21) would not be affected by this substitution, (21) is also the ALM of the economy. (In Section 3.2.2, when a contemporaneous expectations specification of the Taylor rule is used by the monetary authority, we will partly focus the presentation on the derivation of the economy's ALM.)

Observe that there is a mapping from the parameter values in the PLM to the parameter values in the ALM

$$\mathbf{M}_{MSV}\begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix} = \begin{pmatrix} \mathbf{\Xi}^{-1}\cdot\mathbf{\Pi}+\mathbf{\Gamma}\cdot\widehat{\mathbf{\Theta}} \\ \mathbf{\Gamma}\cdot\widehat{\mathbf{\Lambda}}\cdot\rho+\mathbf{\Xi}^{-1}\cdot\mathbf{\Upsilon} \end{pmatrix}, \tag{24}$$

and consider the matrix differential equation

$$\begin{aligned} \frac{\partial}{\partial\tau}\begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix} &= \mathbf{M}_{MSV}\begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix} - \begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{\Xi}^{-1}\cdot\mathbf{\Pi}+\mathbf{\Gamma}\cdot\widehat{\mathbf{\Theta}} \\ \mathbf{\Gamma}\cdot\widehat{\mathbf{\Lambda}}\cdot\rho+\mathbf{\Xi}^{-1}\cdot\mathbf{\Upsilon} \end{pmatrix} - \begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix}, \end{aligned} \tag{25}$$

where τ is "artificial" time. Then, the MSV solution in (22) is E-stable, if the parameter vectors $\widehat{\mathbf{\Theta}}$ and $\widehat{\mathbf{\Lambda}}$ are locally asymptotically stable under (25). This is also the case if all eigenvalues of the following matrixes have negative real parts (see, e.g., Evans and Honkapohja [9]):

$$\begin{cases} \partial\left(\frac{\partial\widehat{\mathbf{\Theta}}}{\partial\tau}\right)/\partial\widehat{\mathbf{\Theta}} = \mathbf{\Gamma}-\mathbf{I} \\ \partial\left(\frac{\partial\widehat{\mathbf{\Lambda}}}{\partial\tau}\right)/\partial\widehat{\mathbf{\Lambda}} = \mathbf{\Gamma}\cdot\rho-\mathbf{I}. \end{cases} \tag{26}$$

Since $0 \leq \rho < 1$, we can limit our attention to the first row in (26).

It is clear that when there is a unique REE and the state of the economy is in the neighborhood of the REE, the agents that use fundamental analysis, which also includes the monetary authority, will learn this REE. To see this result explicitly, note that the characteristic equation for the determinacy problem is (see Section 3.1.1)

$$|\mathbf{\Gamma} - \lambda_d \cdot \mathbf{I}| = 0, \tag{27}$$

where λ_d is the eigenvalue (that has three solutions), and that the characteristic equation for the learnability problem is

$$\begin{aligned} |\mathbf{\Gamma} - \mathbf{I} - \lambda_l \cdot \mathbf{I}| &= \\ |\mathbf{\Gamma} - (1 + \lambda_l) \cdot \mathbf{I}| &= 0, \end{aligned} \tag{28}$$

where λ_l is the eigenvalue (that also has three solutions). Thus,

$$\mathrm{Re}\,(\lambda_l) = \mathrm{Re}\,(\lambda_d - 1)\,, \tag{29}$$

which means that when λ_d is inside the unit circle, λ_l has a negative real part. Therefore, we refer to Figures 1-5 and the discussion around them for the regions in the parameter space of $(\omega, \zeta_x, \zeta_\pi, \zeta_e)$ for which we have a unique and an adaptively learnable inflation rate in equilibrium. Be aware that even though there is a REE that is adaptively learnable in recursive least squares sense, this REE does not have to be unique.

It is not easy to give the intuition behind the result that an increase in technical trading induces a larger learnability region. However, there is a similar result in Bullard and Mitra [5] that we discussed in Section 3.1.1. Specifically, policy inertia not only induces a larger determinacy region in their model, it also induces a larger learnability region. Thus, since chartism is a form of inertia, it is reasonable to expect a larger learnability region when the degree of technical trading in the foreign exchange market increases.

Sensitivity analysis Since a unique REE always is adaptively learnable, we get the same general results when the index of openness of the economy increases from $\delta = 0.2$ to $\delta = 0.4$ as when increasing the same index when we investigated the conditions for determinacy.

3.2.2 Contemporaneous expectations in the Taylor rule

Let us continue with the contemporaneous expectations specification of the interest rate rule as it is presented in (11).

First, using matrices and vectors, the model in (15) can be written as follows

$$\boldsymbol{\Xi}_0 \cdot \mathbf{y}_t = \boldsymbol{\Pi} + \boldsymbol{\Xi}_1 \cdot \mathbf{y}_t^e + \boldsymbol{\Sigma} \cdot \mathbf{y}_{t+1}^e + \boldsymbol{\Upsilon} \cdot \overline{rr}_t, \tag{30}$$

where $\boldsymbol{\Xi}_0 - \boldsymbol{\Xi}_1 = \boldsymbol{\Xi}$. Therefore, and guided by structure of the model's MSV solution, we assume that the PLM of the economy is

$$\mathbf{y}_t = \widehat{\boldsymbol{\Theta}} + \widehat{\boldsymbol{\Lambda}} \cdot \overline{rr}_{t-1} + \boldsymbol{\Phi} \cdot \varepsilon_t, \tag{31}$$

from which we calculate the mathematically expected state of the economy in the current time period

$$\mathbf{y}_t^e = \widehat{\boldsymbol{\Theta}} + \widehat{\boldsymbol{\Lambda}} \cdot \overline{rr}_{t-1}, \tag{32}$$

as well as in the next time period

$$\begin{aligned} \mathbf{y}_{t+1}^e &= \widehat{\boldsymbol{\Theta}} + \widehat{\boldsymbol{\Lambda}} \cdot \overline{rr}_t^e \\ &= \widehat{\boldsymbol{\Theta}} + \widehat{\boldsymbol{\Lambda}} \cdot \rho\overline{rr}_{t-1}. \end{aligned} \tag{33}$$

Recall that the dating of expectations in (32) is time period $t-1$. Moreover, to have an exact correspondence with Bullard and Mitra [4], we assume that the dating of expectations in (33) is also time period $t-1$. (Recall that the dating of expectations when contemporaneous data are used in the Taylor rule is time period t.)

Then, if we substitute the expected states of the economy in (32)-(33) into the PLM of the economy in (31), we get the economy's ALM

$$\begin{aligned} \boldsymbol{\Xi}_0 \cdot \mathbf{y}_t &= \boldsymbol{\Pi} + \boldsymbol{\Xi}_1 \cdot \left(\widehat{\boldsymbol{\Theta}} + \widehat{\boldsymbol{\Lambda}} \cdot \overline{rr}_{t-1}\right) + \\ &\quad \boldsymbol{\Sigma} \cdot \left(\widehat{\boldsymbol{\Theta}} + \widehat{\boldsymbol{\Lambda}} \cdot \rho\overline{rr}_{t-1}\right) + \boldsymbol{\Upsilon} \cdot \overline{rr}_t \\ &= \boldsymbol{\Pi} + \boldsymbol{\Xi}_1 \cdot \left(\widehat{\boldsymbol{\Theta}} + \widehat{\boldsymbol{\Lambda}} \cdot \overline{rr}_{t-1}\right) + \\ &\quad \boldsymbol{\Sigma} \cdot \left(\widehat{\boldsymbol{\Theta}} + \widehat{\boldsymbol{\Lambda}} \cdot \rho\overline{rr}_{t-1}\right) + \boldsymbol{\Upsilon} \cdot \left(\rho\overline{rr}_{t-1} + \varepsilon_t\right), \end{aligned} \tag{34}$$

or, if solved for the contemporaneous values of the model's variables,

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{\Xi}_0^{-1} \cdot \mathbf{\Pi} + \mathbf{\Xi}_0^{-1} \cdot \mathbf{\Xi}_1 \cdot \left(\widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \overline{rr}_{t-1} \right) + \\
&\quad \mathbf{\Xi}_0^{-1} \cdot \mathbf{\Sigma} \cdot \left(\widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \rho \overline{rr}_{t-1} \right) + \mathbf{\Xi}_0^{-1} \cdot \mathbf{\Upsilon} \cdot (\rho \overline{rr}_{t-1} + \varepsilon_t) \\
&= \mathbf{\Xi}_0^{-1} \cdot \left(\mathbf{\Pi} + \mathbf{\Xi}_1 \cdot \widehat{\mathbf{\Theta}} + \mathbf{\Sigma} \cdot \widehat{\mathbf{\Theta}} \right) + \\
&\quad \mathbf{\Xi}_0^{-1} \cdot \left(\mathbf{\Xi}_1 \cdot \widehat{\mathbf{\Lambda}} + \mathbf{\Sigma} \cdot \widehat{\mathbf{\Lambda}} \cdot \rho + \mathbf{\Upsilon} \cdot \rho \right) \cdot \overline{rr}_{t-1} + \mathbf{\Xi}_0^{-1} \cdot \mathbf{\Upsilon} \cdot \varepsilon_t.
\end{aligned} \tag{35}$$

