Public pensions in the age of automation*

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Abstract

We analyze the impact of improved automation on the size and distribution of pension benefits, and on the optimal size of public pension systems. To this end, we build an overlapping generations model with heterogeneous agents. Automation is either conceptualized in a capital-skill complementarity (CSC) or task-based (TB) fashion. We find that any productivity gains of automation realized as increased returns to savings disproportionately benefit high-skilled workers who are less dependent on illiquid public pensions. A redistributive pension system can reduce public pension inequality but increase inequality in private retirement savings. The optimal size of the pension system is larger in the TB specification where displacement effects of automation are accounted for. We do not find that automation-driven growth warrants any change to the optimal size of the public pension system.

Keywords: Automation, General Equilibrium, Overlapping Generations, Public Pensions

JEL Codes: H55, J22, J26

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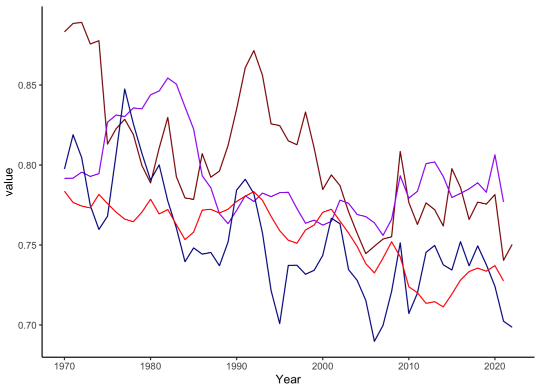
1 Introduction

Much empirical evidence indicates that advances in automation have played a significant role in explaining the decline of the labor share of national income, the limited wage growth across various sectors, and the increased skill premium observed over the past three to four decades (e.g., Acemoglu and Restrepo, 2019, 2022; Autor et al., 2003; Goos et al., 2014; Graetz and Michaels, 2018).[1] Concurrently, the pension literature has documented a rise in public pension expenditures in many developed economies, mainly driven by aging demographics (e.g., Coile et al., 2020). Figure 1 illustrates the recent evolution for a selection of countries that are representative of a broader global pattern. In particular, the decline in fertility rates and the continuous improvements in mortality suggest that the ongoing aging of the population will extend to the next three to four decades. Consequently, public pension expenditures are projected to increase, with average spending among OECD economies increasing from the current 8.9% to 10.3% by 2060 (OECD, 2023).

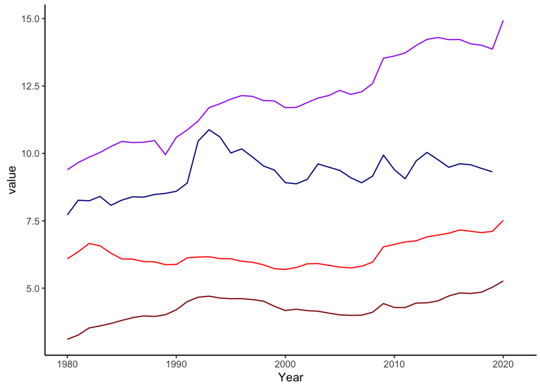
We argue that these trends constitute a source of concern for the future provision of public pensions. This follows from the observation that most public pension systems share two fundamental features that likely make them vulnerable to the impact of extensive automation. First, most systems are pay-as-you-go (PAYG), meaning that the benefits of current pensioners are financed by today's workers. Second, contributions follow almost exclusively from explicit or implicit labor income taxation. Consequently, if automation leads to a reduced labor share and potentially lower equilibrium wages, ceteris paribus, the contributions to the pension system are likely to decline. To maintain solvency, reductions in benefits will be necessary. Such adjustments could have significant negative implications for the welfare of individuals who rely heavily on public pension benefits for their retirement consumption, whether due to behavioral shortcomings or because their income barely covers minimum subsistence levels.

Even in the favorable scenarios where the productivity gains of automation boost

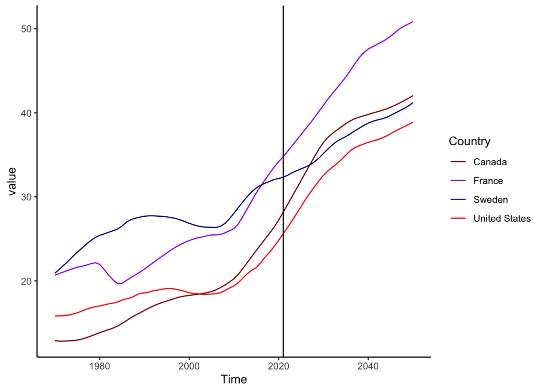
[1] Importantly, these findings are not limited to the United States (see, e.g., Acemoglu et al., 2023, 2020).



(a) Labor share of GDP



(b) Pension spending (% of GDP)



(c) Actual and projected old-age dep. ratios

Figure 1: Trends in labor shares, public pension spending, and old-age dependency ratios for a selection of countries.

Note: Data collected from The World Inequality Database (a), the OECD Social Expenditure Database (b), and the World Bank (c).

average wages and total contributions to the pension system, automation can still negatively impact the distribution of pension benefits. For instance, in countries like Sweden, Italy, and Latvia, where the pension systems display a strong link between contributions and benefits, automation is likely to benefit high-skilled workers more than low-skilled workers during their working lives. Consequently, the increased pension contributions from high-skilled workers relative to those of lower-skilled workers could exacerbate income inequality, as the earnings differential will systematically carry over to the pension income distribution. If the use of robots causes a decrease in the demand for low-skilled workers, this effect would be further amplified.

Furthermore, the design of PAYG pension systems implies that the contributions and therefore the income they generate only appreciate by population and wage growth.

In contrast, advances in automation are expected to boost investment returns. Consequently, in a dynamically efficient economy, the disparity between pensioners who save privately for retirement and those who depend on the illiquid public pension system is likely to widen. This, in turn, exacerbates the automation-driven inequality between capital owners and non-capital owners, as discussed in Moll et al. (2022).

This introduces a potentially complex dilemma for policymakers: to curb increased income inequality, policymakers might be tempted to reduce the earnings-dependence of public pensions in favor of a more redistributive system. However, such a reform would create larger disincentives for both labor supply and private savings among low-income workers as their replacement rate increases (see, e.g., Gustafsson, 2023a).

Ultimately, this paper examines the impact of advances in automation on the provision and distribution of pension benefits. To this end, we develop a general equilibrium overlapping generations (OLG) model that incorporates automation on the production side. The OLG framework is particularly suitable for analyzing retirement behavior and pension policy, while the general equilibrium mechanism is essential for capturing changes to the skill premium and investment returns. The theoretical impact of automation can differ significantly depending on the specific framework used. To understand how automation affects wage and pension benefit levels, and their distributions, we explore both the capital-skill-complementarity framework (as in Krusell et al. (2000)) and the task-based framework (as in Acemoglu and Restrepo (2018b)).

For the model to constitute an environment in which public pensions can be meaningful from a welfare perspective, we populate the model with agents that differ both in terms of savings behavior, where some save rationally and some are hand-to-mouth consumers, and also in their skill level. In this context, the pension system can potentially enhance welfare by mandating savings for hand-to-mouth consumers and by reducing economic inequality between high- and low-skilled workers, as suggested by World Bank (1994).

We calibrate the model to match the macroeconomic regularities representative of an average OECD economy. The analysis proceeds in two steps. First, we analyze the comparative static effects of increased automation on the levels and the distribution of pension benefits. We also decompose the relative importance of public vs private pension income for retirement consumption across the different worker types. In the second step, we assess whether improved automation warrants changes to the size of the public pension system by solving the social planner's problem to determine the optimal pension contribution rate.

Regardless of the model specifications, our comparative statics analysis reveals that advances in automation lead to increased wage and pension inequality, higher returns on investments, thereby leading to a higher appreciation of private pension savings, a larger capital share, and a reduced low-skilled labor share of national income. In our preferred calibration, when accounting for displacement effects in the task-based specification, we find that the equilibrium wage for low-skilled workers decreases. The intra-generational redistribution inherent in the pension system mitigates this effect but also generates larger disincentives for private savings among low-skilled workers. Consequently, the higher equilibrium interest rate exacerbates inequality between workers who save privately for retirement and those who live hand-to-mouth.

The welfare analysis suggests that the optimal size of the public pension system is larger in the model economy with task-based production. Based on our baseline calibration, when welfare is evaluated across a cross-section of the population, the optimal contribution rate is approximately 19.5% for the CSC case and 29% for the TB economy. Conversely, when welfare is assessed based on the present value of the lifetime utility of a newly born generation, the optimal size is 6% for the CSC case and 11% for the TB economy. Interestingly, despite the inclusion of a basic income component in the pension system, which works to redistribute income intragenerationally, we do not find that the optimal size of the pension system is influenced by automation-driven growth, even though automation increases lifetime inequality under both technology specifications.

Our findings demonstrate that technology can have substantial implications for welfare analyses of pension reform. The increased returns to private savings suggest additional long-term benefits of capitalizing pension systems, as the inequality between capital owners and hand-to-mouth households grows. Furthermore, as (r-g) increases, the effective taxation of illiquid PAYG pension systems increases due to the additional foregone compound interest from being forced to save in a scheme that offers returns below those of financial markets. Finally, although we do not find that automation demands a change in the size of the pension system, it may still warrant redesigning specific aspects of public pensions unrelated to their size, such as the degree of redistributiveness and overall funding.

