Adaptation, Anticipation-Bias and Optimal Income Taxation**

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Abstract
Adaptation is omnipresent but people systematically fail to correctly anticipate the degree to which they adapt. This leads individuals to make inefficient intertemporal decisions. This paper concerns optimal income taxation to correct for such anticipation-biases in a framework where consumers adapt to earlier consumption levels through a habit-formation process. The analysis is based on a general equilibrium OLG model with endogenous labor supply and savings where each consumer lives for three periods. Our results show how a paternalistic government may correct for the effects of anticipation-bias through a combination of time-variant marginal labor income taxes and savings subsidies. Furthermore, the optimal policy mix remains the same, irrespective of whether consumers commit to their original life-time plan for work hours and savings decided upon in the first period of life or re-optimize later on when realizing the failure to adapt.

Keywords: Optimal taxation, adaptation, habit-formation, anticipation-bias, paternalism.

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1. Introduction

Adaptation is omnipresent as people adapt to almost all life circumstances. For instance, when income and thus the consumption possibilities rise, adaptation implies that the increase in utility may only be transitory and potential utility gains evaporate over time. Indeed, empirical evidence suggests that adaptation eliminates 60 percent of the initial positive effect of an increase in the individual income level on happiness within two years (cf. Clark, Frijters and Shields 2008).

These psychological processes of adaptation have not played any important role in economic analysis. Becker and Murphy (1988) concede that adaptation is important but emphasize that its presence causes no problem for normative economic theory. Adaptation, habituation or, in its strongest form, addiction, can easily be incorporated into the rational choice framework as long as people can foresee the mechanisms of adaptation and addiction. With perfect anticipation, agents will, irrespectively of the degree of adaptation, always act in their own best interest.

However, recent psychological research provides overwhelming evidence that people do not anticipate correctly how they adapt to changes in life circumstances. Wilson and Gilbert (2003) summarize the evidence and argue that while people can correctly anticipate the valence of future feelings and specific emotions (such as joy or sadness) they make systematic errors in predicting the intensity and durability of future feelings. If the person today cannot predict the feeling of the person tomorrow, the consequences are very similar to the consequences of externalities. The cost one’s decision imposes on others – in this case on one’s own future selves – is not adequately accounted for in one’s decision. This behavioral failure might be called adaptation internality. In as far as a person today does not adequately project the consequences of her decisions for the person she will be tomorrow, it follows that the person today makes a decision that may harm her future self. We may end up wanting things that do not make us happier or we may not want things that would make us happier (Dolan and Kahneman 2008). So far, such anticipation-biases have hardly been incorporated into economic modeling. A noteworthy exception is the study by Loewenstein, O'Donoghue and Rabin (2003), being the first to analyze how systematic errors in anticipating adaptation processes may affect the choices people make and lead to economic outcomes that are not in their own best interest.
If individuals’ decisions lead to suboptimal outcomes for themselves, market corrections by the government may improve the individuals’ welfare. Note, however, that the normative concept of methodological individualism would not justify such government intervention. But behavioral economics has identified many situations where people do not appear to do what is best for them. Therefore, since interventions may actually lead people to do things that improve their own well-being, many economists have nowadays become supportive of paternalistic interventions that help people to avoid systematic decision errors without curtailing individual autonomy (cf. Loewenstein and Ubel 2008). We follow this approach by referring to the notion of optimal paternalism: the overall purpose of our study is to examine how the government could use income tax policy to correct for behavioral mistakes caused by anticipation-bias. Taxes are particularly interesting as a means of correction because they leave people with the freedom of choice, while at the same time altering individual incentives to eliminate the behavioral failure. In this paper, we provide such a normative-paternalistic approach to deal with the effects of anticipation-bias of adaptation. To our knowledge, there are no earlier studies dealing with optimal taxation in this particular context.1

Our model is based on an extension of the framework used by Loewenstein, O’Donoghue and Rabin (2003), who develop a simple overlapping-generations—“eat-the-cake” model with habit-formation to illustrate how anticipation-bias leads to misallocation of the consumption of a given resource stock over time. We extend their model by incorporating production and labor supply decisions. Each consumer is assumed to live for three periods, which is the minimum number of periods required to distinguish between a commitment solution in which the consumer follows the original life-time plan for consumption and work hours, and a solution where the consumer re-optimizes in the second period of life when realizing the failure to adapt. As such, we are also able to address the policy implications of a potential time-inconsistency problem caused by anticipation-bias. The tax instruments faced by the government are nonlinear taxes on labor income and capital income. A major advantage with non-linear taxation is that tax distortions will be a consequence of optimization by the government and not due to the necessity to raise revenue per se, allowing us to focus on the policy incentives created by anticipation-bias in the simplest possible way. The optimal tax policy is derived by comparing the utility individuals derive from their biased decisions with

1 Other literature on optimal paternalism has typically focused on the policy implications of self-control problems (e.g., Gruber and Köszegi, 2004; O'Donoghue and Rabin, 2003, 2006; Aronsson and Thunström, 2008; Aronsson and Granlund, 2011).
the potentially optimal utility level they could achieve in the absence of anticipation-bias – which we consider as the benchmark of an intervening paternalistic government.

We show that a paternalistic government has an incentive to use both tax instruments to correct for the anticipation-bias, as there are two channels by which the utility from future consumption is affected. Adaptation implies that if people work more and save less when they are young, they get used to higher consumption levels, which means that the utility derived from any given level of future consumption will fall. If people correctly foresee this adaptation to higher consumption, they will take into account this fall in future utility and make optimal intertemporal labor supply and savings decisions. In the presence of an anticipation-bias, however, they are likely to work too much and save too little when young and middle-aged, respectively. Our results show that the simultaneous use of marginal labor income taxes and savings subsidies may correct for the anticipation-bias. Furthermore, this holds irrespective of whether consumers commit to their original life-time plans for work hours and savings (decided upon when young) or re-optimize later on when realizing the failure to adapt.

We proceed as follows. After a brief review on the psychological and economic literature on adaptation and the anticipation-bias of adaptation in section 2, we introduce the model in section 3. In section 4 we then derive the main results and show how an optimal income tax system should be designed in the presence of anticipation-bias. Section 5 summarizes.

2. Adaptation and anticipation-biases: a brief literature review

Adaptation matters in many domains of life. When hit by a negative life event, adaptation may come as a relief as people adapt, at least partially, to calamities such as the death of a beloved partner, long-lasting illnesses or the loss of one’s wealth position (see e.g. Oswald and Powdthavee 2008, Diener, Lucas and Scollon 2006). For positive events, adaptation appears less desirable because it bereaves people from the permanent joy of positive life events such as the honeymoon after marriage, the birth of a child or an unexpected lottery win. Adaptation may be physiological, i.e. sensory in the sense that one becomes less exposed to repeated stimuli over time or may “involve cognitive changes in interests, values, goals, attention, or characterization of a situation” (Frederick and Loewenstein 1999 p. 302, also see Dolan und Kahneman 2008).
The hedonic threatmill theory (Brickman and Cambell 1971) claims that adaptation is inevitable and that it is almost complete. People tend to quickly return to their baseline level of happiness or utility following any change in life circumstances. Brickman, Coates and Janoff-Bulman (1978) provided evidence by comparing the happiness levels of lottery winners with the happiness level of people with spinal-cord injuries and a control group and found no significant differences.