Observe again that there is a mapping from the parameter values in the PLM to the parameter values in the ALM

$$\mathbf{M}_{MSV} \begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix} = \begin{pmatrix} \mathbf{\Xi}_0^{-1} \cdot \left(\mathbf{\Pi} + \mathbf{\Xi}_1 \cdot \widehat{\mathbf{\Theta}} + \mathbf{\Sigma} \cdot \widehat{\mathbf{\Theta}} \right) \\ \mathbf{\Xi}_0^{-1} \cdot \left(\mathbf{\Xi}_1 \cdot \widehat{\mathbf{\Lambda}} + \mathbf{\Sigma} \cdot \widehat{\mathbf{\Lambda}} \cdot \rho + \mathbf{\Upsilon} \cdot \rho \right) \end{pmatrix}, \tag{36}$$

and consider the matrix differential equation

$$\begin{aligned}
\frac{\partial}{\partial \tau} \begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix} &= \mathbf{M}_{MSV} \begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix} - \begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix} \\
&= \begin{pmatrix} \mathbf{\Xi}_0^{-1} \cdot \left(\mathbf{\Pi} + \mathbf{\Xi}_1 \cdot \widehat{\mathbf{\Theta}} + \mathbf{\Sigma} \cdot \widehat{\mathbf{\Theta}} \right) \\ \mathbf{\Xi}_0^{-1} \cdot \left(\mathbf{\Xi}_1 \cdot \widehat{\mathbf{\Lambda}} + \mathbf{\Sigma} \cdot \widehat{\mathbf{\Lambda}} \cdot \rho + \mathbf{\Upsilon} \cdot \rho \right) \end{pmatrix} - \begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix},
\end{aligned} \tag{37}$$

where the equation's fix point is the MSV solution of the model in (30). Hence, if the parameter vectors $\widehat{\mathbf{\Theta}}$ and $\widehat{\mathbf{\Lambda}}$ are locally asymptotically stable under (37), the MSV solution is E-stable, which is the case when all eigenvalues of the following matrixes have negative real parts

$$\begin{cases} \partial \left(\frac{\partial \widehat{\mathbf{\Theta}}}{\partial \tau} \right) / \partial \widehat{\mathbf{\Theta}} = \mathbf{\Xi}_0^{-1} \cdot (\mathbf{\Xi}_1 + \mathbf{\Sigma}) - \mathbf{I} \\ \partial \left(\frac{\partial \widehat{\mathbf{\Lambda}}}{\partial \tau} \right) / \partial \widehat{\mathbf{\Lambda}} = \mathbf{\Xi}_0^{-1} \cdot (\mathbf{\Xi}_1 + \mathbf{\Sigma} \cdot \rho) - \mathbf{I}. \end{cases} \tag{38}$$

Due to the fact that $0 \leq \rho < 1$, we can limit our attention to the first row in (38), meaning that the relevant characteristic equation is

$$\left| \mathbf{\Xi}_0^{-1} \cdot (\mathbf{\Xi}_1 + \mathbf{\Sigma}) - \mathbf{I} - \lambda_l \cdot \mathbf{I} \right| = 0, \tag{39}$$

where λ_l is the eigenvalue (that has three solutions).

It turns out that the regions in the parameter space of $(\omega, \zeta_x, \zeta_\pi, \zeta_e)$ for which we have a unique and an adaptively learnable inflation rate in equilibrium are not the same as when a contemporaneous data specification of the Taylor rule is used by the monetary authority. See Figures 2-9. To be more precise, the learnability regions are exactly the same for both types of rules when $\zeta_x > 0$, but slightly different when $\zeta_x = 0$. In the latter case, the regions are slightly smaller when contemporaneous expectations of the variables are used in the interest rate rule than when contemporaneous data are used in the rule (e.g., compare Figure 4 a and Figure 8 as well as Figure 5 a and Figure 9). Since the determinacy regions for both types of rules are exactly the same, this means that there are parametrizations of the contemporaneous expectations specification of the interest rate rule that give rise to a unique REE that is not adaptively learnable. Note that a factor in common for most of these parametrizations is that $\zeta_e > 0$.

Sensitivity analysis When increasing the index of openness of the economy from $\delta = 0.2$ to $\delta = 0.4$, none of the findings are affected. For both parameter settings, this means that when at least half of the trade in the currency market is driven by technical analysis, the learnability region increases when the monetary authority reacts stronger to the output gap, whereas the opposite is true when less than half of the trade is driven by chartism. Moreover, given the output gap reaction in the Taylor rule, the learnability region gets larger when the proportion of chartists in currency trade increases. Finally, when $\zeta_x = 0$, the learnability region is smaller when the economy is more open.

3.3 Robust and desirable Taylor rules

As already discussed in Section 1, our paper fills the gap between papers that derive optimal policy rules in the new Keynesian framework and papers that focus on determinacy and adaptive learnability of the REE in the same framework. For this task, we use a loss-function as our metric that takes the expected CPI inflation rate and the conditional volatility of the CPI inflation rate in equilibrium as arguments.

Moreover, to find robust parametrizations of the Taylor rules, the properties of a rule should be relatively unaffected by the degree of technical trading in the foreign exchange market. That is, the rule should give rise to determinacy, adaptive learnability of the REE, and a desirable outcome according to the loss-function for most proportions of chartism in currency trade.

3.3.1 Contemporaneous data in the Taylor rule

Starting with the contemporaneous data specification of the interest rate rule in (10), the expected CPI inflation rate in equilibrium is, according to the MSV solution in (22),

$$E_t(\pi_t) = \left[(\mathbf{I}-\mathbf{\Gamma})^{-1}\cdot\mathbf{\Xi}^{-1}\cdot\mathbf{\Pi} + (\mathbf{I}-\mathbf{\Gamma}\cdot\rho)^{-1}\cdot\mathbf{\Xi}^{-1}\cdot\mathbf{\Upsilon}\cdot E_t(\overline{rr}_t)\right]_{(2)}, \tag{40}$$

and the conditional volatility of the CPI inflation rate in equilibrium is

$$var_t(\pi_t) = \left[\left((\mathbf{I}-\mathbf{\Gamma}\cdot\rho)^{-1}\cdot\mathbf{\Xi}^{-1}\cdot\mathbf{\Upsilon}\right)\cdot\left((\mathbf{I}-\mathbf{\Gamma}\cdot\rho)^{-1}\cdot\mathbf{\Xi}^{-1}\cdot\mathbf{\Upsilon}\right)'\right]_{(2,2)}\cdot\sigma_\varepsilon^2, \tag{41}$$

where (2) and $(2,2)$ refer to the second element in the vector and the second element along the diagonal in the matrix, respectively. Thereafter, substitute the assumed values of the structural parameters and the exogenous variables in the model into the expressions in (40)-(41). Thus, we make use of the parameter values in (14) in the evaluation of the model's outcome in equilibrium.

When it comes to the exogenous variables in the model, these are π_t^*, $\pi_{t+1}^{e,*}$ and r_t^*. In addition, we treat $\overline{rr}_t$ as an exogenous variable. To make the analysis as simple as possible, we set $\pi_t^* = \pi_{t+1}^{e,*} = 0$. Moreover, when the variables in the Taylor rule are at their target values, i.e., $x_t = \pi_t = \Delta e_t = 0$ in (10), and the economy is in a stationary equilibrium, the domestic interest rate is equal to the foreign interest rate due to the parity condition in (5) that holds at the international asset market. Thus, the constant ζ_c in the Taylor rule must be equal to the foreign interest rate. Finally, we set $E_t(\overline{rr}_t) = r_t^*$, because the natural rate of interest is, in a stationary equilibrium, equal to the nominal interest rate due to the first equation in (2), which, in turn, is equal to the foreign interest rate due to the aforementioned parity condition.

The loss-function that we use as our metric to evaluate the desirability of a specific REE is formulated as follows

$$L = H(|E_t(\pi_t)| - 0.01) + H\left(var_t(\pi_t) - 0.2\sigma_\varepsilon^2\right), \tag{42}$$

where $H(\cdot)$ is the Heaviside step function.[10] Thus, the loss-function in (42) is minimized and equal to 0 when the expected CPI inflation rate is within $\pm$ 0.01, and the conditional volatility of the CPI inflation rate is at most $0.2\sigma_{\varepsilon}^{2}$. The motivation of the limits for a desirable inflation rate is that they are typical in established inflation rate targeting regimes, whereas the choice of the limit for the variability of the inflation rate is somewhat arbitrary. As an example of an established inflation rate targeting regime, the Swedish Riksbank has defined price-stability as an increase in the CPI of two per cent, but with a tolerance margin of plus/minus one percentage point around this target.

Needless to say, the interest rate set by the monetary authority must be non-negative. Moreover, we are searching for robust and desirable parametrizations of the Taylor rule in the sense that the desirable properties of the rule should be relatively unaffected by the degree of technical trading in the foreign exchange market, which implies that a robust parametrization of the rule should satisfy $L = 0$ for a range of values of ω. This is because the exact proportion of chartists in currency trade is not known when conducting monetary policy.

In Tables 1 a-c, the degree of technical trading in the foreign exchange market is 25, 35, 45, 55, 65, 75, 85 and 95 percent, and the interest rate abroad is 0.01, 0.02 and 0.03, respectively. Moreover, we are performing a grid search for desirable parametrizations of the Taylor rule in which the parameter values in the rule are whole numbers. (Throughout this section, the choices of sets of ω in the grid search are to avoid matrices that are singular.) Two results are found in the tables: (i) the monetary authority should increase (decrease) the interest rate when the CPI inflation rate increases (decreases) and when the currency gets stronger (weaker), but not care about the output gap, to have a desirable rule that also is robust; and (ii) the number of interest rate rules with these properties decreases with increases in the foreign interest rate.

In Tables 2 a-c, we have repeated the same procedure with the exception that the parameter ζ_e in the Taylor rule is restricted to 0. As a consequence, the proportion of chartists in currency trade is limited to 55, 65, 75, 85 and 95 percent since there are no desirable parametrizations of the rule when the

[10] The Heaviside step function has the following property: $H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$.

proportion is 45 percent or lower, having restricted the parameter values in the interest rate rule to whole numbers. As before, the number of desirable interest rate rules that are robust decreases with increases in the foreign interest rate. Further on, which is true irrespective of whether the parameter ζ_e in the Taylor rule is restricted to 0 or not, the monetary authority should react strongly to changes in the inflation rate to have an outcome that is desirable in terms of the expected inflation rate and the conditional volatility of the inflation rate in equilibrium.