The rest of the paper is organized as follows. We review the literature in section 2. The model is introduced and solved in section 3, followed by analytical results. Section 4 contains the calibration exercises and numerical analyses. Section 5 concludes.

2 Literature review

The present paper contributes to a mature literature on the economic implications of improved automation. Within this literature, automation has been conceptualized in the production process in various ways. The traditional view, based on neoclassical production theory, suggests that improved automation, as a general part of technological progress, is either Hicks neutral, leading to a proportional increase in marginal products, or factor augmenting, increasing the marginal product of capital or labor (see, e.g., Bessen, 2019; Graetz and Michaels, 2018; Nordhaus, 2021). For both labor- and capital-augmenting options, automation always increases labor demand and the equilibrium wage—outcomes that are contested by some recent evidence that suggest the opposite (see, e.g., Acemoglu and Restrepo, 2020).

Others have conceptualized improvements to automation as part of skill-biased technological change à la Krusell et al. (2000), where automation complements high-skilled workers more than their low-skilled counterparts (see, e.g., Prettner, 2019; Prettner and Strulik, 2020). The capital-skill complementarity hypothesis has gained popularity as it can explain the recent evolution of the wage structure, in particular the increased skill premium observed in the US[2], and also a non-trivial share of the observed decline in the

[2]See Autor et al. (2008) for a discussion of the wider question.

labor share of US national income (Prettner, 2019).

A third, and currently very prominent approach, is to model automation within a task-based framework (see, e.g., Acemoglu and Restrepo, 2018b; Kina et al., 2024; Zeira, 1998). Improved automation increases the number of assembly tasks where capital (or robots) have a comparative advantage over human labor, thus replacing or displacing workers on the margin. In the latter, as shown in Acemoglu and Restrepo (2018a), automation always reduces the labor share, but can also reduce labor demand and equilibrium wages if the productivity gains are not large enough to dominate the displacement effect. The task-based framework has been shown to explain several labor market trends that have arisen since at least the 1980s in several developed economies, such as decreasing labor shares and wage growth across mainly manufacturing sectors, and also an increased skill premium (e.g., Acemoglu et al., 2023; Acemoglu and Restrepo, 2019, 2020, 2022).

Following this empirical evidence, the taxation literature has studied the potential role of a robot tax to reduce automation-driven inequality. Under the conventional assumption that the government cannot use lump-sum or skill-indexed income taxes to achieve redistribution, a general conclusion of this literature is that automation should be subject to a positive marginal tax to compress the wage structure and reduce automation-driven inequality (Guerreiro et al., 2022; Kına, 2024; Thuemmel, 2023). Guerreiro et al. (2022) find that this "robot tax" should be positive as long as the current generations of routine workers, who are unable to move to non-routine occupations, are active in the labor force. Once these generations are retired, the tax should be abandoned. Kına (2024) find that the optimal tax policy should not only include a positive tax on automation that mainly substitutes for low-skilled workers, such as industrial robots, but also a small subsidy on automation that replaces high-skilled workers, such as artificial intelligence and machine learning. Akar et al. (2023) show that a transition from an economy with a traditional production structure to an automatized economy is characterized by reduced wages and declining labor shares. Taxing capital and robots can slow down this process, but not halt it as long as the exogenous growth rate is positive.

The role of automation and robot taxation has also been explored in other public finance contexts, as well as in the growth literature. Within an overlapping generations model, Prettner and Strulik (2020) find that automation increases educational attainment, inequality, and leads to a declining labor share. As the skill premium increases, tertiary education becomes more attractive. However, due to ability constraints, enrolling in higher education is only a feasible option for high-ability workers. Ultimately, low-skilled workers "lose the race" against automation. Ultimately, this evolution merits educational subsidies to reduce inequality.

However, the consequences of automation for public pensions have been largely overlooked in the modeling and quantitative literature. A notable exception is Kim and Lee (2024), who study how improvements to longevity, or decreased fertility, affect pension benefits in an economy with automation capital. In the case of increased longevity, both direct and indirect subsidies targeting low-skilled workers adversely affected by automation can be Pareto improving. In the case of reduced fertility, however, only indirect subsidies are found to achieve this improvement. There are, however, several important differences between our paper and theirs. First, we endogenize labor supply and consider the effects of automation on retirement behavior. This is an important feature since we want to capture that a reform to the design of the pension system will change the implicit taxation of contributing to the pension system. Indeed, it is well-known that the efficiency costs of public pensions follow from their degree of actuarial fairness (see, e.g., Gustafsson, 2023a). Our focus on retirement timing follows from micro-econometric evidence suggesting larger labor supply elasticities along the extensive margin compared to the intensive margin.

Second, our analysis includes individuals that deviate from the rational, forward-looking agent paradigm as we postulate that a fraction of the population lives hand-to-mouth. This aligns with the current frontier of heterogeneous agent macroeconomic modeling (see, e.g., Krueger et al. (2016), Kaplan and Violante (2018), Aguiar et al. (2024)). Third, while Kim and Lee (2024) include automation through a capital-skill-complementarity specification, their analysis focuses on the effects of aging demographics. The main focus of the current paper is instead on the implications of the technology specification for the provision and distribution of pension benefits. In particular, we also study the effects of a task-based production function, allowing for displacement effects of automation.

3 Model

Time is continuous and denoted t. The time endowment in each period is normalized to one. Consider an economy in steady state with a constant population size of unit mass, populated by overlapping generations. The reason why we abstract from population growth is that we want to isolate the effects of automation on the performance of public pensions from those of population aging, where the latter has been extensively explored.[3]

On the production side, a representative firm produces a final good that can be either consumed or saved, in which case it is realized one-to-one as physical capital. Any savings then appreciate by the real rate of return to capital investments r.

On the consumer side, each worker type lives with certainty for T periods and replicates themselves identically. The life cycle can be divided into two distinct phases: working life and retirement. Throughout the working life, the worker supplies labor inelastically. The retirement age R is endogenous. No bequest motive exists.

Workers differ in labor productivity (skill) and optimization behavior. Let $j = H, L$ denote skill level, and $i = RS, HTM$ the type of optimizing behavior. All high-skilled workers are rational savers, while a fraction of low-skilled workers live hand-to-mouth and do not save by assumption. Let Λ_H^{RS} denote the share of high-skilled workers, Λ_L^{RS} the share of rational low-skilled workers, and the complement $1 - \Lambda_H^{RS} - \Lambda_L^{RS}$ the share of low-skilled workers that live hand-to-mouth. Since all HTM households are low-skilled by construction, we drop the skill indexation for these workers. Let w_i denote the wage rate of each skill type.

The government administers a public pension system financed by payroll taxes. In the absence of mortality risk and aggregate risk, the pension system can serve two

[3] For papers that study the effects of aging populations on public pensions, see, e.g., Marchand and Pestieau (1991), Disney (2000), Heijdra and Romp (2009), Poterba (2014), Kudrna et al. (2022).

meaningful purposes within this model framework. The first is to mandate savings, given that HTM workers do not save privately at all. As such, redistribution at any age prior to retirement will not help these individuals to smooth consumption. The second is to reduce intragenerational inequality, as promoted by World Bank (1994). To accommodate both these purposes, we follow Casamatta et al. (2000) and model a two-pillar pension system: one earnings-related (Bismarckian) pillar to mandate savings, and one common benefit (Beveridgean) pillar that aims to reduce intragenerational inequality.

3.1 Aggregate output

Aggregate output is produced by a representative firm using a CES technology with constant returns to scale. Inputs are high-skilled labor L_H, and a composite G that is assembled by low-skilled labor L_L and capital/machines K:

$$Y = \Omega \left[L_H^{\rho_F} + B\Big(G(K, L_L)\Big)^{\rho_F} \right]^{\frac{1}{\rho_F}}, \tag{1}$$

where Ω is total factor productivity (TFP) and B the factor productivity of $G(.)$. $G(.)$ is a function that describes the substitutability between capital and low-skilled labor, to be specified in the following subsections. $\rho_F = \frac{\sigma_F - 1}{\sigma_F}$, where σ_F is the elasticity of substitution between high-skilled labor and $G(.)$.

We now proceed to fully specify the production function for two different cases. First, we consider the case when $G(K, L_L)$ is produced by another CES function, resulting in an aggregate production function with capital-skill complementarity. In the second case, we assume that $G(K, L_L)$ is the output resulting from the assembly of a range of tasks produced by either capital or low-skilled labor.

3.1.1 Case A: capital-skill-complementarity

First, we follow the capital-skill-complementarity specification in Prettner and Strulik (2020) and specify $G(.)$ as follows:

$$G(K, L_L) = (AK^{\rho_G} + L_L^{\rho_G})^{\frac{1}{\rho_G}}, \tag{2}$$

so that aggregate output becomes:

$$Y = \Omega\left[L_H^{\rho_F} + B\Big(AK^{\rho_G} + L_L^{\rho_G}\Big)^{\frac{\rho_F}{\rho_G}}\right]^{\frac{1}{\rho_F}}. \tag{3}$$

Here, A is the capital factor productivity which we, like Prettner and Strulik (2020), interpret as the state of automation. $\rho_G = \frac{\sigma_G - 1}{\sigma_G}$, where σ_G is the elasticity of substitution between capital and low-skilled labor.