However, complete adaptation has been rejected in many other studies. Diener et al. (2009) reinterpret the data from Brickman, Coates and Janoff-Bulman (1978) and argue that although the gap in reported well-being between the control group and the spinal-cord-injured group is smaller than one might have expected, it is nevertheless the case that 79% of the spinal-cord-injured group were less happy than the average control participant. In general, the literature now assumes that with respect to some events and life circumstances complete adaptation may indeed occur, but several surveys in recent years (see e.g. Lucas 2007, Diener et al. 2009 and Luhmann et al. 2011) find that adaptation varies between different life domains. Often, adaptation is slow and changes in life satisfaction can indeed have long-lasting effects on the well-being of individuals. For instance, the average person quickly adapts to marriage after a short time-period, while adaptation to the death of one’s spouse may proceed very slowly. Unemployment has long-lasting negative effects on well-being and people with severe disabilities also continue to report life-satisfaction levels that are more than a full standard deviation below their baseline levels (cf. Lucas 2007). Furthermore, individuals differ substantially in how they adapt to changes in life circumstances (Diener, Lucas and Scollon 2006 p. 311f).

Adaptation may not be complete, but it is important nevertheless. A striking example seems to be adaptation to income changes. Besides relative comparisons with others (e.g. Johansson-Stenman, Carlsson and Daruvala 2002, Frank 2005, Luttmer 2005, Solnick and Hemenway 2005, Carlsson, Johansson-Stenman and Martinsson 2007), adaptation may explain a good deal of the so-called Easterlin (1974) paradox. While cross section analyses show that people with higher income are happier than people with lower income, panel studies show that over time, higher income hardly improves the subjective well-being when the income is above a certain limit that is necessary to meet the basic needs. This limit rises as income rises. For Switzerland, Stutzer (2004) reports that a 10 percent increase in household income raises this income limit by 4.5 percent. This is suggestive evidence for a shift in the reference point to
which people compare their actual income, which in turn can be interpreted as cognitive adaptation. Van Praag and van der Sar (1988) find similar results as they show that the reference point shift eliminates about 60 percent of the expected welfare effect of increases in income. Clark, Frijters and Shields (2008 p. 110), by referring to several empirical results, thus suggest a utility function where 60 percent of the effect of an increase in the individual income level evaporates within two years due to adaptation.

The permanent adjustment of one’s reference income induces people to accomplish more and more as they will never be satisfied (Frey and Stutzer 2002). If people adapt, both the utility level and the marginal utility of income depend on factors that affect the reference points such as one’s own previous income. Economic theory has taken this into account (whereby adaptation is normally analyzed under the heading of habit formation). Among the first economists who dealt with adaptation were Edgeworth (1881), Marshall (1890) and Duesenberry (1949). Pollak (1970) was the first to formulate a model of consumer behavior and derive demand functions that included adaptation (habit formation), in particular by allowing some or all parameters of the utility function to depend on past consumptions. He thus provides a theoretical framework of adaptation for positive economic analysis.

Becker and Murphy (1988), however, argue that adaptation causes no problem for normative economic analysis. Adaptation or, in its stronger form, addiction to alcohol, cocaine, and cigarettes, but also to work, eating, music, television, the standard of living, other people, religion, and many other activities can be analyzed within the rational choice framework if one assumes that people can foresee the mechanisms of adaptation and addiction. Thus, including adaptation causes no threat to standard economic reasoning and economic theory can deal with it in an appropriate way and show that rational consumers will always act in their best interest.²

Recent research in psychology challenges the idea of rational addiction. There is overwhelming evidence that people do not correctly foresee their ability to adapt to new circumstances. According to Wilson and Gilbert (2003), while people correctly anticipate the valence of our future feelings and specific emotions, they make systematic errors in predicting

² One application of this approach to rational addiction is found in Carroll, Overland and Weil (2000) who use a model of adaptation to reconcile the empirical findings that high growth leads to higher savings. While standard economic theory would predict savings to fall because people in countries with high growth rates will anticipate that they become richer even when they save little, anticipating adaptation to income may lead people to save more because this would not only leave more income for tomorrow but would also lower the reference point and thus increase utility from future consumption.
the intensity and durability of future feelings: “It is useful to know that we will feel happy on our first day at a new job, but better to know how happy and how long this feeling will last, before committing ourselves to a lifetime of work as a tax attorney” (Wilson and Gilbert 2003 p. 349). To correctly foresee our future feelings it is important not only to foresee particular changes in life circumstances but also to “have some degree of insight into a set of dynamic psychological processes … that produce a change in the relationship between what happens and how one feels” (Ubel, Loewenstein and Jepson 2005 p. 113). Indeed, the psychological literature shows that “people’s mental simulations of future events are almost always imperfect” and that people are “insufficiently sensitive to the fact that mental simulations lack important details.” (Dunn, Gilbert and Wilson 2011 p. 115f, for recent reviews also see Gilbert and Wilson 2007, 2009). The lack of such insights may be of an evolutionary advantage for the survival of genes. Overestimating the intensity and duration of negative outcomes motivates people to work harder in the present to prevent negative outcomes and exaggerating the intensity and duration of reactions to positive events can be a motivator for people to work harder to obtain these outcomes (cf. Wilson and Gilbert 2003 p. 399).

Anticipation-biases may be extremely large. For instance, healthy people seem to underestimate the extent to which they adjust to health problems. Sackett and Torrance (1978) show that healthy people evaluate one additional life year as dialysis patient as being equivalent of living 0.39 additional years as a healthy person while actual dialysis patients evaluate their current life as equivalent of living 0.56 additional years as a healthy person.

While, as mentioned above, adaptation has been analyzed in economic theory, anticipation-bias, by contrast, has hardly been incorporated into economic modeling. To the best of our knowledge, Loewenstein, O'Donoghue and Rabin (2003) were the first to analyze how systematic errors in anticipating adaptation processes affect the choices people make and may lead to economic outcomes that are not in their own best interest. Their paper thus will be the starting point for our analysis of tax policy implications of anticipation-bias from the point of view of a paternalistic government.

3. The model

In this section, we present the model and analyze the behavior of private agents, i.e. consumers and firms, taking into account that private agents adapt to higher income and do
not anticipate their ability to adapt. We also present the decision-problem faced by a
paternalistic government that maximizes the aggregate intertemporal private agents’ utility.
The outcome in terms of optimal taxation will be addressed in section 4 below.