In Tables 3 a-c, the grid search for desirable Taylor rules has been refined in the sense that the parameter values in the rules are multiples of 0.1. This also means that we restrict the presentation to the share of rules that satisfy $L = 0$ for different sets of ω. Three results are found in the tables: (i) the number of desirable rules decreases when the range of values of ω increases, and irrespective of whether the value of ζ_e is restricted to 0 or not; (ii) the number of robust and desirable rules decreases with increases in the foreign interest rate, and also irrespective of the value of ζ_e (as also noted above); and (iii) the number of rules that give rise to a unique and adaptively learnable REE, but not restricted to $L = 0$, decreases when the range of values of ω increases. Of course, that the share of rules that are associated with determinacy and learnability is not affected by the foreign interest rate is not surprising since this variable is not part of the coefficient matrix in (13).

But, then, which parametrization of the Taylor rule is the best rule? When the interest rate abroad is 0.01 and 0.02, respectively, it is the same 16 rules that satisfy $L = 0$ for the set of ω that includes 5, 15, 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade. Of these 16 parametrizations of the interest rate rule, the following rule is desirable down to 2 percent technical trading in the foreign exchange market

$$r_t = r_t^* + 9.9\pi_t - 3.2\Delta e_t. \tag{43}$$

The Taylor rule in (43) fails to be desirable when the interest rate abroad is 0.03, and this is because the expected inflation rate is not within $\pm$ 0.01 when a large proportion of currency trade is driven by technical analysis. Concerning the other 15 parametrizations of the interest rate rule, the parameters belong to the sets $\zeta_x \in [0, 0.1]$, $\zeta_\pi \in [9.2, 10]$ and $\zeta_e \in [-3.4, -2.9]$. Thus, in principle, the monetary authority should not care about the output gap when setting the interest rate since the value of the currency is a better substitute.

Sensitivity analysis We have again performed a sensitivity analysis of the numerical findings in which the index of openness of the economy has been increased from $\delta = 0.2$ to $\delta = 0.4$. Basically, the findings that we reported when this index was equal to 0.2 are not affected. However, in comparison, the share of rules that give rise to a unique and adaptively learnable REE is larger when the smallest proportions of chartism in currency trade is excluded in the grid search, whereas the opposite is true when the smallest proportions of technical trading is included. We also found the same results in our search for desirable parametrizations of the Taylor rule.

3.3.2 Contemporaneous expectations in the Taylor rule

Continuing with the contemporaneous expectations specification of the interest rate rule in (11), the expected CPI inflation rate in equilibrium is[11]

$$E_{t-1}(\pi_t) = \left[(\mathbf{I} - \mathbf{\Gamma})^{-1} \cdot \mathbf{\Xi}^{-1} \cdot \mathbf{\Pi} + (\mathbf{I} - \mathbf{\Gamma} \cdot \rho)^{-1} \cdot \mathbf{\Xi}^{-1} \cdot \mathbf{\Upsilon} \cdot E_{t-1}(\overline{rr}_t) \right]_{(2)}, \tag{44}$$

and the conditional volatility of the CPI inflation rate in equilibrium is

$$\begin{aligned} & var_{t-1}(\pi_t) \\ = \; & \rho^2 \cdot \left[\left((\mathbf{I} - \mathbf{\Gamma} \cdot \rho)^{-1} \cdot \mathbf{\Xi}^{-1} \cdot \mathbf{\Upsilon} \right) \cdot \left((\mathbf{I} - \mathbf{\Gamma} \cdot \rho)^{-1} \cdot \mathbf{\Xi}^{-1} \cdot \mathbf{\Upsilon} \right)' \right]_{(2,2)} \cdot \sigma_\varepsilon^2 + \\ & \left[\left(\mathbf{\Xi}_0^{-1} \cdot \mathbf{\Upsilon} \right) \cdot \left(\mathbf{\Xi}_0^{-1} \cdot \mathbf{\Upsilon} \right)' \right]_{(2,2)} \cdot \sigma_\varepsilon^2. \end{aligned} \tag{45}$$

Except for a difference in the dating of expectations, the expected CPI inflation rate in equilibrium in (44) is the same as in (40). This means that it does not matter if contemporaneous data are used in the Taylor rule or if contemporaneous expectations, formed in the previous time period, of the variables are used in the Taylor rule, the expected CPI inflation rate in equilibrium is exactly the same.

[11] See the Appendix for the derivations of (44)-(45).

Even though the conditional volatility of the CPI inflation rate in equilibrium in (45) is not the same as in (41), there is a linear relationship between them. To be more precise, the former quantity can be written as $\rho^2\sigma_\pi^2 + \alpha^2\gamma(1-\delta)\sigma_\varepsilon^2 = 0.1225\sigma_\pi^2 + 0.77894\sigma_\varepsilon^2$, where σ_π^2 is the latter quantity, and where we have substituted the parameter values in (14) into the expression.[12] This means that the ordering from the best interest rate rule to the worst rule is the same, irrespective of whether the specification of the rule includes contemporaneous data or contemporaneous expectations of the included variables.

However, none of the parametrizations of the Taylor rule are satisfactory from the point of view of the variability of the CPI inflation rate. This is because the conditional volatility of the CPI inflation rate in equilibrium always is larger than (the somewhat arbitrary limit) $0.2\sigma_\varepsilon^2$. Specifically, the variability of the CPI inflation rate can never be below $0.77894\sigma_\varepsilon^2$. Of course, this does not mean that one should never adopt a contemporaneous expectations specification of the Taylor rule in monetary policy-making. The reason is that it may be the case that the contemporaneous data specification of the Taylor rule is not accessible due to data revisions. Consequently, one is forced to use a rule that includes contemporaneous expectations of the variables, which also means that one must accept a higher conditional volatility of the CPI inflation rate in equilibrium.

Sensitivity analysis Obviously, the ordering of the Taylor rules are not affected when the index of openness of the economy increases from $\delta = 0.2$ to $\delta = 0.4$. It is also still true that none of the parametrizations of the Taylor rule are satisfactory from the point of view of the variability of the CPI inflation rate, because this variability can never be below $\alpha^2\gamma(1-\delta)\sigma_\varepsilon^2 = 0.5842\sigma_\varepsilon^2$.

4 Concluding discussion

We do not repeat our findings in this discussion. Instead, we conclude with a few remarks on the model developed and shortly discuss the claims in Taylor [18] that the monetary authority's interest rate rule should not include a reaction to an exchange rate index to be favorable, which is in contrast with our finding.

[12] See the Appendix for the derivation of this result.

Our model Firstly, a few words about the technical trading technique in (7) are in place. It is clear that questionnaire surveys made at currency markets around the world not only confirm that chartism is extensively used in currency trade, but they also confirm that some variant of a moving average technique is the most commonly used technical trading technique. This means that exchange rates in the more distant past also should affect the decision to trade, and not only the exchange rates in time periods t and $t-1$.

In Bask [1], an asset pricing model for the exchange rate is developed in which the current rate is affected by an exponentially weighted moving average of all past exchange rates. When analyzing the effects of changes in monetary fundamentals, it is clear that the exchange rate in time period $t-1$ has a first-order effect on the current rate, while rates in the more distant past have a second-order effect on the current exchange rate. Encouraged be this finding, we restricted the technique in (7) to only include the exchange rates in time periods t and $t-1$. An advantage of this restriction is that the complete model would, otherwise, be too cumbersome to analyze, even numerically. This is because we would have to work with extremely large matrixes when investigating if a certain parametrization of a Taylor rule is associated with a unique, adaptively learnable and desirable inflation rate in equilibrium.

Secondly, we could formulate the interest rate rules in (10)-(11) in terms of the level of the nominal exchange rate; the actual level of the exchange rate in (10), and the mathematically expected level of the exchange rate in (11). However, having in mind that there have been several monetary arrangements throughout history aiming at achieving less variable exchange rates, we stick with the formulations of the interest rate rules in (10)-(11) and focus on the change in the nominal exchange rate. Of course, it is part of future research to search for robust and desirable parametrizations of the Taylor rules that take current and past levels of the exchange rate as arguments.

Thirdly, the dating of expectations might be important for the findings in this paper. Recall that when contemporaneous data are used in the Taylor rule as in (10), the dating of expectations is time period t, whereas when contemporaneous expectations of the variables are used in the Taylor rule as in (11), the dating of expectations is time period $t-1$. As was explained in Section 3.2.2, the reason for the latter assumption is that we would like to have an exact correspondence with Bullard and Mitra [4]. For the same reason, one should also investigate the case when the Taylor rule is (10) and the dating of expectations is time period $t-1$. In Bullard and Mitra [4], the findings are not affected by this change of dating of expectations.

Finally, the recursive least squares learning algorithm that is used by the fundamentalists is a decreasing gain algorithm. It would, therefore, be interesting to complement the analysis in this paper with the case in which the learning algorithm is a constant gain algorithm, especially when the fundamentalists, including the monetary authority, is using a PLM of the economy that does not include any REE. This is because it might open up for so-called escape dynamics in the inflation rate from a self-confirming equilibrium (see, e.g., Cho et al. [8] and Williams [20] for an introduction to this recent literature, and Bullard and Cho [3] for an example of escape dynamics in a closed economy like the one in Bullard and Mitra [4]).

Taylor's [18] claim The vigilant reader might object that we, in this paper, are not really meeting the claim in Taylor [18]. This is because we investigate the properties of the model developed using specifications of the Taylor rule that include the change in the nominal exchange rate, while Taylor [18] is discussing interest rate rules that include the current and past levels of the real exchange rate. To be more specific, Taylor [18] is discussing the following rule

$$r_t = \zeta_c + \zeta_x x_t + \zeta_\pi \pi_t + \zeta_q q_t + \zeta_{q'} q_{t-1}, \tag{46}$$

where q_t is the real exchange rate.

In fact, the investigation in this paper is adequate, and there are two reasons for this. Firstly, by assuming that $\zeta_{q'} = -\zeta_q$, we turn our focus from levels of the real exchange rate to the change in the real exchange rate. This also means, since the real exchange rate is $q_t = e_t + p_t^* - p_t$, where p_t is the CPI, that the Taylor rule in (46) can be written as follows

$$r_t = \zeta_c + \zeta_x x_t + \left(\zeta_\pi - \zeta_q\right) \pi_t + \zeta_q \Delta e_t + \zeta_q \pi_t^*. \tag{47}$$

Secondly, since we assume that $\pi_t^* = 0$ in the numerical analysis, the interest rate rule in (47) is, in principle, exactly the same as the rule in (10). Note that the assumption $\zeta_{q'} = -\zeta_q$ is necessary to transform the Taylor rule in (46) to the rule in (47).