Operating under perfect competition, the firm employs high- and low-skilled labor, and rents capital, so that their marginal products equal their factor prices respectively:

$$L_H^d : \Omega L_H^{\rho_F - 1}\left[L_H^{\rho_F} + B\Big(AK^{\rho_G} + L_L^{\rho_G}\Big)^{\frac{\rho_F}{\rho_G}}\right]^{\frac{1-\rho_F}{\rho_F}} = MP_{L_H} = w_H, \tag{4}$$

$$L_L^d : \Omega B L_L^{\rho_g - 1}\left[L_H^{\rho_F} + B\Big(AK^{\rho_G} + L_L^{\rho_G}\Big)^{\frac{\rho_F}{\rho_G}}\right]^{\frac{1-\rho_F}{\rho_F}}\Big(AK^{\rho_G} + L_L^{\rho_G}\Big)^{\frac{\rho_F - \rho_G}{\rho_G}} = MP_{L_H} = w_L, \tag{5}$$

$$\begin{aligned} K^d : \Omega B A K^{\rho_G - 1}\left[L_H^{\rho_F} + B\Big(AK^{\rho_G} + L_L^{\rho_G}\Big)^{\frac{\rho_F}{\rho_G}}\right]^{\frac{1-\rho_F}{\rho_F}}\Big(AK^{\rho_G} + L_L^{\rho_G}\Big)^{\frac{\rho_F - \rho_G}{\rho_G}} - \delta = \\ MP_K - \delta = r, \end{aligned} \tag{6}$$

where δ is the depreciation rate of capital.

3.1.2 Case B: task-based

We now consider a scenario where $G(.)$ quantifies the output of an integration of outputs from individual tasks $y(u)$, where u denotes a particular task. Normalizing the amount of tasks to unity, and assuming imperfect substitution between tasks in the production, let $G(.)$ be specified as follows:

$$G(y(u)) = \left(\int_0^1 y(u)^{\rho_G} du \right)^{\frac{1}{\rho_G}}, \tag{7}$$

where $\rho_G = \frac{\sigma_G - 1}{\sigma_G}$ and σ_G now is the elasticity of substitution between different tasks in production.

Substituting the above expression into the aggregate production function yields:

$$Y = \Omega \left[L_H^{\rho_F} + B \left(\int_0^1 y(u)^{\rho_G} du \right)^{\frac{\rho_F}{\rho_G}} \right]^{\frac{1}{\rho_F}}. \tag{8}$$

Following Acemoglu and Restrepo (2018b) we assume that labor and capital are perfect substitutes in the production of any task $u \in [0, 1]$. Let $k(u)$ and $l_L(u)$ denote the amount of capital and labor used for a specific task so that:

$$y(u) = \psi(u)k(u) + l_L(u). \tag{9}$$

To ensure an interior solution where a fraction $a \in (0, 1)$ of tasks are automatized, we specify the marginal productivity of capital $\psi(u)$ such that capital has a comparative advantage over labor for a range $u \in [0, a)$ of tasks, and where labor has a comparative advantage for $u \in (a, 1]$. Following Kına (2024), we specify $\psi(u)$ as follows:

$$\psi(u) = D(u^{-\eta} - 1), \tag{10}$$

where $\eta > 0$ and $D > 0$. This functional form ensures that (i) $\psi(0) \approx \infty$, (ii) $\psi(u)$ is continuous and decreasing in u, and (iii) $\psi(1) = 0$ [4]. The extensive margin of automation

[4] If we denote $\bar{F}(x)$ the survival function for some distribution function, then $\psi(u)$ can be defined as the inverse of $\bar{F}(x)$ so that $\psi(u) \equiv \bar{F}(u)^{-1}$. Kına (2024)'s choice correspond to the productivity of

a is then the unique solution to the no-arbitrage condition $\psi(a) = \frac{r+\delta}{w_l}$ [5]. Importantly, anytime a change in $\psi(u)$ (either resulting from a change in D or η) leads to a change in the r/w_l, the extensive margin of automation is also affected in general equilibrium.

Following Acemoglu and Restrepo (2018b), the production function $G(.)$ can then be written in terms of the aggregate use of the different sources of input in the following fashion:[6]

$$G(K, L_L, a) = (1-a)^{\frac{1-\rho_G}{\rho_G}} S_G(K, L_L, a)^{\frac{1}{\rho_G}}, \tag{11}$$

with $S_G(K, L_L, a)$ such that:

$$S_G(K, L_L, a) \equiv (1-a)^{\rho_G - 1}\Big(\int_0^a \psi(u)^{\frac{\rho_G}{1-\rho_G}} du\Big)^{1-\rho_G} K^{\rho_G} + L_L^{\rho_G}.$$

The aggregate production function then takes the form:

$$Y = \Omega\Bigg[L_H^{\rho_F} + \tilde{B}\Bigg[(1-a)^{\rho_G-1}\Big(\int_0^a \psi(u)^{\frac{\rho_G}{1-\rho_G}} du\Big)^{1-\rho_G} K^{\rho_G} + L_L^{\rho_G}\Bigg]^{\frac{\rho_F}{\rho_G}}\Bigg]^{\frac{1}{\rho_F}}, \tag{12}$$

where a can be conceptualized as the extensive margin of automation and where $\tilde{B} \equiv B(1-a)^{\frac{\rho_F}{\rho_G}(1-\rho_G)}$ [7]. The aggregates K and L_L are the aggregated input uses across tasks:

$$K \equiv \int_0^a k(u)du,$$

$$L_L \equiv \int_a^1 l(u)du.$$

Clearly the production function described by (12) shares some of the CES\CES characteristics that describe the technology with the capital skill complementarities as it is given in equation (3). The factor demands can then be derived and will satisfy expressions very similar (given a) to the expressions given in (4),(5) and (6):

capital being distributed according to a specific Lomax distribution, Lomax (1954), such that $F(x) = 1-(x+1)^{-k}$. Obviously other choices are possible.

[5] $r+\delta$ is the gross interest rate, including depreciation.

[6] see appendix A.3

[7] Appendix 4 shows how to calibrate the production described in 12 to match the production function described in 3 for some values $(Y, L_H, L_L, K, w_H, w_L, r)$

$$L_H^d : \Omega L_H^{\rho_F - 1} \left(L_H^{\rho_F} + \tilde{B} S_G(K, L_L, a)^{\frac{\rho_F}{\rho_G}} \right)^{\frac{1}{\rho_F} - 1} = MP_{L_H} = w_H, \tag{13}$$

$$L_L^d : \Omega \tilde{B} L_L^{\rho_G - 1} S_G(K, L_L, a)^{\frac{\rho_F}{\rho_G} - 1} \left(L_H^{\rho_F} + \tilde{B} S_G(K, L_L, a)^{\frac{\rho_F}{\rho_G}} \right)^{\frac{1}{\rho_F} - 1} = MP_{L_L} = w_L, \tag{14}$$

$$\begin{aligned} K^d : &\Omega \tilde{B} (1-a)^{\rho_G - 1} \Big(\int_0^a \psi(u)^{\frac{\rho_G}{1-\rho_G}} du \Big)^{1-\rho_G} K^{\rho_G - 1} \\ &\left(L_H^{\rho_F} + \tilde{B} S_G(K, L_L, a)^{\frac{\rho_F}{\rho_G}} \right)^{\frac{1}{\rho_F} - 1} S_G(K, L_L, a)^{\frac{\rho_F}{\rho_G} - 1} - \delta = MP_K - \delta = r. \end{aligned} \tag{15}$$

3.2 Pension system

In line with the recommendations of the World Bank (World Bank, 1994), many economies operate multi-pillar PAYG pension systems which both mandate savings over the life-cycle and redistribute income intragenerationally.[8]

Since our model includes both income heterogeneity and non-rational savings behavior, both of these mechanisms could hypothetically increase welfare. Therefore, we model a stylized pension system that features both earnings-based (Bismarckian) and common-benefit (Beveridgean) pillars. Each worker type recognizes that any contributions to the Bismarckian pillar constitute forced savings (albeit in a technology return dominated by capital investments) and thus accounts for it in their labor supply decision. However, any benefits from the Beveridgean pillar are treated as exogenous. Let κ denote the fraction of contributions that goes toward the Bismarckian pillar (henceforth the "Bismarckian factor"). Reforming κ creates an equity-efficiency trade-off: the effective taxation of the Bismarckian scheme is lower, while the Beveridgean scheme offers an explicit mechanism

[8]For studies that analyze the redistributive role of public pensions, see, e.g., Huggett and Ventura (1999), Fehr and Habermann (2008), Cremer and Pestieau (2011), Gustafsson (2023a), Gustafsson (2023b).

for redistributing income intragenerationally.[9]

$$b_j^i = \frac{\tau}{T - R_j^i}(\kappa w_j R_j^i + (1-\kappa)\Psi), \tag{16}$$

where:

$$\Psi = \Lambda_H^{RS} w_H R_H^{RS} + w_L[\Lambda_L^{RS} R_L^{RS} + (1 - \Lambda_H^{RS} - \Lambda_L^{RS})R^{HTM}], \tag{17}$$

measures the aggregate labor income in the economy at any moment in time.