**Consumer behavior**

Consider an overlapping-generations-economy where each consumer lives for three periods,
works in the first two and is retired in the third. We will refer to an individual born in the
beginning of period \( t \) as being part of “generation \( t \)” in what follows; since population growth
is not important for the qualitative results to be derived below, we normalize the number of
individuals of each such generation to one. An individual of generation \( t \) faces the following
instantaneous utility functions:

\[
\begin{align*}
    u_{0,t} &= u(c_{0,t} - c^R_{0,t}, z_{0,t}) \\
    u_{1,t+1} &= u(c_{1,t+1} - c^R_{1,t+1}, z_{1,t+1}) \\
    u_{2,t+2} &= u(c_{2,t+2} - c^R_{2,t+2}, H)
\end{align*}
\]

where \( c \) is private consumption and \( z \) leisure. An individual of generation \( t \) is young in
period \( t \) (subindex 0), middle-aged in period \( t+1 \) (subindex 1) and old in period \( t+2 \) (subindex
2). For the young and middle-aged, leisure is defined as a time endowment, \( H \), less the hours
of work, \( l \); when old, all available time is leisure. The variable \( c^R \) is an internal reference
point for consumption to which we return below. The intertemporal objective facing the
individual can then be written as

\[
U_{0,t} = u_{0,t} + u_{1,t+1} \Theta + u_{2,t+2} \Theta^2,
\]

in which \( \Theta \) indicates the utility discount factor, i.e. \( \Theta = 1/(1+\theta) \), where \( \theta \) is the utility
discount rate.

The individual budget constraints for each of the three living periods are given by

\[
\begin{align*}
    w_{0,t}l_{0,t} - T_0(w_{0,t}l_{0,t}) - s_{0,t} &= c_{0,t}, \\
    s_{0,t}(1+r_{t+1}) + w_{1,t+1}l_{1,t+1} - T_1(w_{1,t+1}l_{1,t+1}, s_{0,t}r_{t+1}) - s_{1,t+1} &= c_{1,t+1}, \\
    s_{1,t+1}(1+r_{t+2}) - T_2(s_{1,t+1}r_{t+2}) &= c_{2,t+2},
\end{align*}
\]
where \( w \) denotes the before-tax wage rate and \( s \) savings, while the functions \( T_0(\cdot) \), \( T_1(\cdot) \) and \( T_2(\cdot) \) are age-specific income tax payments, e.g., tax functions with age-specific intercepts and slope parameters. The individual pays labor income taxes when young and middle-aged, and capital income taxes when middle-aged and old (the purposes of which are to affect the savings behavior when young and middle-aged, respectively). For further use, we introduce the following notation for marginal income tax rates (where super-index “\( l \)” refers to labor income and super-index “\( s \)” to capital income):

\[
\tau^l_{0,t} = \frac{\partial T_0(w_{0,t},l_{l,t})}{\partial (w_{0,t},l_{l,t})}, \quad \tau^l_{1,t+1} = \frac{\partial T_1(w_{1,t+1},l_{l,t+1},s_{0,t},r_{s,t})}{\partial (w_{1,t+1},l_{l,t+1})}, \quad \tau^s_{1,t+1} = \frac{\partial T_1(w_{1,t+1},l_{l,t+1},s_{0,t},r_{s,t})}{\partial (s_{0,t},r_{s,t})}, \\
\tau^s_{2,t+2} = \frac{\partial T_1(s_{1,t+2},r_{s,t+2})}{\partial (s_{1,t+2},r_{s,t+2})}.
\]

We consider the case where people’s utility of current consumption depends on the previous consumption level. We model this type of adaptation or (internal) habit formation such that last period’s consumption level serves as a reference measure (or “habit stock”, cf. Loewenstein, O’Donoghue and Rabin 2003) with which the current consumption is compared.\(^3\) The consumer thus derives utility from the difference between the current consumption and last period’s consumption as well as from leisure. To be able to concentrate on adaptation, we abstract from the possibility that the individual also compares his/her own consumption with the consumption among other people.\(^4\)

Following the discussion in the last section, we further assume that the adaptation process is subject to an anticipation-bias in the sense that the individual may underestimate how much he/she will adapt to a change in the private consumption. The reference measure is determined as follows:

\[
\begin{align*}
\zeta^R_{0,t} &= 0, \quad (4a) \\
\zeta^R_{1,t+1} &= (1 - \alpha)\zeta^R_{0,t}, \quad (4b)
\end{align*}
\]

\(^3\) Following Loewenstein, O’Donoghue and Rabin (2003), we abstract from the possibility of adaptation with respect to leisure. See also Dunn et al. (2011) who argue that adaptation to experiences is lower than to material purchases.

\[ c_{2,t+2}^R = \alpha c_{1,t+1}^R + (1-\alpha)c_{1,t+1} = (1-\alpha)\left[ \alpha c_{0,t} + c_{1,t+1} \right]. \] (4c)

The parameter \( \alpha \) represents anticipation-bias where, \( \alpha = 0 \) means no bias, and \( \alpha = 1 \) means full bias. Therefore, without an anticipation-bias, the second and third equations simplify to \( c_{1,t+1}^R = c_{0,t} \) and \( c_{2,t+2}^R = c_{1,t+1} \), respectively, and the model boils down to a standard model with habit formation. As such, we have full adaptation within one period. This assumption is made for analytical convenience since the focus of our interest is on the anticipation-bias.

Consider first the situation where the individual decides upon the whole life-time plan for consumption and work hours when young and then commits to this plan throughout life. Later, we will relax this assumption and, instead, assume that the individual may revise his/her consumption and hours of work plan when middle-aged.