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Appendix

Derivation of (4) Firstly, shift the first equation in (3) one time period forward in time, and rearrange terms

$$\pi^e_{H,t+1} = \pi^e_{t+1} - \delta \Delta s^e_{t+1}. \tag{A.1}$$

Secondly, shift the second equation in (3) one time period forward in time, and take differences

$$\begin{aligned} \Delta s^e_{t+1} &= \Delta e^{e,m}_{t+1} + \Delta p^{e,*}_{t+1} - \Delta p^e_{H,t+1} \\ &= \Delta e^{e,m}_{t+1} + \pi^{e,*}_{t+1} - \pi^e_{H,t+1}. \end{aligned} \tag{A.2}$$

Thirdly, substitute (A.2) into (A.1), and solve for $\pi^e_{H,t+1}$

$$\pi^e_{H,t+1} = \frac{1}{1-\delta} \cdot \left(\pi^e_{t+1} - \delta \left(\Delta e^{e,m}_{t+1} + \pi^{e,*}_{t+1}\right)\right). \tag{A.3}$$

Fourthly, shift (A.3) one time period backward in time

$$\pi_{H,t} = \frac{1}{1-\delta} \cdot \left(\pi_t - \delta \left(\Delta e_t + \pi^*_t\right)\right). \tag{A.4}$$

Fifthly, substitute (A.3) into the first equation in (2), and the first equation in (4) is derived. Finally, substitute (A.3)-(A.4) into the second equation in (2), solve for π_t, and the second equation in (4) is derived.

Derivation of (12) Firstly, substitute aggregated expectations at the foreign exchange market in (9) and the Taylor rule in (10) into the dynamic IS-type equation in (4), and rearrange terms

$$\begin{aligned} & \left(1 + \alpha\zeta_x\right) x_t + \alpha\zeta_\pi \pi_t + \alpha \left(\frac{\delta\omega}{1-\delta} + \zeta_e\right) \Delta e_t \\ = \; & x^e_{t+1} + \frac{\alpha}{1-\delta} \cdot \pi^e_{t+1} + \frac{\alpha\delta\left(\omega-1\right)}{1-\delta} \cdot \Delta e^e_{t+1} \\ & -\alpha\zeta_c - \frac{\alpha\delta}{1-\delta} \cdot \pi^{e,*}_{t+1} + \alpha\overline{rr}_t. \end{aligned} \tag{A.5}$$

Secondly, substitute aggregated expectations at the foreign exchange market in (9) into the new Keynesian Phillips curve in (4), and rearrange terms

$$\begin{aligned} & \gamma\left(\delta-1\right) x_t + \pi_t + \delta\left(\beta\omega-1\right)\Delta e_t \\ = \; & \beta\pi^e_{t+1} + \beta\delta\left(\omega-1\right)\Delta e^e_{t+1} + \delta\pi^*_t - \beta\delta\pi^{e,*}_{t+1}. \end{aligned} \tag{A.6}$$

Thirdly, substitute aggregated expectations at the foreign exchange market in (9) and the Taylor rule in (10) into the UIP condition in (5), and rearrange terms

$$\zeta_x x_t + \zeta_\pi \pi_t + (\zeta_e - \omega)\,\Delta e_t = (1-\omega)\,\Delta e^e_{t+1} - \zeta_c + r^*_t. \tag{A.7}$$

Finally, put (A.5)-(A.7) into matrix form, and (12) is derived.

Derivation of (15) Firstly, substitute aggregated expectations at the foreign exchange market in (9) and the Taylor rule in (11) into the dynamic IS-type equation in (4), and rearrange terms

$$\begin{aligned} & x_t + \frac{\alpha\delta\omega}{1-\delta}\cdot\Delta e_t \\ = \; & -\alpha\zeta_x x^e_t - \alpha\zeta_\pi \pi^e_t - \alpha\zeta_e \Delta e^{e,f}_t + \\ & x^e_{t+1} + \frac{\alpha}{1-\delta}\cdot\pi^e_{t+1} + \frac{\alpha\delta\,(\omega-1)}{1-\delta}\cdot\Delta e^e_{t+1} \\ & -\alpha\zeta_c - \frac{\alpha\delta}{1-\delta}\cdot\pi^{e,*}_{t+1} + \alpha\overline{rr}_t. \end{aligned} \tag{A.8}$$

Secondly, substitute aggregated expectations at the foreign exchange market in (9) and the Taylor rule in (11) into the UIP condition in (5), and rearrange terms

$$-\omega\Delta e_t = -\zeta_x x^e_t - \zeta_\pi \pi^e_t - \zeta_e \Delta e^{e,f}_t + (1-\omega)\,\Delta e^e_{t+1} - \zeta_c + r^*_t. \tag{A.9}$$

Finally, put (A.6) and (A.8)-(A.9) into matrix form, and (15) is derived. Note that (A.6) is unaffected by the type of Taylor rule that is used by the monetary authority.

Derivation of (13) when the Taylor rule is (11) Substitute (16) into (15), and note that the coefficient matrix for the vector $[x_t, \pi_t, \Delta e_t]'$ at the left-hand side of (15) is now

$$\begin{bmatrix} 1 & 0 & \frac{\alpha\delta\omega}{1-\delta} \\ \gamma(\delta-1) & 1 & \delta(\beta\omega-1) \\ 0 & 0 & -\omega \end{bmatrix} - \tag{A.10}$$
$$\begin{bmatrix} -\alpha\zeta_x & -\alpha\zeta_\pi & -\alpha\zeta_e \\ 0 & 0 & 0 \\ -\zeta_x & -\zeta_\pi & -\zeta_e \end{bmatrix}$$
$$= \begin{bmatrix} 1+\alpha\zeta_x & \alpha\zeta_\pi & \alpha\left(\frac{\delta\omega}{1-\delta}+\zeta_e\right) \\ \gamma(\delta-1) & 1 & \delta(\beta\omega-1) \\ \zeta_x & \zeta_\pi & \zeta_e-\omega \end{bmatrix},$$

which is the same coefficient matrix for the vector $[x_t, \pi_t, \Delta e_t]'$ at the left-hand side of (12). Consequently, the relevant coefficient matrix when deriving the conditions for determinacy is (13).

Derivations of (44)-(45) The fix point in (37) is the MSV solution of the model in (30). Hence,

$$\begin{cases} \widehat{\mathbf{\Theta}} = \mathbf{\Xi}_0^{-1} \cdot \left(\mathbf{\Pi} + \mathbf{\Xi}_1 \cdot \widehat{\mathbf{\Theta}} + \mathbf{\Sigma} \cdot \widehat{\mathbf{\Theta}}\right) \\ \widehat{\mathbf{\Lambda}} = \mathbf{\Xi}_0^{-1} \cdot \left(\mathbf{\Xi}_1 \cdot \widehat{\mathbf{\Lambda}} + \mathbf{\Sigma} \cdot \widehat{\mathbf{\Lambda}} \cdot \rho + \mathbf{\Upsilon} \cdot \rho\right), \end{cases} \tag{A.11}$$

which means that the MSV solution is

$$\begin{aligned} \mathbf{y}_t &= \widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \overline{rr}_{t-1} + \mathbf{\Phi} \cdot \varepsilon_t \\ &= (\mathbf{I}-\mathbf{\Gamma})^{-1} \cdot \mathbf{\Xi}^{-1} \cdot \mathbf{\Pi} + \\ &\quad (\mathbf{I}-\mathbf{\Gamma}\cdot\rho)^{-1} \cdot \mathbf{\Xi}^{-1} \cdot \mathbf{\Upsilon} \cdot \rho\overline{rr}_{t-1} + \mathbf{\Xi}_0^{-1} \cdot \mathbf{\Upsilon} \cdot \varepsilon_t, \end{aligned} \tag{A.12}$$

where $\mathbf{\Phi} = \mathbf{\Xi}_0^{-1} \cdot \mathbf{\Upsilon}$ follows from comparing the PLM in (31) with the ALM in (35) since the PLM of the economy is guided by the structure of the model's MSV solution. Recall that $\mathbf{\Gamma} = \mathbf{\Xi}^{-1} \cdot \mathbf{\Sigma}$ and $\mathbf{\Xi}_0 - \mathbf{\Xi}_1 = \mathbf{\Xi}$. Thereafter, take the conditional expectations and volatility of (A.12), note that $E_{t-1}(\rho\overline{rr}_{t-1}) = E_{t-1}(\overline{rr}_t)$ due to the stochastic process for the natural rate of interest in (6), and (44)-(45) follows.