3.3 Workers

Following a standard in the quantitative macroeconomic and public finance literature, we assume CRRA per-period utility functions over consumption for all workers. For retirement leisure preferences, we convert the CRRA stock specification used in, e.g., Jacobs (2009), Gustafsson (2023a), and Gustafsson (2023b), into a flow specification. In that way, the discounting of retirement leisure enters our model formulation explicitly, and we are able to turn discounting on or off in a simple fashion. This stock-to-flow conversion is described in Appendix A.2.

3.3.1 Rational savers

A rational worker decides on consumption $c_j^{RS}(t)$ and retirement age R_j^{RS} to maximize her lifetime utility:

$$U_j(c_j^{RS}(t), R_j^{RS}) = \int_0^T \frac{c_j^{RS}(t)^{1-\sigma} - 1}{1-\sigma} e^{-\theta t} dt + \beta \int_{R_j^{RS}}^T e^{\gamma t}(T-t)^{-\phi} e^{-\theta t} dt, \tag{18}$$

subject to the intertemporal budget constraint:

$$w_j(1-\tau)\int_0^{R_j^{RS}} e^{-rt} dt + b_j^{RS} \int_{R_j^{RS}}^T e^{-rt} dt = \int_0^T c_j^{RS}(t) e^{-rt} dt, \tag{19}$$

[9]However, Sommacal (2006) and Gustafsson (2023a) show that this trade-off can collapse once the model allows for endogenous labor supply effects. Since this model includes endogenous retirement, the distributive effects of reforming this parameter are not trivial.

and subject to the initial and terminal conditions $k(0) = k(T) = 0$. Here, θ is the discount rate, β is the utility weight attached to retirement leisure, σ is the inverse of the elasticity of intertemporal substitution, and ϕ is the inverse of the retirement elasticity with respect to the replacement income rate.

We write the corresponding Lagrangian as follows:

$$\begin{aligned}\mathcal{L} = &\int_0^T \frac{c_j^{RS}(t)^{1-\sigma}-1}{1-\sigma} e^{-\theta t} + \beta \frac{(T-R_j^{RS})^{1-\phi}-1}{1-\phi} + \\ &\mu_j \left\{ w_j(1-\tau)\int_0^{R_j^{RS}} e^{-rt}dt + b_j^{RS}\int_{R_j^{RS}}^T e^{-rt}dt - \int_0^T c_j^{RS}(t)e^{-rt}dt \right\}.\end{aligned} \tag{20}$$

The solution of the optimization problem provides an expression for the optimal consumption profile:

$$c_j^{RS}(t) = \left[\frac{e^{(r-\theta)t}}{\mu_j}\right]^{\frac{1}{\sigma}}, \tag{21}$$

where μ_j is an unknown constant shadow-price term that satisfy equation 19.

Optimal retirement, R_j^{RS} is the solution to:

$$\beta\big(T-R_j^{RS}\big)^{-\phi} e^{(\gamma-\theta)R_j^{RS}} = \mu_j\left[\big(w_j(1-\tau_l)-b_j^{RS}\big)e^{-rR_j^{RS}} + \frac{\tau_l\big(\kappa w_j T + (1-\kappa)\Psi\big)}{(R_j^{RS}-T)^2}\int_{R_j^{RS}}^T e^{-rt}dt\right]. \tag{22}$$

Furthermore, from the expression given in Equation (21), we can analytically solve for the unknown constant term μ_j by exploiting the terminal condition that $k(T) = 0$, we find:

$$\mu_j = \left\{\frac{\sigma\big(e^{\frac{r(1-\sigma)-\theta}{\sigma}}-1\big)}{\frac{r(1-\sigma)-\theta}{r}\big(w_j(1-\tau_l)(1-e^{-rR_j^{RS}}) + b_j^{RS}(e^{-rR_j^{RS}}-e^{-rT})\big)}\right\}^{\sigma}. \tag{23}$$

By substituting the expression in Equation (23) into (22), the optimization problem for the rational savers is then reduced to identifying R_j^{RS} that solves (22) and (23) simultaneously.

Given R_j^{RS*} and μ_j, the optimal savings profile is then described over the working life and the retirement period as:

$$k_j(t) = \begin{cases} e^{rt}\left[\frac{w_j(1-\tau_l)}{r}(1-e^{-rt}) - \frac{\sigma}{\mu_j^{\frac{1}{\sigma}}(r(1-\sigma)-\theta)}\left(e^{\frac{r(1-\sigma)-\theta}{\sigma}t}-1\right)\right], & \text{for } t \in [0, R_j^{RS}); \\ e^{rt}\left[k_j(R_j^{RS})e^{-rR_j^{RS}} + \frac{b_j^{RS}}{r}(e^{-rR_j^{RS}} - e^{-rt}) - \right. & \\ \left.\frac{\sigma}{\mu_j^{\frac{1}{\sigma}}(r(1-\sigma)-\theta)}\left(e^{\frac{r(1-\sigma)-\theta}{\sigma}t} - e^{\frac{r(1-\sigma)-\theta}{\sigma}R_j^{RS}}\right)\right], & \text{for } t \in [R_j^{RS}, T]. \end{cases} \tag{24}$$

3.3.2 Hand-to-mouth workers

While HTM workers do not save, we assume that they optimize over the retirement margin. Since their consumption $c^{HTM}(t)$ at any point is equal to their income in the same period:

$$c^{HTM}(t) = \begin{cases} w_L(1-\tau_l), & \text{for } t \in [0, R^{HTM}); \\ b^{HTM}, & \text{for } t \in [R^{HTM}, T], \end{cases} \tag{25}$$

their optimization problem reduces to identifying the retirement age R^{*HTM} that solves the following problem:

$$\max_{R^{HTM}} U^{HTM} = \int_0^{R^{HTM}} \frac{(w_L(1-\tau_l))^{1-\sigma}-1}{1-\sigma}e^{-\theta t}dt + \int_{R^{HTM}}^T \frac{(b^{HTM})^{1-\sigma}-1}{1-\sigma}e^{-\theta t}dt + \beta\int_{R_j^{HTM}}^T e^{\gamma t}(T-t)^{-\phi}e^{-\theta t}dt. \tag{26}$$

The condition for optimal retirement becomes:

$$R^{HTM}: \frac{(w_L(1-\tau_l))^{1-\sigma}-1}{1-\sigma}e^{-\theta R^{HTM}} - \frac{b^{1-\sigma}-1}{1-\sigma}e^{-\theta R^{HTM}} + \int_{R^{HTM}}^T (b^{HTM})^{-\sigma}\frac{\tau_l(\kappa w_L T + (1-\kappa)\Psi)}{(R^{HTM}-T)^2}e^{-\theta t}dt - \beta(T-R^{HTM})^{-\phi}e^{(\gamma-\theta)R^{HTM}} = 0. \tag{27}$$

3.4 Aggregation and closing the model

In general equilibrium, all markets clear. The clearing conditions for the market for high-skilled labor:

$$L_H^d = \Lambda_H^{RS} R_H^{RS}, \tag{28}$$

for low-skilled labor:

$$L_L^d = \Lambda_L^{RS} R_L^{RS} + (1 - \Lambda_H^{RS} - \Lambda_L^{RS}) R^{HTM}, \tag{29}$$

and capital:

$$K^d = \int_0^T \left[\Lambda_H^{RS} k_H(t) + \Lambda_L^{RS} k_L(t) \right] dt. \tag{30}$$

The solution also satisfies the aggregate resource constraint:

$$Y = C + I, \tag{31}$$

where

$$C = \int_0^T \left[\Lambda_H^{RS} c_H^{RS} + \Lambda_L^{RS} c_L^{HS} + (1 - \Lambda_H^{RS} + \Lambda_L^{RS}) c^{HTM} \right] dt, \tag{32}$$

is the aggregate consumption, and

$$I = \delta \int_0^T \left[\Lambda_H^{RS} k_H(t) + \Lambda_L^{RS} k_L(t) \right] dt = \delta K, \tag{33}$$

aggregate investments in the steady state.

4 Calibration and Simulations

We proceed to study the effects of increased automation on the provision and distribution of public pension benefits numerically, for each of the model specifications. Since we explore two different production function specifications, scale becomes an issue. To ensure that both model economies are comparable in terms of marginal returns to inputs and input/output ratios, we scale the task-based economy so that it matches the CSC model, not only in terms of the object described in Table 1, but also in terms of equilibrium output, capital, labor, and wage levels. The steps of this process are: *Step 1*: We first calibrate the CSC specification. This involves both an external and an internal calibration. For the external calibration, we lift values for policy parameters from relevant documentation and data provided by `https://www.oecd.org` and `https://databank.worldbank.org`. We then consult the empirical literature for reasonable values for the elasticities governing both demand- and supply-side trade-offs. In particular, we fix the elasticity of substitution between capital and unskilled labor to $\rho_G = 5/3$, which implies substantial substitution between the two inputs.[10] Third, we discipline the remaining parameters for the model to match the capital-output ratio (K/Y), interest rate r, skill premium w_H/w_L, and the average retirement age $\bar{R}$ of an average OECD economy.