An individual of generation \( t \) chooses \( l_{0,t}, \ s_{0,t}, \ l_{1,t+1} \) and \( s_{1,t+1} \) to maximize the life-time utility given by equation (2) subject to the life-time budget constraint presented in equations (3). Thereby, individuals only partially anticipate the adaptation process in private consumption as summarized by equations (4). To shorten the notation, let \( \Delta_{0,t} = c_{0,t} - c_{0,t}^R \), \( \Delta_{1,t+1} = c_{1,t+1} - c_{1,t+1}^R \) and \( \Delta_{2,t+2} = c_{2,t+2} - c_{2,t+2}^R \). The first order conditions for \( l_{0,t}, \ s_{0,t}, \ l_{1,t+1} \) and \( s_{1,t+1} \) can then be written as

\[
\begin{align*}
\frac{\partial u_{0,t}}{\partial \Delta_{0,t}} w_{0,t} [1-r_{0,t}^l] - \frac{\partial u_{0,t}}{\partial z_{0,t}} + \frac{\partial u_{1,t+1}}{\partial \Delta_{1,t+1}} \frac{\partial c_{1,t+1}^R}{\partial \Delta_{1,t+1}} + \frac{\partial u_{2,t+2}}{\partial \Delta_{2,t+2}} \Theta \frac{\partial c_{2,t+2}^R}{\partial \Delta_{2,t+2}} \bigg|_{\Theta = 0}, \\
\frac{\partial u_{0,t}}{\partial \Delta_{0,t}} w_{0,t} [1-r_{0,t}^l] + \frac{\partial u_{1,t+1}}{\partial \Delta_{1,t+1}} \left[ 1 + r_{1,t+1}^l + r_{1,t+1}^{s,t} \right] - \frac{\partial u_{1,t+1}}{\partial \Delta_{1,t+1}} \frac{\partial c_{1,t+1}^R}{\partial \Delta_{1,t+1}} + \frac{\partial u_{2,t+2}}{\partial \Delta_{2,t+2}} \Theta \frac{\partial c_{2,t+2}^R}{\partial \Delta_{2,t+2}} \bigg|_{\Theta = 0}, \\
\frac{\partial u_{1,t+1}}{\partial \Delta_{1,t+1}} w_{1,t+1} [1-r_{1,t+1}^l] - \frac{\partial u_{1,t+1}}{\partial \Delta_{1,t+1}} \Theta \frac{\partial c_{2,t+2}^R}{\partial \Delta_{2,t+2}} = 0, \\
\frac{\partial u_{2,t+2}}{\partial \Delta_{2,t+2}} w_{2,t+2} [1-r_{2,t+2}^l + r_{2,t+2}^{s,t}] - \frac{\partial u_{2,t+2}}{\partial \Delta_{2,t+2}} \Theta \frac{\partial c_{2,t+2}^R}{\partial \Delta_{2,t+2}} = 0.
\end{align*}
\] (5a, 5b, 5c, 5d)

The final term on the left-hand side of equations (5a) and (5b) reflects the effects of \( l_{0,t} \) and \( s_{0,t} \), respectively, on the reference consumption measures \( c_{1,t+1}^R \) and \( c_{2,t+2}^R \), whereas the final part of equations (5c) and (5d) appear because \( l_{1,t+1} \) and \( s_{1,t+1} \) affect the reference measure.
faced by the old consumer, $c_{2,t+2}^R$. Also, notice that the derivatives of $c_{1,t+1}^R$ and $c_{2,t+2}^R$ with respect to $l_{0,t}$, $s_{0,t}$, $l_{1,t+1}$ and $s_{1,t+1}$ – which are part of equations (5a)–(5d) – can be written as follows:

\[
\frac{\partial c_{1,t+1}^R}{\partial l_{0,t}} = (1 - \alpha)w_{0,t}(1 - \tau_{0,t}'), \\
\frac{\partial c_{1,t+1}^R}{\partial s_{0,t}} = -(1 - \alpha), \\
\frac{\partial c_{2,t+2}^R}{\partial l_{0,t}} = \alpha(1 - \alpha)w_{0,t}(1 - \tau_{0,t}'), \\
\frac{\partial c_{2,t+2}^R}{\partial s_{0,t}} = (1 - \alpha)\{1 + r_{t+1} - \tau_{1,t+1}'r_{t+2}'\}, \\
\frac{\partial c_{1,t+1}^R}{\partial l_{1,t+1}} = \frac{\partial c_{1,t+1}^R}{\partial s_{1,t+1}} = 0, \\
\frac{\partial c_{2,t+2}^R}{\partial l_{1,t+1}} = (1 - \alpha)w_{1,t+1}(1 - \tau_{1,t+1}'), \\
\frac{\partial c_{2,t+2}^R}{\partial s_{1,t+1}} = -(1 - \alpha).
\]

The analysis so far assumes that the young consumer makes a plan for consumption and work hours over the whole life-cycle, which his/her future selves will follow. An alternative is that the individual revises the plan when middle-aged, i.e. that generation $t$ re-optimizes by revising the choices of $l_{1,t+1}$ and $s_{1,t+1}$ in period $t+1$. The rationale for this change of assumption is that the individual, when reaching middle-age, realizes that he/she has already adapted to the new consumption level. For the young consumer, the decision-problem is the same as before: he/she chooses $l_{0,t}$, $s_{0,t}$, $l_{1,t+1}$ and $s_{1,t+1}$ to maximize the life-time utility given by equation (2) subject to the budget constraint and the process for how the reference points are formed given in equations (3) and (4), respectively. Therefore, $l_{0,t}$ and $s_{0,t}$ will take the same values as under the commitment solution analyzed above and this yields equations (5a)–(5d).
When reaching middle-age in period $t+1$, however, the individual realizes that $c^R_{t+1} = c_{0,t}$ and will then choose $l_{l,t+1}$ and $s_{l,t+1}$ to maximize $u_{l,t+1} + u_{2,t+2} \Theta$ subject to the second and third parts of the budget constraint given in equations (3b) and (3c), respectively, as well as subject to

$$c^R_{t+1} = c_{0,t}, \quad (4b\text{-new})$$

$$c^R_{2,t+2} = \alpha c_{0,t} + (1-\alpha)c_{l,t+1}, \quad (4c\text{-new})$$

while treating $l_{0,t}$ and $s_{0,t}$ as exogenous. Therefore, if the choices of $l_{0,t}$ and $s_{0,t}$ are suboptimal for the consumer in retrospect due to anticipation-bias, and if the consumer re-optimizes when reaching middle-age, the choices of $l_{l,t+1}$ and $s_{l,t+1}$ will differ from the outcome of the commitment solution discussed above. As such, and given the original consumption path, $\Delta_{1,t+1}$ and $\Delta_{2,t+2}$ are smaller if $c^R_{l,t+1}$ and $c^R_{2,t+2}$ are determined by equations (4b-new) and (4c-new) than if they are determined by equations (4b) and (4c). The effects on $l_{l,t+1}$ and $s_{l,t+1}$ of assuming that the individual re-optimizes, instead of choosing the commitment solution, then depend on whether the incentives to work more and save less due to the increased marginal utility of consumption in period $t+1$ dominates, or is dominated by, the incentives to work less and save more, following the higher marginal utility of consumption in period $t+2$. None of these effects can be signed unambiguously. Yet, notice that the first order conditions for $l_{l,t+1}$ and $s_{l,t+1}$ take the same form as in equations (5c) and (5d), since the individual still makes the same behavioral mistake as under the commitment solution, i.e. $l_{l,t+1}$ and $s_{l,t+1}$ affect $c^R_{2,t+2}$ in the same general way irrespective of whether this reference measure is determined by equation (4c) or equation (4c-new).