Derivation of the relationship between (41) and (45) Since

$$\Xi_0^{-1} \cdot \Upsilon = \begin{bmatrix} 1 & 0 & \frac{\alpha\delta\omega}{1-\delta} \\ \gamma(\delta-1) & 1 & \delta(\beta\omega-1) \\ 0 & 0 & -\omega \end{bmatrix}^{-1} \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix} \tag{A.13}$$

$$= \begin{bmatrix} 1 & 0 & \frac{\alpha\delta}{1-\delta} \\ \gamma(1-\delta) & 1 & \frac{\delta(\alpha\gamma\omega+\beta\omega-1)}{\omega} \\ 0 & 0 & -\frac{1}{\omega} \end{bmatrix}^{-1} \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha \\ \alpha\gamma(1-\delta) \\ 0 \end{bmatrix},$$

it follows that

$$\left[\left(\Xi_0^{-1} \cdot \Upsilon\right) \cdot \left(\Xi_0^{-1} \cdot \Upsilon\right)'\right]_{(2,2)} \tag{A.14}$$

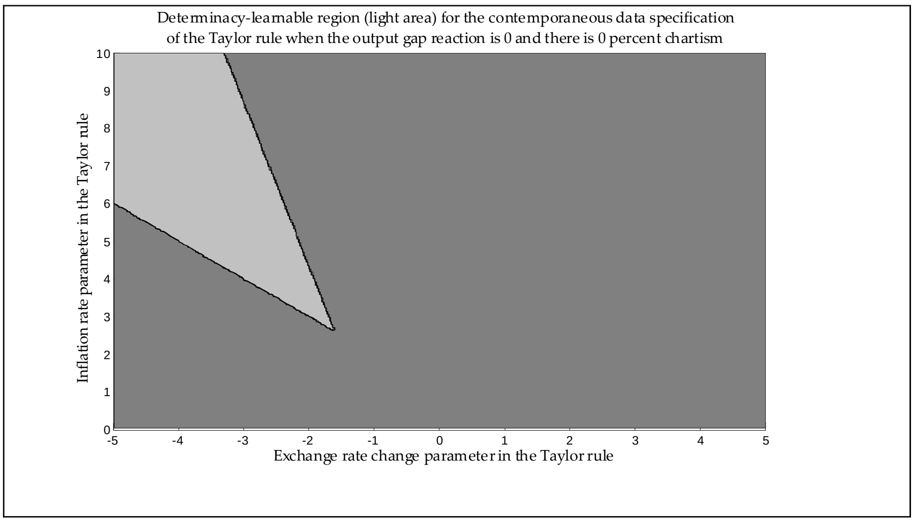
$$= \left[\begin{bmatrix} \alpha \\ \alpha\gamma(1-\delta) \\ 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \alpha\gamma(1-\delta) \\ 0 \end{bmatrix}'\right]_{(2,2)}$$

$$= \begin{bmatrix} \alpha^2 & \alpha^2 & 0 \\ \alpha^2\gamma(1-\delta) & \alpha^2\gamma(1-\delta) & 0 \\ 0 & 0 & 0 \end{bmatrix}_{(2,2)}$$

$$= \alpha^2\gamma(1-\delta),$$

and the postulated linear relationship between (41) and (45) follows.

Figures and Tables



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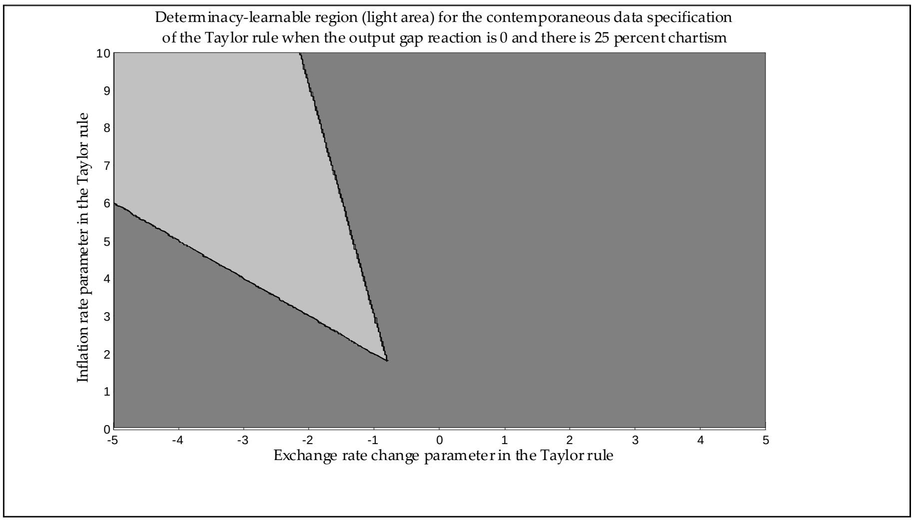

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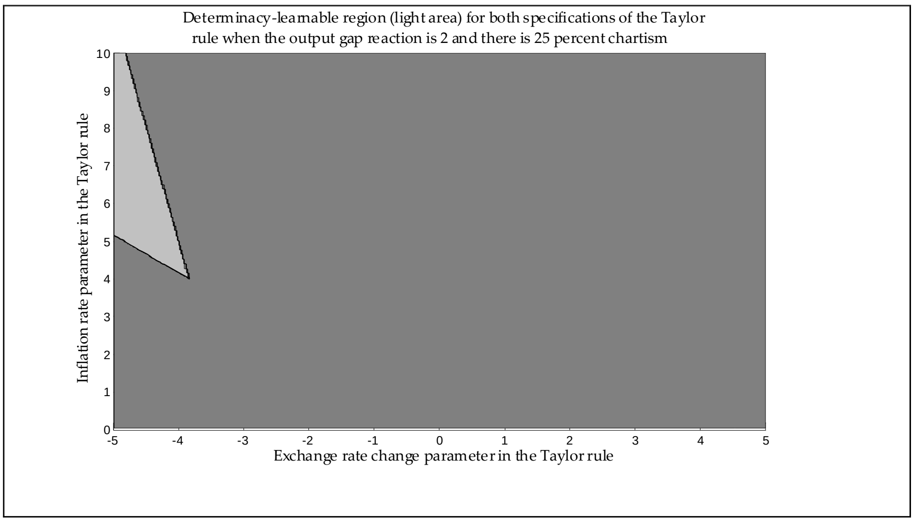

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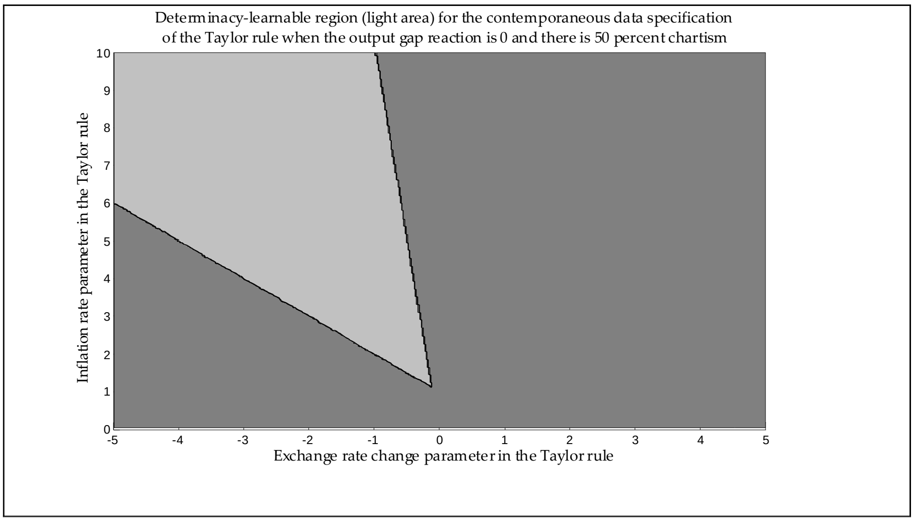

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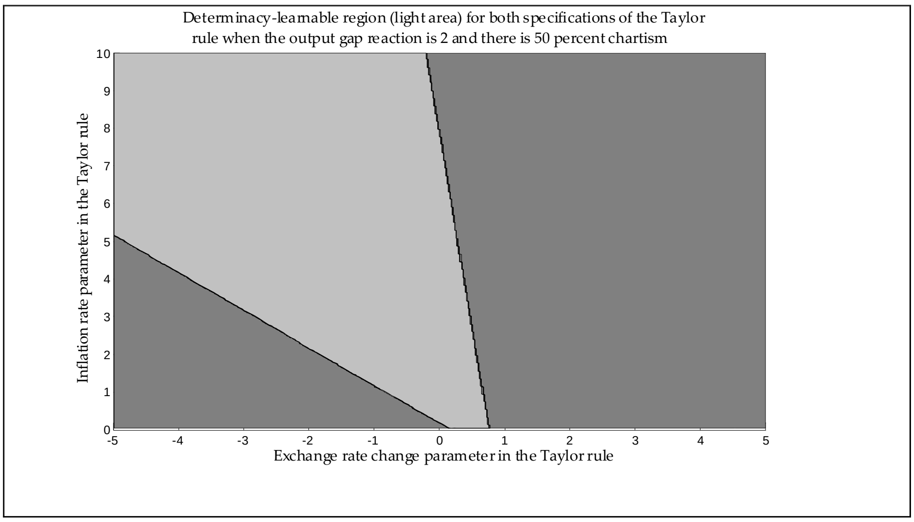

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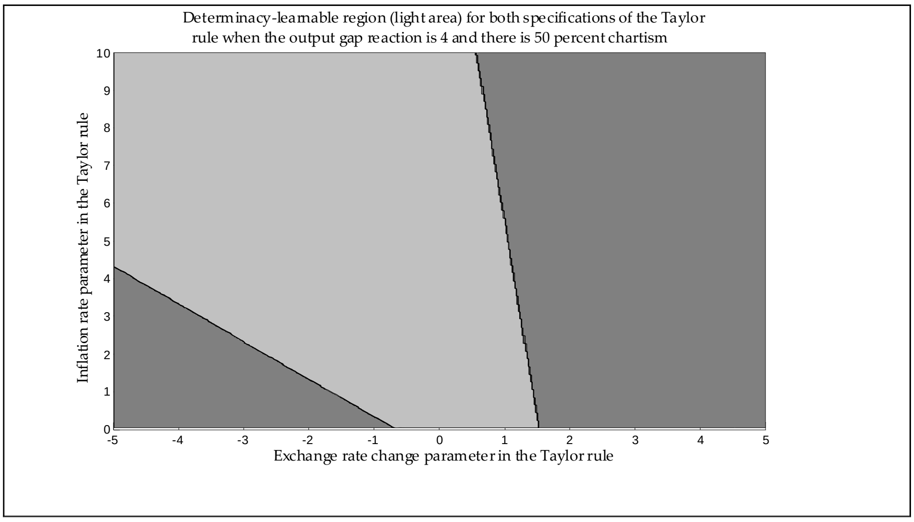