Figure 2 shows that the calibrated model is able to fit precisely the three targets that are determined by the form of the technology. Assuming $\sigma_G = 1.4$, the figure compares the loci where the three targets are met exactly for two sets of values: first, for the approximate value for the quantity of input labor where we weight the average amount of labor in the economy (the average retirement age minus twenty-five) by the share of skilled and unskilled workers in the economy. This sets $L_L = 25.4$ and $L_H = 12.1$. The second sets of values for the input labor are the ones we obtain from the internal calibration process, in this case $L_L = 26.0$ and $L_H = 12.5$. The first two targets are consistent with a parameter value ρ_F around -0.3 over a wide range of values for the amount of capital in the economy (say between 10 and 30). The skill premium target appears to be the most sensitive to small differences in the input labor values. It is therefore this target that, in the end, determines precisely the calibrated value for the level of capital in the economy (in this case, $K = 18.0$ and $\rho_F = -0.31$, which corresponds to $\sigma_F = 0.763$, see Table 1).

Step 2: we adjust the baseline of the TB model to the calibrated CSC model to ensure that the economies are identical in equilibrium. To achieve this, we first, we observe in particular that the quantities $(Y^*, L_H^*, L_L^*, K^*, w_H^*, w_L^*, r^*)$ are determined in the baseline

[10]In particular, it is almost identical to the value Krusell et al. (2000) find for a different specification of the CSC technology.

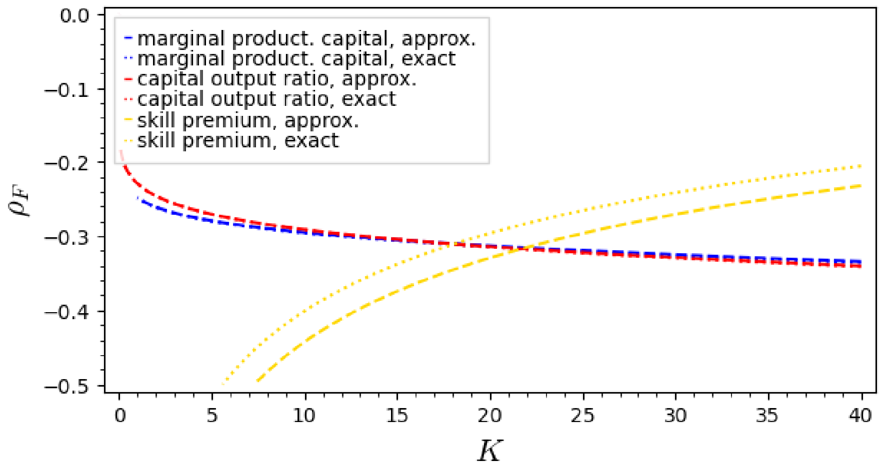


Figure 2: Sensitivity of calibration to K and ρ_F, $\sigma = 1.4$

Note: The figure shows in the (K,ρ_F) plane the locii where each of the targets are met, all else equal. The dotted lines show the evidence at approximate values for L_L and L_H, such that $L_L = (1 - \Lambda_H)(63.5 - 25) = 25.4$ and $L_H = \Lambda_H(63.5 - 25) = 12.1$. The dashed lines present the same loci at the calibrated value when $\sigma = 1.4$.

CSC case such that:

$$Y^* = \Omega\left[L_H^{*\rho_F} + B\Big(AK^{*\rho_G} + L_L^{*\rho_G}\Big)^{\frac{\rho_F}{\rho_G}}\right]^{\frac{1}{\rho_F}}, \tag{34}$$

and:

$$\begin{aligned} MP^*_{L_H} &= w^*_H, \\ MP^*_{L_L} &= w^*_L, \\ MP^*_K &= r^* - \delta, \end{aligned} \tag{35}$$

for the calibrated values of the constants A, B, ρ_F and ρ_G.

Given these constants and for a chosen value for the parameter η, we determine the parameter values $\check{a}$, $\check{B}$ and $\check{D}$ such that:

$$\begin{aligned} \psi(\check{a}) &= \frac{r^*}{w^*_L}, \\ \check{B} &= B(1-\check{a})^{\frac{\rho_F}{\rho_G}(\rho_G - 1)}, \\ A &= \check{D}^{\frac{\rho_G}{1-\rho_G}}(1-\check{a})^{(\rho_G-1)}\left(\int_0^{\check{a}} \psi(u)^{\frac{\rho_G}{1-\rho_G}} du\right)^{(1-\rho_G)}. \end{aligned} \tag{36}$$

Observe that since the function $\psi()$ depends on D the last equation is non-linear in D. If a solution of the previous equation system exists then the list $(Y^*, L_H^*, L_L^*, K^*, w_H^*, w_L^*, r^*)$ satisfies:

$$Y^* = \Omega\left[L_H^{*\rho_F} + \check{B}\left[(1-\check{a})^{\rho_G - 1}\Big(\int_0^{\check{a}} \psi(u)^{\frac{\rho_G}{1-\rho_G}} du\Big)^{1-\rho_G} K^{*\rho_G} + L_L^{*\rho_G}\right]^{\frac{\rho_F}{\rho_G}}\right]^{\frac{1}{\rho_F}}, \tag{37}$$

as well as equations (13), (14), (15) and $\psi(\check{a}) = \frac{w_L + r}{w_L^*}$ which determine the marginal products and the optimal share $\check{a}$ of tasks that are completed by automation. The parameter values for the external calibration, and their sources, are reported in Table 1, and the parameters obtained by the internal calibration are shown in Table 2. The model fit is documented in Table 3. Finally, Table 4 describes the equilibrium quantities and prices under two specifications for the inverse EIS, i.e., the log utility when $\sigma = 1.0$ or $\sigma = 1.4$. This parameter is important since its inverse describes the elasticity of intertemporal substitution, and thus the sensitivity of savings decisions to a change in the real interest rate. In particular, it determines the resource cost of increasing the size of the public pension system given its capital crowding-out effect.

Step 3: Our analysis proceeds by analyzing the consequences to either economy of growth-generating changes: on the one hand, changes that affect TFP and, on the other hand, changes that are specific to the marginal productivity of capital. Specifically, we ensure that all independent changes yield the same increase in total output.[11] Therefore, the change to output is identical in all scenarios, whether we compare the different economies or whether, given a particular technology, we compare alternative sources of change. The focus of our analysis is mostly on the interactions between the sources of growth and the distributional effects of the public pension system across the population.

[11] In practice, we start with the CSC economy and we increase the marginal product of capital by increasing the parameter A by 10%. We then determine the change to the TFP parameter Ω which produces the equivalent increase in total output. We then turn to the TB economy and determine the value of the parameter η which yields once again an equivalent change to total output all else equal. Finally, we determine the value of the TFP parameter in the TB economy in the same fashion.

Table 1: External calibration

	Parameter	Source/target
Population		
Time horizon	$T = 55$	Economic lifespan ages 25-80;
High-skilled population share	$\Lambda_H^{RS} = 0.34$	OECD (2023);
Low-skilled rational savers population share	$\Lambda_L^{RS} = 0.46$	20 % live hand to mouth;
Behavioral		
Inverse EIS	$\sigma = 1$	Standard;
Inverse retirement elasticity	$\phi = 1.5$	Ret. elasticity around 0.3;
Common production		
Total factor productivity	$\Omega = 25$	
Elasticity of substitution L_h vs $G(K, L_l)$	$\sigma_F = 0.763$	
Elasticity of substitution between L_L and K	$\sigma_G = 1.66$	Krusell et al. (2000);
Capital depreciation rate	$\delta = 0.08$	Standard;
Policy		
Public pension contribution rate	$\tau^s = 0.154$	OECD data ;
Bismarckian factor	$\kappa = 0.67$	Bourlés and López-Cantor (2024).

Table 2: Calibration

	Parameter	CSC	TB
θ	Discount rate	0.032	0.032
β	Retirement utility parameter	14.431	14.431
B	Capital-low skilled productivity	5.074	3.627
A	Capital productivity	1.167	
η	Curvature capital productivity		0.295
D	Scale capital productivity		2.072
α^*	Marginal task		0.513

Note: The Table shows the calibrated values

Table 3: Equilibrium objects for calibration

	Object	Target	CSC	Source
1	Capital–Output ratio	3.00	3.00	Standard
2	Real interest rate (%)	4.00	4.00	Standard
3	Avg. Retirement age	63.80	63.50	OECD data
4	Skill premium	1.75	1.75	Standard

Note: The Table shows the model fit to a set of aggregate moment targets.

5 Numerical analysis

Following the baseline calibration, we now proceed to analyze numerically the effects of automation-driven growth on the level and distribution of pension benefits. First, we perform a comparative static analysis on key equilibrium objects such as wage rates and pension benefits, conditional on a 10% growth of the productivity of capital. We compare

Table 4: Equilibrium objects in alternative specifications

Object	$\sigma = 1.0$	$\sigma = 1.4$
Y^*	6.676	6.008
w_H	0.155	0.141
w_L	0.887	0.080
K^*	20.000	18.000
L_H^*	12.900	12.500
L_L^*	25.600	26.000

Note: The Table compares the quantities and prices of the calibrated model for the two different values of the inverse elasticity of intertemporal substitution considered.

the consequences of this change in the productivity of capital to a change in the total factor productivity that yields an identical effect on total output. Second, we proceed to derive the optimal size of the public pension system to study whether automation-driven growth merits any reform to the contribution rate.

5.1 Comparative analysis

First we study how increased automation affects labor supply, pension benefits, labor income, and lifetime income, across the different worker types. The income and inequality measures are defined in Table 5.