Production

The production sector is characterized by identical competitive firms producing a homogenous good under constant returns to scale. Since the number of such firms is not important, it will be normalized to one for notational convenience. The objective function of the representative firm is written as

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5 Technically, this means calculating the derivatives of $l_{l,t+1}$ and $s_{l,t+1}$ with respect to $c_{u,t}$ in equation system (5c)-(5d). These comparative statics derivatives are ambiguous.
where $K$ denotes the capital stock. The firm obeys the first order conditions

$$F_{l_{0,t}} - w_{0,t} = 0,$$  \hspace{1cm} (8a)

$$F_{l_{1,t}} - w_{1,t} = 0,$$  \hspace{1cm} (8b)

$$F_{K_t} - r_t = 0.$$  \hspace{1cm} (8c)

**The government**

Following earlier literature on optimal paternalism (e.g., O'Donoghue and Rabin, 2003, 2006; Aronsson and Thunström, 2008; Aronsson and Granlund, 2011), we assume that the government attempts to correct for behavioral mistakes made by the consumers. In the present context, the anticipation-bias constitutes the only behavioral mistake. This bias leads individuals, when they are young, to make decisions that do not take account of the full cost these decisions involve for the same individual when middle-aged and old. By analogy to the theory of externalities, one might call this an internality problem (Herrnstein et al. 1993), which the government can solve for the individual’s own best.

In our framework, this means that from the perspective of the government, it is optimal to set $\alpha = 0$, although the government accepts all other aspects of consumer preferences. Therefore, the government attaches the following instantaneous utilities to generation $t$,

$$u_{0,t} = u(c_{0,t}, z_{0,t}),$$  \hspace{1cm} (9a)

$$u_{1,t+1} = u(c_{1,t+1} - c_{0,t}, z_{1,t+1}),$$  \hspace{1cm} (9b)

$$u_{2,t+2} = u(c_{2,t+2} - c_{1,t+1}, H),$$  \hspace{1cm} (9c)

and imposes the following adjusted life-time utility function:

$$W_{0,t} = u(c_{0,t}, z_{0,t}) + u(c_{1,t+1} - c_{0,t}, z_{1,t+1}) \Theta + u(c_{2,t+2} - c_{1,t+1}, H) \Theta' .$$  \hspace{1cm} (10)

The social welfare function is then given by the discounted sum of adjusted life-time utilities over generations

$$W = \sum_t W_{0,t} \Theta'. \hspace{1cm} (11)$$
The resource constraint for the economy as a whole takes the following form:

\[ K_{t+1} - K_t = F(l_{0,t}, l_{1,t}, K_t) - c_{0,t} - c_{1,t} - c_{2,t}, \]  

(12)

for all \( t \). Equation (12) means that the output in any period is used for consumption and net investment.

The public decision-problem is written as a direct decision-problem in terms of \( c_{0,t}, c_{1,t}, c_{2,t}, l_{0,t}, l_{1,t} \) and \( K_t \), which are chosen for all \( t \) to maximize the social welfare function in equation (11), subject to the resource constraint given by equation (12). The Lagrangean corresponding to this decision-problem can be written as

\[ L = W + \sum_t \gamma_t \left[ F(l_{0,t}, l_{1,t}, K_t) + K_t - c_{0,t} - c_{1,t} - c_{2,t} - K_{t+1} \right] \]

The allocation for generation \( t \) preferred by the government, here represented by the first order conditions for \( c_{0,t}, l_{0,t}, c_{1,t+1}, l_{1,t+1}, c_{2,t+2} \) and \( K_{t+1} \), are given as follows:

\[ \left[ \frac{\partial u_{0,t}}{\partial \Delta_{0,t}} - \frac{\partial u_{t,t+1}}{\partial \Delta_{t,t+1}} \right] \Theta' - \gamma_t = 0, \]  

(13a)

\[-\frac{\partial u_{0,t}}{\partial z_{0,t}} \Theta' + \gamma_t w_{0,t} = 0, \]  

(13b)

\[ \left[ \frac{\partial u_{t,t+1}}{\partial \Delta_{t,t+1}} \Theta - \frac{\partial u_{2,t+2}}{\partial \Delta_{2,t+2}} \Theta^2 \right] \Theta' - \gamma_{t+1} = 0, \]  

(13c)

\[-\frac{\partial u_{t,t+1}}{\partial z_{t,t+1}} \Theta' + \gamma_{t+1} w_{t,t+1} = 0, \]  

(13d)

\[ \frac{\partial u_{2,t+2}}{\partial \Delta_{2,t+2}} \Theta'^2 - \gamma_{t+2} = 0, \]  

(13e)

\[-\gamma_t + \gamma_{t+1}(1 + r_{t+1}) = 0. \]  

(13f)

In equations (13), we have used the first order conditions of the firm. As before, \( \Delta \) measures the difference between the current consumption and the reference consumption level (although the reference levels used by the government differ from those of the individual).

Suppose that the vector
solves the public decision-problem, where $\Delta_{1,t}^* = c_{1,t}^* - c_{0,t-1}^*$ and $\Delta_{2,t}^* = c_{2,t}^* - c_{1,t}^*$. The optimal marginal income tax rates that will implement the allocation preferred by the government in a decentralized setting can then be calculated by combining equations (13a) – (13f) with the first order conditions characterizing the private sector. This is the issue to which we turn next.

4. Optimal income taxation and anticipation-bias

This section analyzes the optimal tax policy that the government implements in order to correct for the effects of anticipation-bias. Since the policy rules implemented for the middle-aged are technically simpler than the corresponding policy rules used to influence the behavior of the young, we start by presenting the optimal tax policy implemented for the middle-aged of any generation, $t$, and then continue with the marginal income tax rates implemented for the young generation, $t$.

**Tax policy to affect the choices made by the middle-aged**

Since the government has access to nonlinear taxes, it may raise tax revenue (for redistribution) through the lump-sum components of the tax system. As a consequence, the use of marginal labor and capital income taxation is in this model due solely to the presence of an anticipation-bias, causing a negative internality that our paternalistic government wants to correct for. We have derived the following results with respect to marginal income taxation of the middle-aged:

**Proposition 1.** Suppose that $0 < \alpha \leq 1$. To correct for the effects of anticipation-bias of one’s own ability adaptation behavior, the labor income faced by the middle-aged of any generation $t$ should be taxed and the savings subsidized at the margin, irrespective of whether the consumer commits to the life-time plan for consumption and work hours made in period $t$ or re-optimizes when becoming middle-aged in period $t+1$. The policy rules for marginal labor and capital income taxation can be written as

$$ P_t^* = (c_{0,t}^*, l_{0,t}^*, c_{1,t}^*, l_{1,t}^*, c_{2,t}^*, K_{t}^*, \Delta_{1,t}^*, \Delta_{2,t}^*) \forall t $$

where $\Delta_{1,t}^* = c_{1,t}^* - c_{0,t-1}^*$ and $\Delta_{2,t}^* = c_{2,t}^* - c_{1,t}^*$. The optimal marginal income tax rates that will implement the allocation preferred by the government in a decentralized setting can then be calculated by combining equations (13a) – (13f) with the first order conditions characterizing the private sector. This is the issue to which we turn next.
\( \tau'_{2,t+2} = -\frac{\alpha}{r_{t+2}} < 0, \)  