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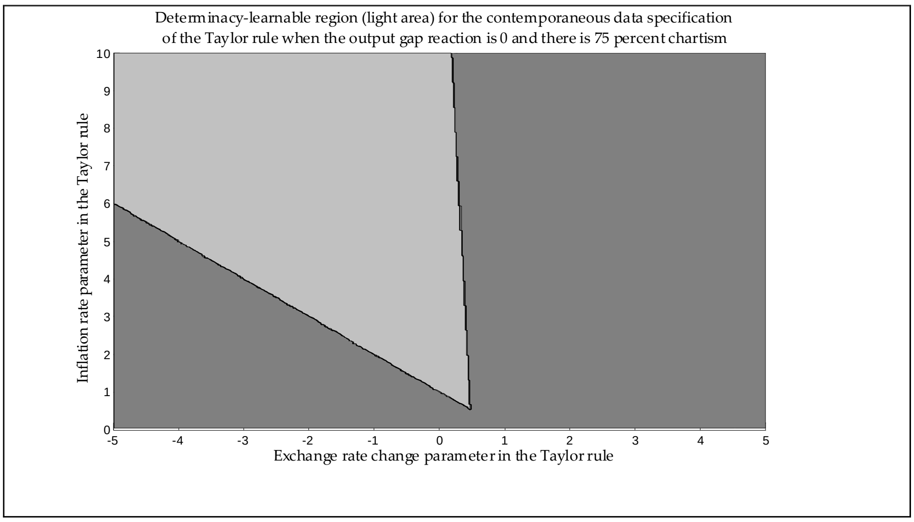

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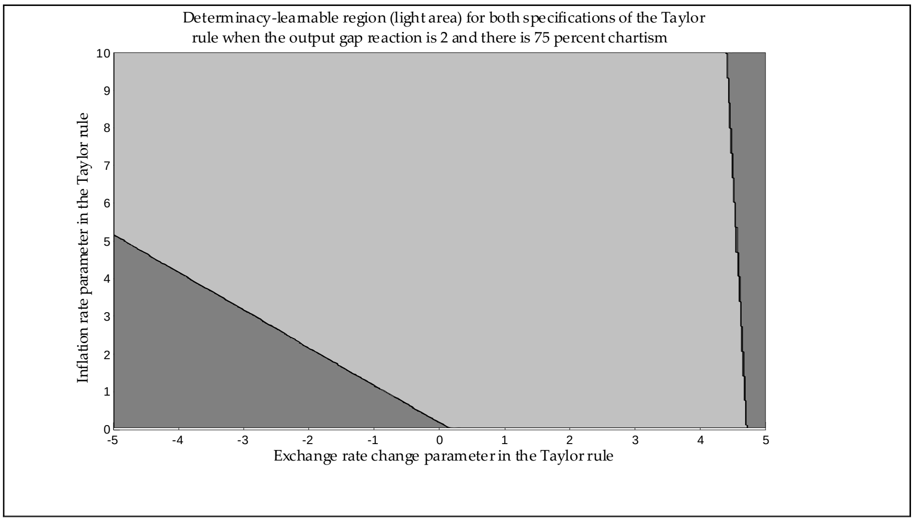

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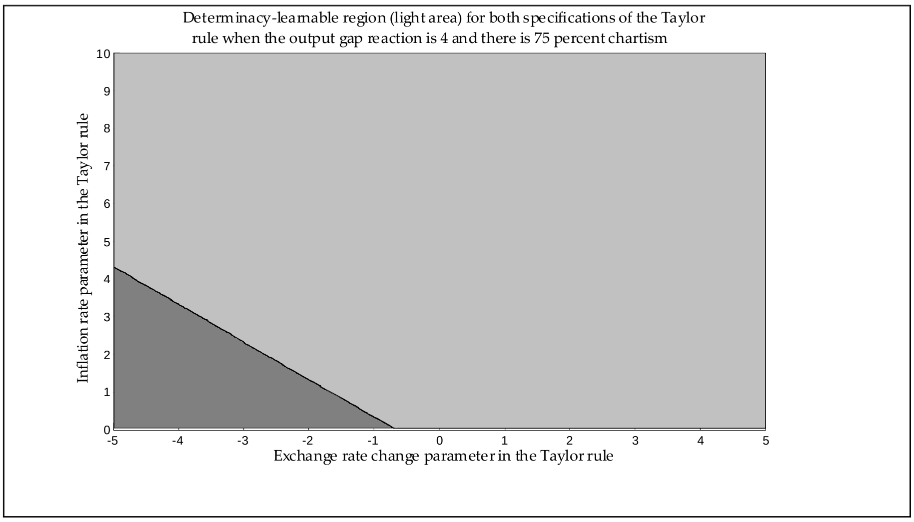

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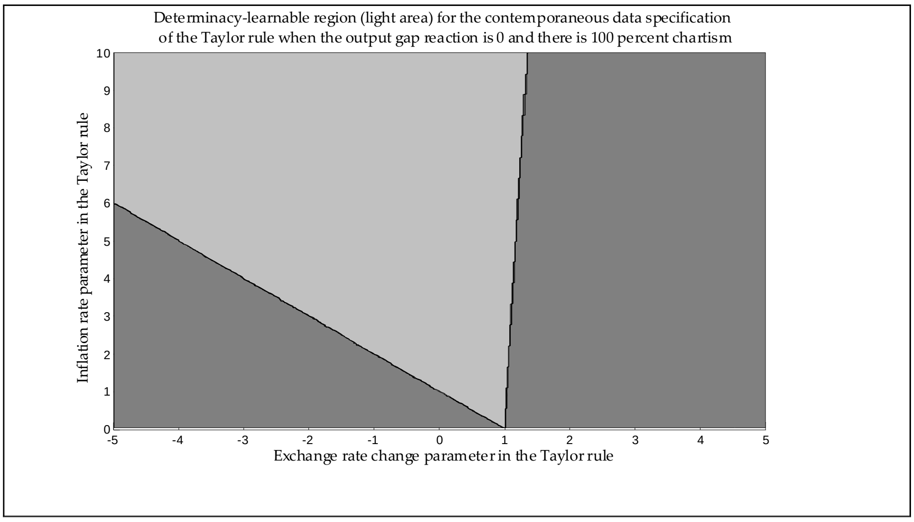
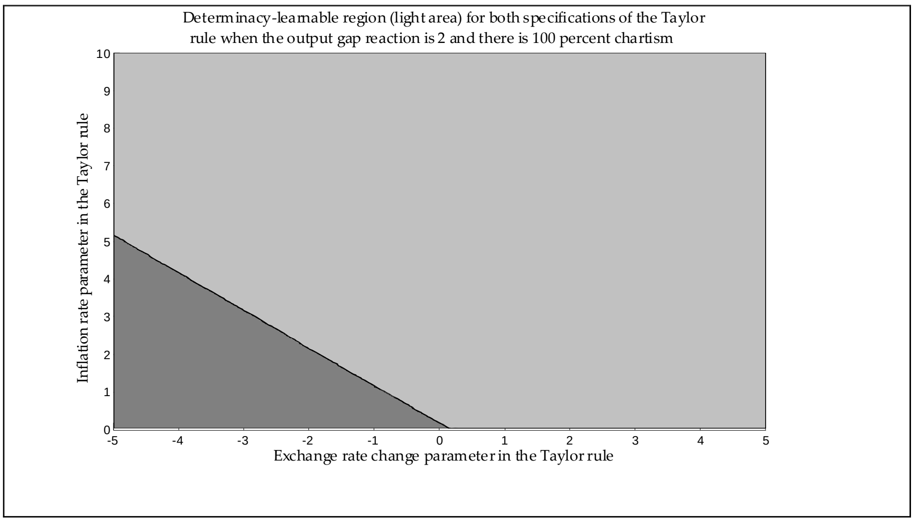


Figure 5 a.



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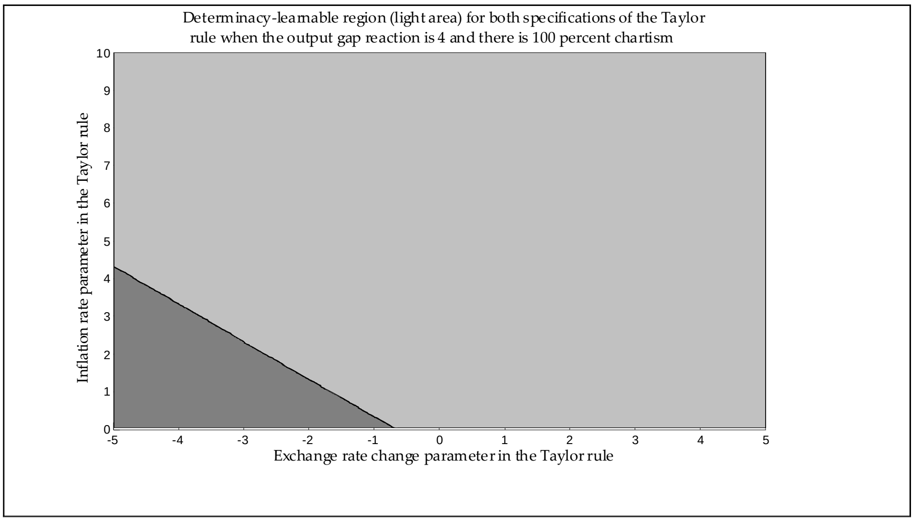

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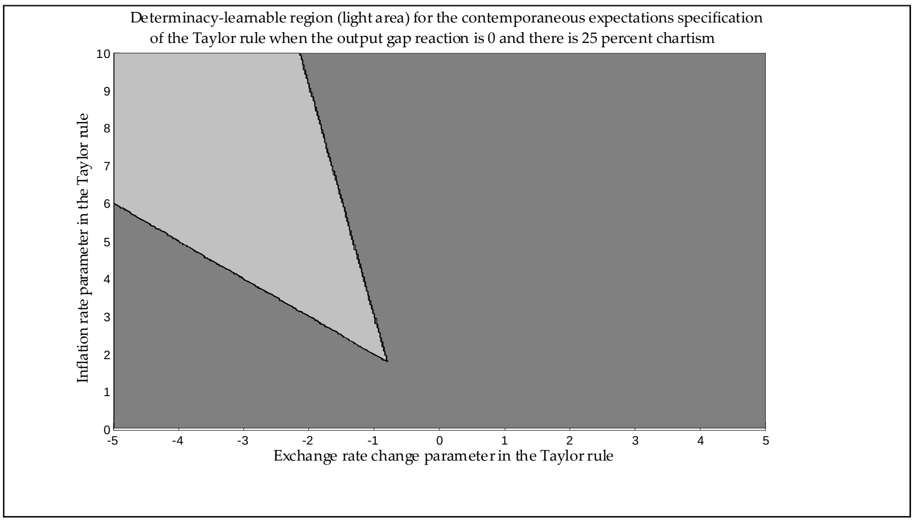