Table 5: Income and inequality measures

Earnings income (EI)	Pension income (PI)	Lifetime income (LI)
$w_j^i(1-\tau_l)\int_0^{R_j^i} e^{-rt}dt$	$b_j^i \int_0^{R_j^i} e^{-rt}dt$	EI+PI
Earnings inequality	**Pension income inequality**	**Lifetime income inequality**
$\frac{\Lambda_H^{RS} EI_H^{RS}}{\Lambda_L^{RS} EI_L^{RS} + (1-\Lambda_H^{RS}-\Lambda_L^{RS})EI^{HTM}}$	$\frac{\Lambda_H^{RS} PI_H^{RS}}{\Lambda_L^{RS} PI_L^{RS} + (1-\Lambda_H^{RS}-\Lambda_L^{RS})PI^{HTM}}$	$\frac{\Lambda_H^{RS} LI_H^{RS}}{\Lambda_L^{RS} LI_L^{RS} + (1-\Lambda_H^{RS}-\Lambda_L^{RS})LI^{HTM}}$

Note: The Table contains the formulas for computing the various income and inequality measures used in the comparative analyses. Lifetime income can also be interpreted as lifetime consumption.

The effects of the different growth scenarios on factor shares of production are presented in Table 6. These results are expected: under both model specifications, automation leads to an increase in the capital share of income and, given q-complementarity, also an increase in the high-skilled labor share. Meanwhile, the low-skilled labor share falls. When comparing the automation-driven growth scenarios between the CSC and TB specifications, we find that the increase in the capital share is more than twice as large under the TB specification. This is a consequence of the displacement effect: as capital grows more productive, it not only becomes more productive in the tasks it's already performing, but it also gains a comparative advantage in more tasks than before. On the flip side, we observe that the fall in the low-skilled labor share is also substantially larger for the TB specification due to capital replacing labor in some tasks.

Table 6: Comparative statics on input shares, elasticities w.r.t. a change in output Y

Model	K/Y	L_H/Y	L_L/Y
$\sigma = 1.0$			
TFP (CSC)	0.163	0.131	-0.164
Automation (CSC)	0.372	0.320	-0.387
TFP (TB)	0.396	0.125	-0.245
Automation (TB)	0.835	0.328	-0.554
$\sigma = 1.4$			
TFP (CSC)	0.173	0.148	-0.173
Automation (CSC)	0.387	0.351	-0.396
TFP (TB)	0.387	0.142	-0.259
Automation (TB)	0.855	0.363	-0.565

Note: The Table shows the percentage change for the factor shares of production, given a percentage increase in output Y - conditional on either automation-driven or TFP-driven growth for both the CSC and TB specifications.

Table 7 documents the elasticity of prices to an increase in total factor productivity or capital productivity for the different technology specifications. Any growth driven by improved automation disproportionately benefits high-skilled individuals, since their wages are more responsive to output growth. This is true whether the source of growth is specific to capital (i.e., automation) or general (i.e., TFP). The direct effect follows from the higher degree of complementarity between high-skilled workers and capital in both specifications. In the post-growth steady states, we observe an increase in the skill premium, which in turn carries over to a more unequal distribution of pension benefits as the pension system is partially earnings-based. Furthermore, as the productivity of capital increases in both scenarios, inequality between capital owners and hand-to-mouth individuals will also increase. This bolsters pension inequality as rational savers top up any forced savings in the illiquid pension system with assets that earn interest.

Table 7: Comparative statics on prices, elasticities w.r.t. a change in output Y

Model	r	w_H	w_L	b_H^{RS}	b_L^{RS}	b^{HTM}
$\sigma = 1.0$						
TFP (CSC)	0.076	1.141	0.394	1.046	0.746	0.763
Automation (CSC)	0.174	1.344	0.153	1.121	0.382	0.417
TFP (TB)	0.149	1.148	0.313	0.994	0.590	0.638
Automation (TB)	0.315	1.377	−0.018	1.034	0.042	0.130
$\sigma = 1.4$						
TFP (CSC)	−0.005	1.234	0.400	0.910	0.651	0.659
Automation (CSC)	0.116	1.472	0.145	0.954	0.340	0.345
TFP (TB)	0.101	1.248	0.309	0.844	0.495	0.540
Automation (TB)	0.307	1.523	−0.036	0.847	0.022	0.085

Note: The Table shows the percentage change for various prices, given a percentage increase in output Y - conditional on either automation-driven or TFP-driven growth for both the CSC and TB specifications.

While growth is unequally distributed, it is important to note that every worker benefits from improved automation under the CSC scenario, as Table 8 illustrates. For automation-driven growth under the task-based technology, however, we find that automation-driven growth results in a lower steady-state wage rate among low-skilled workers. This suggests that the productivity effect is dominated by the displacement effect. The effect of automation on pension income is not trivial: while the wage rate falls for low-skilled workers under the TB specification, the pension system is partially redistributive and thus partially benefits from the increased contributions to the pension system made by high-skilled workers. If only looking at the benefit levels in 7, we see that both low-skilled rational savers and the HTM experience higher annuities following automation-driven growth. However, if one computes total pension income, as presented in Table 8, all low-skilled workers experience lower pension income following TB automation.

Table 8: Comparative statics on income measures, elasticities w.r.t. a change in output Y

Model	EI_H^{RS}	PI_H^{RS}	LI_H^{RS}	EI_L^{RS}	PI_L^{RS}	LI_L^{RS}	EI_L^{HTM}	PI_L^{HTM}	LI_L^{HTM}
$\sigma = 1.0$									
TFP (CSC)	1.093	0.944	1.086	0.666	0.656	0.665	0.668	0.643	0.666
Automation (CSC)	1.234	0.886	1.217	0.178	0.178	0.178	0.181	0.146	0.180
TFP (TB)	1.052	0.796	1.040	0.478	0.410	0.474	0.481	0.385	0.476
Automation (TB)	1.174	0.608	1.147	−0.246	−0.341	−0.251	−0.239	−0.397	−0.247
$\sigma = 1.4$									
TFP (CSC)	1.198	1.163	1.197	0.712	0.818	0.718	0.722	0.812	0.727
Automation (CSC)	1.353	1.095	1.341	0.194	0.273	0.198	0.197	0.244	0.244
TFP (TB)	1.145	0.963	1.136	0.490	0.497	0.490	0.500	0.474	0.498
Automation (TB)	1.281	0.748	1.256	−0.277	−0.360	−0.281	−0.274	−0.422	−0.282

Note: The Table shows the percentage change for various income measures, given a percentage increase in output Y - conditional on either automation-driven or TFP-driven growth for both the CSC and TB specifications.

The inequality effects are presented in Table 10. Comparing the income inequality measures for the baseline scenarios, we find that earnings inequality is higher when compared to pension income inequality. This follows mechanically from the design of the pension system as it is partially redistributive. Following automation-driven growth, under any specification, we find that both earnings inequality and pension inequality increase, but that earnings inequality remains larger throughout. For lifetime income inequality, which can also be interpreted as inequality in lifetime consumption, we see that the pension system does provide some insurance against automation-driven inequality, but this effect is modest. Overall, inequality effects are greater under the TB specification, as is expected given the inclusion of the displacement effect.

In Table 10, we quantify the public pension income share of retirement consumption for both high- and low-skilled rational saving workers. Obviously, this share is one for hand-to-mouth workers since they do not save. This share is also higher for low-skilled workers as the pension system is redistributive, so their replacement rate is higher than that of high-skilled workers. Since low-skilled workers face higher relative pension benefits, the Beveridgean pillar of the pension system causes larger disincentives for private

Table 9: Comparative statics on inequality measures

Model	Earnings ineq.	Pension ineq.	Lifetime ineq.
$\sigma = 1.0$			
Baseline	0.860	0.738	0.853
TFP (CSC)	0.924	0.775	0.915
Automation (CSC)	1.026	0.833	1.015
TFP (TB)	0.946	0.789	0.938
Automation (TB)	1.089	0.869	1.077
$\sigma = 1.4$			
Baseline	0.843	0.744	0.838
TFP (CSC)	0.911	0.787	0.904
Automation (CSC)	1.016	0.852	1.007
TFP (TB)	0.936	0.804	0.929
Automation (TB)	1.083	0.894	1.073

Note: The Table shows how earnings, pension and lifetime income inequality changes with either automation-driven or TFP-driven growth for both the CSC and TB specifications.

savings among these workers. This illustrates the potential dilemma that the designers of public pensions face: if the system is designed to redistribute income, as via the Beveridgean component, it can mitigate some of the increased inequality caused by improved automation, specifically the inequality that carries over from the skill premium via the Bismarckian pillar. Simultaneously, however, as the system becomes more redistributive, it creates larger disincentives for private savings among low-skilled workers. Such a reform therefore bolsters capital inequality, not only between rational savers and hand-to-mouth workers, but also between high-skilled and low-skilled rational savers.

To summarize, the public pension system provides some insurance against automation-driven income inequality through the Beveridgean pillar. However, the same mechanism also reduces the incentives for private retirement savings among high- and low-skilled workers. The pension system can work to reduce inequality via the link between wage and retirement benefits, and at the same time increase inequality via the savings channel.