(15)

where all entities are evaluated at \( P'_t. \)

**Proof.** Let

\[
M_{z,c}^1 = \frac{\partial u_{1,t+1}}{\partial z_{1,t+1}} / \partial \Delta_{1,t+1} - [\partial u_{2,t+2} / \partial \Delta_{2,t+2}] \Theta
\]

(16)

denote an “adaptation-adjusted” measure of marginal rate of substitution between leisure and private consumption in period \( t+1 \) that the consumer would face in the absence of an anticipation-bias. By combining equations (5c), (13c), (13d) and (16), we can then derive (if evaluated at \( P'_t \))

\[
0 = w_{1,t+1} - M_{z,c}^1 = \left[ 1 + \alpha \frac{\partial u_{2,t+2}}{\partial u_{1,t+1}} / \partial \Delta_{1,t+1} - [\partial u_{2,t+2} / \partial \Delta_{2,t+2}] \Theta \right] w_{1,t+1} \tau'_{1,t+1} - \alpha \frac{\partial u_{2,t+2}}{\partial u_{1,t+1}} / \partial \Delta_{1,t+1} - [\partial u_{2,t+2} / \partial \Delta_{2,t+2}] \Theta w_{1,t+1} \tau'_{1,t+1}.
\]

(17)

Solving equation (17) for \( \tau'_{1,t+1} \) gives equation (14). Similarly, let

\[
M_{t_1,t_2}^1 = \frac{\partial u_{1,t+1}}{\partial \Delta_{t_1,-1}} - [\partial u_{2,t+2} / \partial \Delta_{2,t+2}] \Theta > 0
\]

(18)

denote the “adaptation-adjusted” marginal rate of substitution between consumption in periods \( t \) and \( t+1 \). By combining equations (5d), (13c), (13e), (13f) and (18), and evaluating the resulting derivatives at \( P'_t \), we can derive

\[
0 = (1 + r_{t+2}) - M_{t_1,t_2}^1 = \tau'_{2,t+2} r_{t+2} + \alpha.
\]

(19)

Solving equation (19) for \( \tau'_{2,t+2} \) gives equation (15). ■

Consider first the case where \( \alpha = 0. \) In this case, equations (14) and (15) show that neither labor nor capital income should be taxed or subsidized at the margin. As the individual completely foresees that he/she will adapt, it follows that the individual takes into account to
what extent current consumption will lower future utility from consumption. By analogy to Becker and Murphy (1988), this is the case of rational adaptation where individuals make decisions that are in their own best interest.

If individuals face an anticipation-bias, i.e. $\alpha > 0$, the middle-aged consumer will supply more labor and save less because he/she underestimates the effect of the current consumption on the reference consumption level faced by his/her old self. For instance, individuals might overestimate the long-run effects on well-being of additional income because attention eventually shifts to less novel aspects of daily life and underestimate the present benefits from additional leisure time that we could spend e.g. for socializing (see Kahneman et al. 2006 p. 1910).

Taxes can correct for the oversupply of labor as well as for the tendency to undersave. To see the intuition behind the optimal tax policy of a paternalistic government more clearly, consider the special case where individuals completely ignore the fact that they will adapt to changing consumption levels, i.e. $\alpha = 1$. In this case, equations (14) and (15) can be rewritten as

$$
\tau'_{1,t+1} = \frac{\partial u_{2,t+2}}{\partial u_{1,t+1}} \frac{\partial \Delta_{2,t+2}}{\partial \Delta_{1,t+1}} \Theta > 0 \quad (14/\alpha = 1)
$$

$$
\tau^*_{2,t+2} = -\frac{1}{r_{t+2}} < 0 \ . \quad (15/\alpha = 1)
$$

The anticipation bias distorts the decision of the middle-aged at two margins. First, it distorts the consumption-leisure choice. From (5c), using (6f) and (14/\alpha = 1), we obtain the following relationship:

$$
\frac{w_{1,t+1}}{\partial u_{1,t+1} / \partial \Delta_{1,t+1}} = \frac{\partial u_{2,t+2}}{\partial u_{1,t+1}} \frac{\partial \Delta_{2,t+2}}{\partial \Delta_{1,t+1}} \Theta > 0 .
$$

The left-hand side of this expression measures the difference between the before-tax wage rate and the marginal rate of substitution between leisure and private consumption at the social optimum, while the right-hand side represents the value of the marginal internality. The negative internality in giving up one hour of leisure in order to consume more in period $t+1$ increases the reference consumption point of the old by $w_{1,t+1}$ and thus reduces future utility.
by \( w_{t+1} \cdot \frac{\partial u_{t+2}}{\partial \Delta_{t+2}} \). Discounting by \( \Theta \) yields the negative internality that the middle-aged consumer imposes on himself/herself when becoming old. Ignoring this negative internality leads the middle-aged to work too much. A labor income tax according to equation \((14/\alpha = 1)\) serves to correct for this distortion.

However, although the optimal labor income tax leads to an optimal labor supply in period \( t+1 \), this does not guarantee an optimal intertemporal allocation of consumption. If the individual does not anticipate the adaptation process going on, he/she has an incentive to consume too much today and too little when becoming old. This can be seen from \((5d)\).

Anticipating the adaptation process, the first-order condition reads:

\[
- \frac{\partial u_{t+1}}{\partial \Delta_{t+1}} + \frac{\partial u_{t+2}}{\partial \Delta_{t+2}} \Theta(1 + r_{t+2}) + \frac{\partial u_{t+2}}{\partial \Delta_{t+2}} \Theta = 0.
\]

The first two terms cover the standard saving decision. The discounted future marginal utility of consumption must exceed current marginal utility by \( 1 + r_{t+2} \). This standard decision that individuals with full anticipation-bias will make ignores that an additional marginal consumption unit today will increase the reference consumption level by one unit and thus lower discounted future utility by the future marginal utility.