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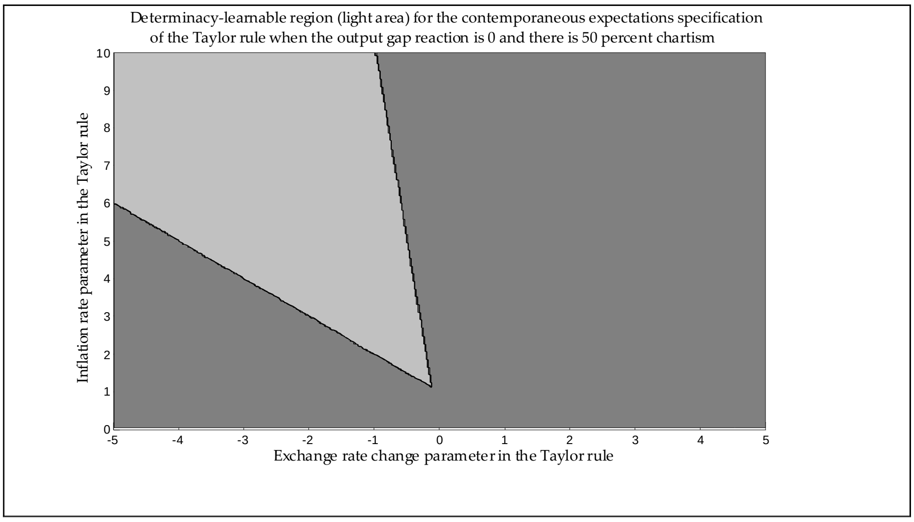

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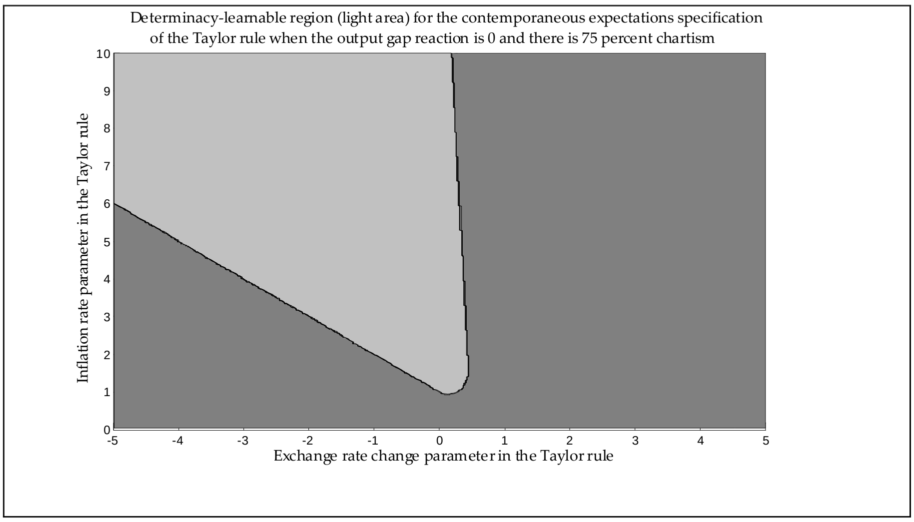

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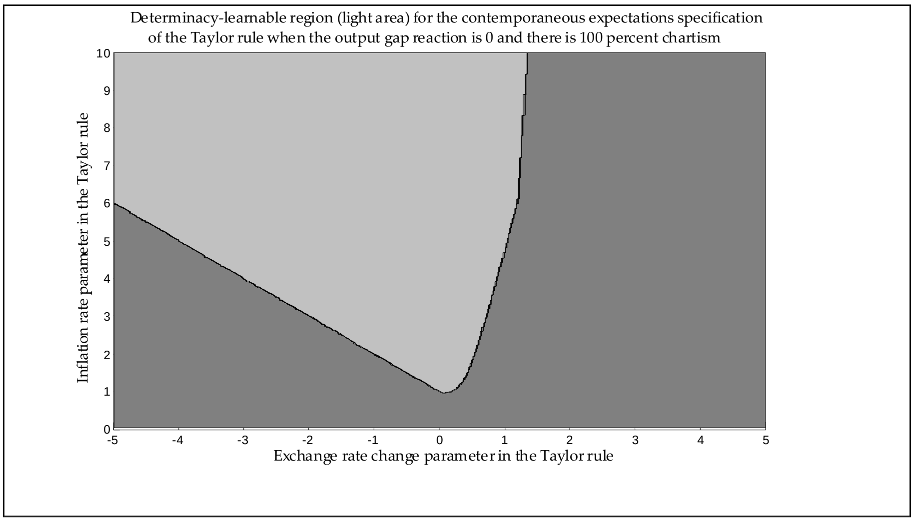

Figure 9.

Table 1 a

The Taylor rules below satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative. The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.01. Consequently, the constant in the Taylor rule is also 0.01.
The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest
Unique and learnable equilibrium:	when 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade

The parameter values in the rules are whole numbers.

Parameter values in the Taylor rule	Nominal interest rate	Expected CPI inflation rate	Conditional volatility of CPI inflation rate
(output gap, inflation rate, exchange rate change, percent chartism)			
(0, 8, -2, 95)	0.0014	-0.0032	0.0262
(0, 8, -2, 25)	0.0020	-0.0019	0.0861
(0, 9, -2, 95)	0.0025	-0.0025	0.0232
(0, 9, -2, 25)	0.0033	-0.0011	0.0838
(0, 10, -3, 95)	7.5059e-4	-0.0037	0.0280
(0, 10, -3, 25)	9.9770e-4	-0.0024	0.0881
(0, 10, -3, 15)[1]	0.0011	-0.0019	0.1156
(1, 10, -4, 95)	7.5506e-4	-0.0067	0.0064
(1, 10, -4, 25)	0.0010	-0.0047	0.0785

[1] The Taylor rule is also desirable when 15 percent chartism in currency trade.

Table 1 b

The Taylor rules below satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative. The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.02. Consequently, the constant in the Taylor rule is also 0.02.
The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest
Unique and learnable equilibrium:	when 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade

The parameter values in the rules are whole numbers.

Parameter values in the Taylor rule	Nominal interest rate	Expected CPI inflation rate	Conditional volatility of CPI inflation rate
(output gap, inflation rate, exchange rate change, percent chartism)			
(0, 8, -2, 95)	0.0028	-0.0064	0.0262
(0, 8, -2, 25)	0.0040	-0.0037	0.0861
(0, 9, -2, 95)	0.0049	-0.0050	0.0232
(0, 9, -2, 25)	0.0066	-0.0022	0.0838
(0, 10, -3, 95)	0.0015	-0.0073	0.0280
(0, 10, -3, 25)	0.0020	-0.0047	0.0881
(0, 10, -3, 15)[2]	0.0022	-0.0039	0.1156

[2]The Taylor rule is also desirable when 15 percent chartism in currency trade.

Table 1 c

The Taylor rules below satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative. The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.03. Consequently, the constant in the Taylor rule is also 0.03.
The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest
Unique and learnable equilibrium:	when 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade

The parameter values in the rules are whole numbers.

Parameter values in the Taylor rule	Nominal interest rate	Expected CPI inflation rate	Conditional volatility of CPI inflation rate
(output gap, inflation rate, exchange rate change, percent chartism)			
(0, 8, -2, 95)	0.0043	-0.0095	0.0262
(0, 8, -2, 25)	0.0060	-0.0056	0.0861
(0, 9, -2, 95)	0.0074	-0.0074	0.0232
(0, 9, -2, 25)	0.0099	-0.0033	0.0838

Table 2 a

The Taylor rules below satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative. The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.01. Consequently, the constant in the Taylor rule is also 0.01. The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest
Unique and learnable equilibrium:	when 55, 65, 75, 85 and 95 percent chartism in currency trade[3]

The parameter values in the rules are whole numbers. Finally, since there are 37 rules that satisfy the criteria, only the 10 best rules according to the conditional volatility of the CPI inflation rate are listed.

Parameter values in the Taylor rule	Nominal interest rate	Expected CPI inflation rate	Conditional volatility of CPI inflation rate	Parameter values in the Taylor rule	Nominal interest rate	Expected CPI inflation rate	Conditional volatility of CPI inflation rate
(output gap, inflation rate, exchange rate change, percent chartism)							
(1, 8, 0, 95)	0.0085	-9.2669e-4	6.9865e-4	(3, 8, 0, 95)	0.0086	-0.0011	2.8590e-4
(1, 8, 0, 55)	0.0092	-1.0997e-4	0.0117	(3, 8, 0, 55)	0.0091	-1.8865e-4	0.0118
(1, 9, 0, 95)	0.0086	-7.9084e-4	6.6905e-4	(3, 9, 0, 95)	0.0088	-9.6749e-4	2.8104e-4
(1, 9, 0, 55)	0.0094	1.4530e-5	0.0113	(3, 9, 0, 55)	0.0093	-5.7619e-5	0.0116
(2, 8, 0, 95)	0.0086	-0.0010	3.8413e-4	(3, 10, 0, 95)	0.0089	-8.5817e-4	2.7630e-4
(2, 8, 0, 55)	0.0092	-1.5080e-4	0.0117	(3, 10, 0, 55)	0.0094	4.6640e-5	0.0115
(2, 9, 0, 95)	0.0087	-9.0074e-4	3.7480e-4	(4, 9, 0, 95)	0.0087	-0.0010	2.3618e-4
(2, 9, 0, 55)	0.0093	-2.0571e-5	0.0115	(4, 9, 0, 55)	0.0092	-9.2913e-5	0.0117
(2, 10, 0, 95)	0.0088	-7.9179e-4	3.6582e-4	(4, 10, 0, 95)	0.0089	-9.0982e-4	2.3311e-4
(2, 10, 0, 55)	0.0095	8.1409e-5	0.0113	(4, 10, 0, 55)	0.0094	1.2024e-5	0.0115

[3] None of the Taylor rules below are desirable when 45 percent chartism in currency trade or lower.