Table 10: Public pension income share of retirement consumption

	CSC		TB	
Object	HS	LS	HS	LS
$\sigma = 1.4$				
Baseline	0.308	0.394	0.308	0.394
TFP growth	0.295	0.391	0.287	0.385
Automation growth	0.282	0.392	0.268	0.384
$\sigma = 1$				
Baseline	0.325	0.380	0.325	0.380
TFP growth	0.315	0.377	0.308	0.371
Automation growth	0.303	0.374	0.289	0.363

Note: The Table shows how the ratio of public pension income to retirement consumption changes for the different growth scenarios under both the CSC and TB specifications.

5.2 Pension policy and welfare

From the previous analysis, we conclude that automation can increase inequality in pension payments between high- and low-skilled individuals, and between rational savers and hand-to-mouth consumers. Furthermore, since public pensions are illiquid, hand-to-mouth consumers will forego any compound interest from not topping up their forced retirement savings with private savings in the capital market. Altogether, this implies an increase in lifetime consumption inequality, as seen in Table 9. In light of these sources of increased inequality, we now pose the question of whether automation-driven growth justifies any reform to the public pension size and design. To this end, we first quantify the optimal size of the public pension system, given by τ^*, in the baseline economy. We then analyze how the optimal value for this parameter changes if the economy is subject to growth, driven by either improvements to automation or to total factor productivity.

The central planner determine the size of the pension system τ so as to maximize social welfare which is:

$$\mathcal{W}(\tau) = \Gamma^{HTM}(1 - \Lambda_H^{RS} - \Lambda_L^{RS})U^{HTM} + \sum_{i \in \{L,H\}} \Gamma_j^{RS} \Lambda_j^{RS} U_j^{RS}, \tag{38}$$

where Γ^{HTM} and Γ_j^{RS} for $j \in \{L, H\}$ are welfare weights attached to the lifetime utility of hand-to-mouth and rational workers respectively. Assuming a standard Utilitarian central planner, these weights are normalized to unity $\Gamma^{HTM} = \Gamma^{RS}_j = 1$.

In turn, the analytical expressions for the optimal utility levels can be written as follows. For rational savers $j \in \{L, H\}$:

$$
\begin{aligned}
U^{RS}_j =& \frac{\mu_0^{-\frac{1-\sigma}{\sigma}}}{1-\sigma} \frac{\sigma}{r(1-\sigma)-\theta}\Big(\exp\big(\frac{r(1-\sigma)-\theta}{\sigma} T\big) - 1\Big) + \frac{1}{(1-\sigma)\theta}\Big(\exp(-\theta T) - 1\Big) + \\
& \beta \int_{R^{RS}_j}^{T} e^{\gamma t}(T-t)^{-\phi} e^{-\theta t} dt,
\end{aligned}
\tag{39}
$$

and HTM workers:

$$
\begin{aligned}
U^{HTM} =& \frac{(w_l(1-\tau_l))^{1-\sigma} - 1}{(1-\sigma)\theta}\Big(1 - \exp(-\theta R^{HTM})\Big) + \\
& \frac{(b_l)^{1-\sigma} - 1}{(1-\sigma)\theta}\Big(\exp(-\theta T) - \exp(-\theta R^{HTM})\Big) + \beta \int_{R^{HTM}}^{T} e^{\gamma t}(T-t)^{-\phi} e^{-\theta t} dt.
\end{aligned}
\tag{40}
$$

Ultimately, let τ_l^* denote the optimal size the pension system such that:

$$
\{\tau_l^*\} = \arg\max_{\tau} \{\mathcal{W}(\tau)\}. \tag{41}
$$

In Figure 3, we illustrate how welfare changes with the size of the pension system for the baseline calibration of both the CSC and the TB specifications. Since the TB specification is calibrated to perfectly match the CSC economy, the welfare level is the same for both model economies at the calibrated value for $\tau = 0.154$.

For both technology specifications, we first note that the optimal contribution rate τ^* is larger for a higher value of σ, which in turn translates to a lower value of the elasticity of intertemporal substitution. This is reasonable since if savings are more inelastic, the capital crowding-out effect of increasing the size of the pension system is smaller.

Comparing the CSC and TB economies, we find that the latter merits a larger public pension system than the former. More generally, increasing the size of the pension system beyond the calibrated value is less costly in terms of reduced welfare under the TB specification compared to the CSC specification. This follows from the displacement

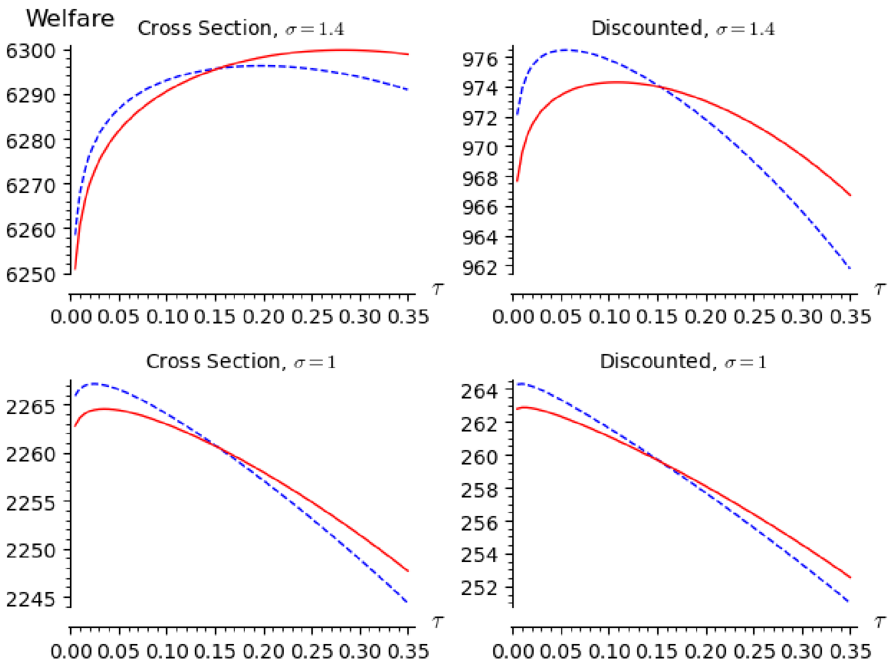


Figure 3: optimal size of the pension system given the baseline calibrations.

Note: The figures show the behavior of overall welfare in the economy under two specifications for the preferences: $\sigma = 1.4$, top row, and $\sigma = 1$, bottom row. The left-most column shows how the instantaneous welfare (i.e. when $\theta = 0$ in the economy , as defined in equation (38), varies with the size of the pension system, τ. The right-most provides the same illustration when the discount rate is set to its calibrated value $\theta = 0.032$.

effect of the TB specification: increasing the size of the pension system crowds out private savings, and therefore robots. As the interest rate increases, robots close to the extensive margin of automation now become relatively costlier to operate relative to low-skilled labor. As such, the increased labor demand for low-skilled labor results in higher equilibrium low-skilled wages.

Interestingly, only under the TB specification, when $\sigma = 1.4$ and we calculate cross-sectional welfare, do we find that the optimal size of τ is larger than the calibrated value. As illustrated in Figures 4 and 5, a somewhat surprising finding is that no growth scenario we consider merits any reform to the size of the public pension system. This holds true for both technology specifications.

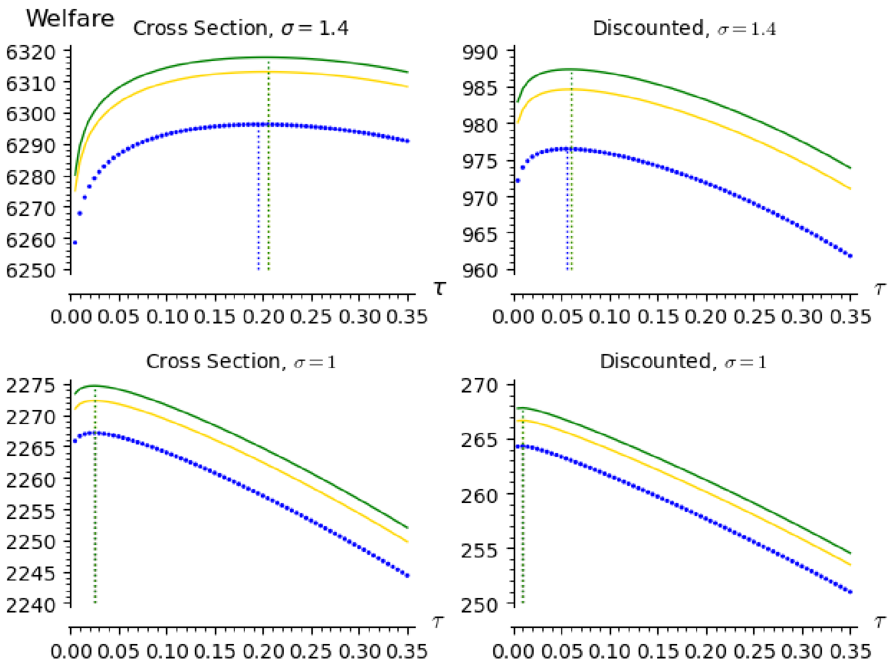


Figure 4: Optimal size of the pension system under CSC-technology.