This provides an incentive for the consumer to consume more today than is optimal for himself/herself in a longer time-perspective. An optimal subsidy serves to correct for this saving-internality. Thus, the optimal marginal rate of substitution between present and future consumption increases by one unit compared to the individual decision in the absence of corrective taxes, i.e. in the optimum, we have:

\[
\frac{\partial u_{t+1}}{\partial \Delta_{t+1}} \Theta = 2 + r_{t+2}.
\]

**Tax policy to affect the choices made by the young**

Having examined the marginal income tax rates implemented for the middle-aged, we continue with the corresponding policies to correct the labor supply and savings behavior of the young. By introducing the short notation

\[
\gamma_{0,t} = \frac{\partial u_{0,t}}{\partial \Delta_{0,t}} - \frac{\partial u_{t+1}}{\partial \Delta_{t+1}} \Theta + \alpha \left[ \frac{\partial u_{t+1}}{\partial \Delta_{t+1}} \Theta - (1 - \alpha) \frac{\partial u_{t+2}}{\partial \Delta_{t+2}} \Theta^2 \right] > 0,
\]

\((20)\).
the marginal income tax structure can be summarized as follows:

**Proposition 2.** Suppose that $0 < \alpha \leq 1$. To correct for the effects of anticipation-bias of one’s own ability adaptation behavior, the labor income faced by the young of any generation $t$ should be taxed at the margin. Savings should be subsidized at the margin, if $\alpha > r_{t+1} - r_{t+2}$, and taxed if $\alpha < r_{t+1} - r_{t+2}$. The policy rules for the marginal labor and capital income tax rates can be written as

$$r^t_{0,t} = \frac{\partial u_{t+1} / \partial \Delta_{t+1} \Theta - (1 - \alpha) \partial u_{t+2} / \partial \Delta_{t+2} \Theta^2}{Y_{0,t}^t} > 0 \quad (21)$$

$$r^t_{1,t+1} = -\frac{\alpha}{r_{t+1}} \left[ 1 - \frac{1 + r_{t+1}}{1 + r_{t+2} + \alpha} \right], \quad (22)$$

where all entities are evaluated at $P^t_t$.

The proof of Proposition 2 is analogous to the proof of Proposition 1 and can be found in the Appendix. The intuition behind equation (21) is the same as that behind equation (14) above: the young consumer supplies more labor than he/she would have done in the absence of the anticipation-bias. The anticipation-bias leads the consumer to underestimate the effects of $l_{0,t}$ on $c^R_{t+1}$ and $c^R_{t+2}$. Accordingly, when deciding on the hours of work, the young consumer overestimates the utility gain for his/her middle-aged and old selves, which the tax serves to correct for. In the case $\alpha = 1$, condition (20) simplifies to $Y_{0,t}^t = \partial u_{0,t} / \partial \Delta_{0,t}$ and the optimal marginal labor income tax becomes

$$r^t_{0,t} = \frac{\partial u_{0,t+1} / \partial \Delta_{0,t+1}}{\partial u_{0,t} / \partial \Delta_{0,t}} \Theta > 0, \quad (21/\alpha = 1)$$

which has the same structure as $(14/\alpha = 1)$. In the special case of a full anticipation-bias, the whole negative internality already becomes present in the next period. Since the marginal labor income tax rate faced by the middle-aged in period $t+1$ $(14/\alpha = 1)$ fully correct for the internality created in period $t+2$, the marginal labor income tax rate implemented for the young in period $t$ only has to take account of the internality occurring in period $t+1$. In the case of a partial anticipation-bias ($\alpha < 1$), however, the reference consumption in period $t+2$
becomes a weighted average of the consumption levels in periods \( t \) and \( t+1 \). Since the marginal labor income tax rate in period \( t+1 \) only corrects for the internality in period \( t+2 \), the marginal labor income tax rate in period \( t \) must take account of the internality created in period \( t+1 \) as well as the weighted impact the consumption decision by the young in period \( t \) has on the utility of the old in period \( t+2 \). Although the optimal tax formula becomes more complicated, the underlying reason remains the same.

Equation (22) differs from equation (15) in the sense that the optimal tax policy does not necessarily imply a savings subsidy to the young consumer. The reason is that \( s_{0,t} \) directly affects two future reference consumption levels: an increase in \( s_{0,t} \) leads to a decrease in \( c_{1,t+1}^R \), while it may either increase or decrease \( c_{2,t+2}^R \), ceteris paribus. The net effect depends on the interest rates in periods \( t+1 \) and \( t+2 \). If interest rates are equal, it is always optimal to subsidize savings when \( \alpha > 0 \). However, if the interest rate in period \( t+2 \) is much smaller than in period \( t+1 \), i.e. \( r_{t+2} << r_{t+1} \), then savings are very attractive in period \( t \) and not attractive in period \( t+1 \). As such, if the individual has an incentive to save as young and to spend the savings as middle-aged, savings by the young consumer will cause a negative internality for his/her old self. Therefore, a tax rather than a subsidy is justified, if this negative internality dominates the effect savings in period \( t \) have on the reference consumption in period \( t+1 \). This is precisely what happens when \( 0 < \alpha < r_{t+1} - r_{t+2} \). On the other hand, if \( \alpha \) exceeds the interest rate difference, i.e. \( \alpha > r_{t+1} - r_{t+2} \), which is likely to hold even for very low \( \alpha \), the optimal policy is a savings subsidy also to the young consumer.\(^6\) Furthermore, note that for \( r_{t+1} = r_{t+2} \) the savings subsidy to the young consumer will always be smaller than the corresponding subsidy to the middle-aged consumer living in the same period.

Notice also that the tax policy faced by the middle-aged, as summarized in Proposition 1, presupposes that the tax policy implemented for the young is optimal in the sense described in Proposition 2. Otherwise, the government would not be able to reach its preferred allocation, since the labor supply and savings behavior implied by equations (21) and (22) affect the tax formulas faced by the middle-aged, i.e. equations (14) and (15), through the measure of

\(^6\) By comparison with the corresponding policy addressed in Proposition 1, notice that an increase in \( s_{0,t} \) unambiguously reduces \( c_{t+2}^R \), while it has no effect on \( c_{t+1}^R \), which explains why the optimal tax policy always implies a savings subsidy to the middle-aged.
reference consumption. As a consequence, it does not matter for public policy in our framework (neither for the policy rules nor for the level of each instrument) whether the consumer commits to the original life-time plan for consumption and work hours made in period $t$ or re-optimizes when reaching middle-age in period $t+1$. The intuition is that the tax policy implemented for the young in period $t$ is optimal: the individuals will, therefore, behave as if $c_{1,t+1}^R = c_{0,t}^*$, in which case re-optimization would not lead to any change in behavior by comparison with commitment. As such, it becomes irrelevant for the optimal tax policy whether or not the consumer re-optimizes, since the effect of anticipation-bias on the formation of $c_{1,t+1}^R$ is already internalized.