Table 2 b

The Taylor rules below satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative. The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.02. Consequently, the constant in the Taylor rule is also 0.02.
The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest
Unique and learnable equilibrium:	when 55, 65, 75, 85 and 95 percent chartism in currency trade[4]

The parameter values in the rules are whole numbers. Finally, since there are 33 rules that satisfy the criteria, only the 10 best rules according to the conditional volatility of the CPI inflation rate are listed.

Parameter values in the Taylor rule	Nominal interest rate	Expected CPI inflation rate	Conditional volatility of CPI inflation rate	Parameter values in the Taylor rule	Nominal interest rate	Expected CPI inflation rate	Conditional volatility of CPI inflation rate
(output gap, inflation rate, exchange rate change, percent chartism)							
(1, 8, 0, 95)	0.0169	-0.0019	6.9865e-4	(3, 8, 0, 95)	0.0172	-0.0022	2.8590e-4
(1, 8, 0, 55)	0.0184	-2.1994e-4	0.0117	(3, 8, 0, 55)	0.0182	-3.7729e-4	0.0118
(1, 9, 0, 95)	0.0172	-0.0016	6.6905e-4	(3, 9, 0, 95)	0.0175	-0.0019	2.8104e-4
(1, 9, 0, 55)	0.0188	2.9060e-5	0.0113	(3, 9, 0, 55)	0.0186	-1.1524e-4	0.0116
(2, 8, 0, 95)	0.0172	-0.0021	3.8413e-4	(3, 10, 0, 95)	0.0177	-0.0017	2.7630e-4
(2, 8, 0, 55)	0.0183	-3.0160e-4	0.0117	(3, 10, 0, 55)	0.0188	9.3280e-5	0.0115
(2, 9, 0, 95)	0.0175	-0.0018	3.7480e-4	(4, 9, 0, 95)	0.0175	-0.0020	2.3618e-4
(2, 9, 0, 55)	0.0187	-4.1142e-5	0.0115	(4, 9, 0, 55)	0.0185	-1.8583e-4	0.0117
(2, 10, 0, 95)	0.0177	-0.0016	3.6582e-4	(4, 10, 0, 95)	0.0177	-0.0018	2.3311e-4
(2, 10, 0, 55)	0.0190	1.6282e-4	0.0113	(4, 10, 0, 55)	0.0187	2.4048e-5	0.0115

[4] None of the Taylor rules below are desirable when 45 percent chartism in currency trade or lower.

Table 2 c

The Taylor rules below satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative. The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.03. Consequently, the constant in the Taylor rule is also 0.03.
The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest
Unique and learnable equilibrium:	when 55, 65, 75, 85 and 95 percent chartism in currency trade[5]

The parameter values in the rules are whole numbers. Finally, since there are 30 rules that satisfy the criteria, only the 10 best rules according to the conditional volatility of the CPI inflation rate are listed.

Parameter values in the Taylor rule	Nominal interest rate	Expected CPI inflation rate	Conditional volatility of CPI inflation rate	Parameter values in the Taylor rule	Nominal interest rate	Expected CPI inflation rate	Conditional volatility of CPI inflation rate
(output gap, inflation rate, exchange rate change, percent chartism)							
(1, 8, 0, 95)	0.0254	-0.0028	6.9865e-4	(3, 8, 0, 95)	0.0258	-0.0033	2.8590e-4
(1, 8, 0, 55)	0.0276	-3.2991e-4	0.0117	(3, 8, 0, 55)	0.0273	-5.6594e-4	0.0118
(1, 9, 0, 95)	0.0258	-0.0024	6.6905e-4	(3, 9, 0, 95)	0.0263	-0.0029	2.8104e-4
(1, 9, 0, 55)	0.0282	4.3591e-5	0.0113	(3, 9, 0, 55)	0.0278	-1.7286e-4	0.0116
(2, 8, 0, 95)	0.0257	-0.0031	3.8413e-4	(3, 10, 0, 95)	0.0266	-0.0026	2.7630e-4
(2, 8, 0, 55)	0.0275	-4.5240e-4	0.0117	(3, 10, 0, 55)	0.0282	1.3992e-4	0.0115
(2, 9, 0, 95)	0.0262	-0.0027	3.7480e-4	(4, 9, 0, 95)	0.0262	-0.0031	2.3618e-4
(2, 9, 0, 55)	0.0280	-6.1713e-5	0.0115	(4, 9, 0, 55)	0.0277	-2.7874e-4	0.0117
(2, 10, 0, 95)	0.0265	-0.0024	3.6582e-4	(4, 10, 0, 95)	0.0266	-0.0027	2.3311e-4
(2, 10, 0, 55)	0.0284	2.4423e-4	0.0113	(4, 10, 0, 55)	0.0281	3.6072e-5	0.0115

[5] None of the Taylor rules below are desirable when 45 percent chartism in currency trade or lower.

Table 3 a

The number of Taylor rules that satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative. The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.01. Consequently, the constant in the Taylor rule is also 0.01. The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest

The parameter values in the rules are multiples of 0.1.

Unique and learnable equilibrium:	when 5, 15, 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	7 122 out of 418 241, i.e., 1.7028 percent of the rules
Number of desirable rules:	16 out of 418 241, i.e., 0.0038 percent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141
Unique and learnable equilibrium:	when 15, 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	12 790 out of 418 241, i.e., 3.0580 percent of the rules
Number of desirable rules:	404 out of 418 241, i.e., 0.0966 percent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141
Unique and learnable equilibrium:	when 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	25 591 out of 418 241, i.e., 6.1187 percent of the rules
Number of desirable rules:	2 482 out of 418 241, i.e., 0.5934 percent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141
Unique and learnable equilibrium:	when 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	65 085 out of 418 241, i.e., 15.5616 percent of the rules
Number of desirable rules:	14 669 out of 418 241, i.e., 3.5073 percent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141
Unique and learnable equilibrium:	when 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	128 699 out of 418 241, i.e., 30.7715 percent of the rules
Number of desirable rules:	60 574 out of 418 241, i.e., 14.4830 percent of the rules
Number of desirable rules when no exchange rate change reaction:	1 out of 4 141, i.e., 0.0241 percent of the rules
Unique and learnable equilibrium:	when 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	199 165 out of 418 241, i.e., 47.6197 percent of the rules
Number of desirable rules:	129 365 out of 418 241, i.e., 30.9307 percent of the rules
Number of desirable rules when no exchange rate change reaction:	3 152 out of 4 141, i.e., 76.1169 percent of the rules

Table 3 b

The number of Taylor rules that satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative. The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.02. Consequently, the constant in the Taylor rule is also 0.02. The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest

The parameter values in the rules are multiples of 0.1.

Unique and learnable equilibrium:	when 5, 15, 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	7 122 out of 418 241, i.e., 1.7028 percent of the rules
Number of desirable rules:	16 out of 418 241, i.e., 0.0038 percent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141
Unique and learnable equilibrium:	when 15, 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	12 790 out of 418 241, i.e., 3.0580 percent of the rules
Number of desirable rules:	241 out of 418 241, i.e., 0.0576 percent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141
Unique and learnable equilibrium:	when 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	25 591 out of 418 241, i.e., 6.1187 percent of the rules
Number of desirable rules:	1 034 out of 418 241, i.e., 0.2472 percent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141
Unique and learnable equilibrium:	when 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	65 085 out of 418 241, i.e., 15.5616 percent of the rules
Number of desirable rules:	4 234 out of 418 241, i.e., 1.0123 percent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141
Unique and learnable equilibrium:	when 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	128 699 out of 418 241, i.e., 30.7715 percent of the rules
Number of desirable rules:	32 809 out of 418 241, i.e., 7.8445 percent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141
Unique and learnable equilibrium:	when 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	199 165 out of 418 241, i.e., 47.6197 percent of the rules
Number of desirable rules:	99 213 out of 418 241, i.e., 23.7215 percent of the rules
Number of desirable rules when no exchange rate change reaction:	2 819 out of 4 141, i.e., 68.0753 percent of the rules

Table 3 c

The number of Taylor rules that satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative. The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.03. Consequently, the constant in the Taylor rule is also 0.03. The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest

The parameter values in the rules are multiples of 0.1.

Unique and learnable equilibrium:	when 5, 15, 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	7 122 out of 418 241, i.e., 1.7028 percent of the rules
Number of desirable rules:	0 out of 418 241
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141
Unique and learnable equilibrium:	when 15, 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	12 790 out of 418 241, i.e., 3.0580 percent of the rules
Number of desirable rules:	16 out of 418 241, i.e., 0.0038 percent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141
Unique and learnable equilibrium:	when 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	25 591 out of 418 241, i.e., 6.1187 percent of the rules
Number of desirable rules:	175 out of 418 241, i.e., 0.0418 percent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141
Unique and learnable equilibrium:	when 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	65 085 out of 418 241, i.e., 15.5616 percent of the rules
Number of desirable rules:	949 out of 418 241, i.e., 0.2269 percent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141
Unique and learnable equilibrium:	when 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	128 699 out of 418 241, i.e., 30.7715 percent of the rules
Number of desirable rules:	14 528 out of 418 241, i.e., 3.4736 percent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141
Unique and learnable equilibrium:	when 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	199 165 out of 418 241, i.e., 47.6197 percent of the rules
Number of desirable rules:	77 188 out of 418 241, i.e., 18.4554 percent of the rules
Number of desirable rules when no exchange rate change reaction:	2 481 out of 4 141, i.e., 59.9131 percent of the rules

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