5. Summary and discussion

As we indicated in the literature review, there is now a substantial amount of evidence suggesting that people do not correctly foresee their ability to adapt to new circumstances. Yet, anticipation-bias and the policy implications that such biases give rise to have so far played a minor role in the economics literature. This paper deals with the policy implications of anticipation-bias in a framework where consumers adapt to earlier consumption levels through a habit-formation process. The analysis is based on a general equilibrium overlapping-generations model with endogenous labor supply and savings. Each consumer lives for three periods, allowing us to distinguish between a commitment solution for the consumer (in which case he/she sticks to the original life-cycle plan for consumption and work hours) and a solution with re-optimization (where the consumer may change his/her plan in the second period of life when realizing the failure to adapt). The consumer supplies labor and makes an active savings decision both when young and when middle-aged; when old the consumer is retired and consumes the remaining wealth. The government is assumed to be paternalistic in the sense of trying to correct for the behavioral mistakes associated with anticipation-bias through nonlinear taxation of labor income and capital income.

Anticipation-bias leads the consumers to underestimate the extent to which they will adapt to changes in consumption in the future and, therefore, a tendency to overestimate the future marginal utility of consumption. Our results show how the government may internalize this internality through a combination of marginal labor income taxation and savings subsidies. More specifically, to correct for the effects of anticipation-bias, the optimal tax policy
includes labor income taxes implemented for the young and the middle-aged – yet at different marginal rates – in combination with a marginal savings subsidy implemented for the middle-aged. The savings by the young consumers should either be subsidized or taxed depending on the future equilibrium path for the interest rate; let be that a subsidy appears to be the most plausible outcome. The intuition is that the savings made when young affects the measure reference consumption faced by the individual’s old self in two opposing directions, and the qualitative net effect depends on the path for the interest rate.

Future research may take several directions, and we briefly discuss three of them here. First, we have neglected the possibility of adaptation also with respect to leisure and the policy implications of anticipation-bias in this particular context. Although our choice to focus on adaptation in terms of consumption can be justified based on earlier research (see Section 3), it would, nevertheless, be interesting to analyze how the results will change if leisure-adaptation in combination with a leisure-related anticipation-bias are added to the analysis. Second, and equally important, we have made no distinction between different consumption goods. Indeed, one can easily imagine that the importance of adaptation and anticipation-bias varies among goods, in which case commodity taxes may be used as a supplemental instrument for correction (in addition to the income taxes). Third, our analysis disregards the possibility of heterogeneity with respect to the anticipation-bias, i.e. that some people make more biased projections than others. If such differences were common knowledge, they would not affect the qualitative results derived above: the only implication would be that the level of each instrument varies among consumers depending on the magnitude of the bias, an issue which can easily be dealt with via the flexible tax instruments set out above. With unobserved heterogeneity, on the other hand, things become much more complex and would necessitate an extension of the model to deal with either asymmetric information or uncertainty. We hope to address these issues in future research.

References


Appendix

Proof of Proposition 2

Let

\[ M^0_{z,c} = \frac{\partial u_{0,t}}{\partial z_{0,t}} / \partial u_{0,t} - \frac{\partial u_{1,t}}{\partial \Delta_{0,1}} - \frac{\partial u_{1,t+1}}{\partial \Delta_{1,1+1}} \Theta. \]  

(A1)

denote an adaptation-adjusted measure of marginal rate of substitution between leisure and private consumption for the young in period \( t \). We can then use equations (13a) and (13b) to derive the following relationship at the socially optimal resource allocation:

\[ M^0_{z,c} = w_{0,t}. \]  

(A2)

The private first order condition for labor supply in equation (5a) implies

\[ w_{0,t} - M^0_{z,c} = \left[ 1 + \alpha \frac{\partial u_{1,t+1}}{\partial \Delta_{1,t+1}} - (1 - \alpha) \frac{\partial u_{2,t+2}}{\partial \Delta_{2,t+2}} \Theta^2 \right] w_{0,t} r_{0,t}^{I}. \]  

(A3)

Define

\[ \gamma_{0,t} = \frac{\partial u_{0,t}}{\partial \Delta_{0,t}} - \frac{\partial u_{1,t+1}}{\partial \Delta_{1,t+1}} + \alpha \left[ \frac{\partial u_{1,t+1}}{\partial \Delta_{1,t+1}} - (1 - \alpha) \frac{\partial u_{2,t+2}}{\partial \Delta_{2,t+2}} \Theta^2 \right] > 0. \]

By combining equations (A2) and (A3), we can then solve for the marginal labor income tax rate

\[ r_{0,t}^{I} = \alpha \frac{\partial u_{1,t+1}}{\partial \Delta_{1,t+1}} - (1 - \alpha) \frac{\partial u_{2,t+2}}{\partial \Delta_{2,t+2}} \Theta^2 > 0. \]  

(A4)
By analogy to equation (16), we define the adaptation-adjusted marginal rate of substitution between present and future consumption that the young consumer would face in the absence of anticipation-bias as follows:

\[
M_{0}^{0} = \frac{\partial u_{t+1} / \partial \Delta_{t+1} - \partial u_{t+1} / \partial \Delta_{t+1} \Theta - \partial u_{t+1+2} / \partial \Delta_{t+1+2} \Theta^{2}}{\partial u_{t+1} / \partial \Delta_{t+1} \Theta - \partial u_{t+1+2} / \partial \Delta_{t+1+2} \Theta^{2}} > 0.
\]  

(A5)

We can then combine equations (13a), (13c) and (13f) to derive

\[
M_{0}^{0} = 1 + r_{t+1}
\]  

(A6)

which reflects the desired tradeoff between present and future consumption for the young consumer from the point of view of the government, while the private first order condition for saving in equation (5b) implies

\[
(1 + r_{t+1}) - M_{0}^{0} = \left[ \alpha + r_{t+1}^{s} \right] \frac{\partial u_{t+1} / \partial \Delta_{t+1} \Theta - (1 - \alpha) \partial u_{t+1+2} / \partial \Delta_{t+1+2} \Theta^{2}}{\partial u_{t+1} / \partial \Delta_{t+1} \Theta - \partial u_{t+1+2} / \partial \Delta_{t+1+2} \Theta^{2}} - \frac{\partial u_{t+1+2} / \partial \Delta_{t+1+2} \Theta^{2}}{\partial u_{t+1} / \partial \Delta_{t+1} \Theta - \partial u_{t+1+2} / \partial \Delta_{t+1+2} \Theta^{2}} (1 + r_{t+1})
\]

(A7)

At the social optimum, the left hand side of equation (A7) is equal to zero. We can then solve for \( r_{t+1}^{s} \)

\[
\tau_{t+1}^{s} = -\frac{\alpha}{r_{t+1}} + \frac{\partial u_{t+1+2} / \partial \Delta_{t+1+2} \Theta^{2}}{\partial u_{t+1} / \partial \Delta_{t+1} \Theta - (1 - \alpha) \partial u_{t+1+2} / \partial \Delta_{t+1+2} \Theta^{2}} \frac{1 + r_{t+1}}{r_{t+1}}
\]

(A8)

Rearrangement gives equation (22).