Note: The figures show the response of overall welfare in the economy assuming the CSC specification in response to alternative changes to the technology. The blue dotted lines correspond to the baseline calibration; the yellow solid lines describe the welfare response to the automation-driven growth scenario, and the green solid lines show the welfare response to the TFP growth scenario. The top row shows the calibration for the preferences such that $\sigma = 1.4$, and $\sigma = 1$ in the bottom row. The left-most column shows how the instantaneous welfare (i.e. when $\theta = 0$ in the economy , as defined in equation (38), varies with the size of the pension system, τ. The right-most provides the same illustration when the discount rate is set to its calibrated value $\theta = 0.032$.

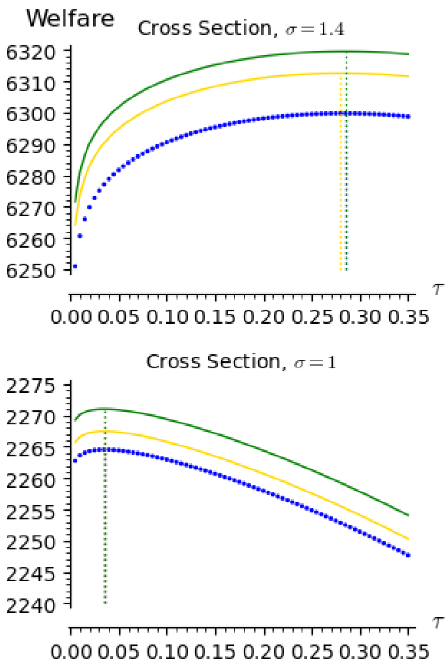


Figure 5: Optimal size of the pension system under TB-technology.

Note: The figures show the response of overall welfare in the economy assuming the TB specification in response to alternative changes to the technology. The blue dotted lines correspond to the baseline calibration; the yellow solid lines describe the welfare response to the automation-driven growth scenario, and the green solid lines show the welfare response to the TFP growth scenario. The top row shows the calibration for the preferences such that $\sigma = 1.4$, and $\sigma = 1$ in the bottom row. The left-most column shows how the instantaneous welfare (i.e. when $\theta = 0$ in the economy , as defined in equation (38), varies with the size of the pension system, τ. The right-most provides the same illustration when the discount rate is set to its calibrated value $\theta = 0.032$.

6 Conclusions

Public pension programs are typically funded through payroll taxes or other labor income-indexed contributions. Since automation has been shown to reduce labor shares, and possibly equilibrium wages, the present paper explores the implications of improved automation on a number of variables related to public pensions, such as the size and distribution of pension benefits. In a second stage, we quantify the optimal size and design of public pensions and study how these characteristics vary when economic growth follows from improvements to automation. These analyses are carried out in an overlapping generations model where workers differ in skill and optimization behavior.

We consider two different production function specifications to model automation. The first exhibits capital-skill-complementarity, where automated capital is more complementary to high-skilled workers than to the low-skilled. The other is a task-based specification in which improved automation displaces low-skilled workers.

Calibrating the model to average OECD macro-regularities, we find that improved automation can increase pension inequality through the earnings-related pillar of the pension system, and that the redistributive pillar provides some insurance against this. However, the redistributive pillar also generates larger disincentives for private savings among low-skilled households. As such, this increases inequality in private pension savings.

From the welfare analysis, we find that the model with task-based production merits a larger public pension system. This follows from the capital crowding out effects of increasing the size of the pension system being smaller under such a system, since capital displaces low-skilled labor. If capital is crowded out, and returns to capital increase, low-skilled labor will gain a comparative advantage in the production of more tasks. This increase in labor demand then results in higher equilibrium wages for these workers.

Ultimately, our results highlight important interaction effects between automation and the performance of public pensions. While public pensions are often designed to reduce intra-generational inequality, this pillar can bolster inequality in capital income as a higher replacement rate for low-skilled workers creates larger disincentives for these individuals to save privately for retirement. More generally, we show that the way technology is specified also matters for the quantitative assessments of the optimal size of public pensions. For example, how much capital displaces human labor affects the resource cost of expanding public pension systems, particularly in terms of crowding out private savings in the economy.

There exist several interesting paths for future work. The focus of this paper is on automation, and specifically capital that can substitute for low-skilled workers. With the recent emergence of AI technologies, it is natural to ask how our current results may change if one includes capital that substitutes for high-skilled workers. If the productivity growth in AI is more rapid than that of automation, it may reduce economic inequality. At the same time, if AI also contributes to the reduction of the labor share in the economy, it could exacerbate the funding issues already faced by public pension systems.

Furthermore, our analysis is restricted to unfunded pay-as-you-go pension systems. While this is by far the most common way of financing public pensions, a major question in the field of pension economics is whether public pensions should or could be privatized, and how such a reform could be made without violating the Pareto criterion. Given that our findings suggest an increased inequality between workers who save privately for retirement and those who only rely on public pensions, automation should increase the long-term gains of capitalizing public pension systems.

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A Appendix

A.1 Calculation of μ_j given R_j^{RS}, equations (19) and (21)

Given a value for R_j^{RS} and keeping all prices as given, we can substitute the expression for the optimal consumption (21) into the intertemproal budget constraint (19) to obtain a general expression for μ_j:

$$\mu_j|_R = \Big(\frac{r}{f}(e^{fT} - 1)\Big)^{\sigma} \Big(\tilde{w}(1 - e^{-rR}) + b_j(e^{-rR} - e^{-rT})\Big)^{-\sigma}, \tag{A1}$$

where $f \equiv \frac{r(1-\sigma)-\theta}{\sigma}$. In the absence of a pension system, that is if $\tau = 0$, the expression simplifies to:

$$\mu_j|_R = \Big(\frac{r}{f}(e^{fT} - 1)\Big)^{\sigma} \Big(w(1 - e^{-rR})\Big)^{-\sigma} = \Big(\frac{r}{wf}\Big)^{\sigma} \Big(\frac{e^{fT} - 1}{1 - e^{-rR}}\Big)^{\sigma}. \tag{A2}$$

A.2 Specifying utility for retirement.

We start from a stock-specification for retirement utility $v(R)$ that builds on the assumption that an individual full-time retires at age R and remains so throughout the rest of its lifetime. Assuming a CRRA functional form, this specification can be written as:

$$v(R) = \frac{(T - R)^{1-\phi}}{1 - \phi}, \tag{A3}$$

as in, e.g., Jacobs (2009), Gustafsson (2023a) and Gustafsson (2023b). From this stock-specification, we specify a constant flow utility function $\Gamma(T - R)$ over the period from R to T such that:

$$\frac{(T - R)^{1-\phi}}{1 - \phi} \approx \int_R^T \Gamma(T - R) e^{-\theta t} dt, \tag{A4}$$

so that the flow utility takes the form:

$$\Gamma(T - R) \approx e^{\gamma t}(T - t)^{-\phi}. \tag{A5}$$

This allows us to include discounting in utility and welfare measures explicitly, rather than assuming any discounting to be implicit to the weight of retirement preferences β.

A.3 Expression for the Aggregate Production Function with Task-Based choice in production, see Equations (7) and (11).

To simplify the discussion we focus on the analysis of the aggregate part of the model which is affected by automation as described in equation (11). For any task $u \in [0, \alpha^*)$ which uses capital it must be the case that at the margin:

$$p_G \psi(u)^{\rho_G} k(u)^{\rho_G - 1} G^{1-\rho_G} = r + \delta, \tag{A6}$$

where p_G measures the value of the aggregate G (we could carry the analysis from first principle detailing how p_G depends on the functions F and G and the derivative $\frac{\partial F}{\partial G}$). Furthermore, for any task $u \in (\alpha^*, 1]$ which uses labour in quantities $l(u)$ it must be the case that:

$$p_G l(u)^{\rho_G - 1} G^{1-\rho_G} = w. \tag{A7}$$

The following observations will be useful in what follows:

$$\begin{aligned} G &= S^{\frac{1}{\rho_G}}, \\ G^{1-\rho_G} &= S^{\frac{1}{\rho_G}-1}, \end{aligned} \tag{A8}$$

where we define $S \equiv \int_0^1 y(u)^{\rho_G} du$.

The first order condition expressed in equation (A6) can be rewritten into two expressions:

$$\begin{aligned} k(u) &= (\frac{r+\delta}{p_G})^{\frac{1}{\rho_G}-1} G \psi(u)^{\frac{\rho_G}{1-\rho_G}}, \\ \psi(u)^{\rho_G} k(u)^{\rho_G} &= \frac{r+\delta}{p_G} G^{\rho_G - 1} k(u). \end{aligned} \tag{A9}$$

From the first of these two expressions we obtain an additional expression for $psi(u)^{\rho_G} k(u)^{\rho_G}$:

$$\psi(u)^{\rho_G} k(u)^{\rho_G} = \psi(u)^{\frac{\rho_G}{1-\rho_G}} \Big[\frac{r+\delta}{p_G} G^{\rho_G-1}\Big]^{\frac{\rho_G}{\rho_G-1}}, \tag{A10}$$

and we observe that the expression within the squared brackets is constant (w.r.t. u). This then allow us to generate two expressions for $\int_0^{\alpha^*} \psi(u)^{\rho_G} k(u)^{\rho_G} du$. We find:

$$\begin{aligned} \mathcal{I}(\alpha^*) &\equiv \int_0^{\alpha^*} \psi(u)^{\rho_G} k(u)^{\rho_G} du = \Big[\frac{r+\delta}{p_G} G^{\rho_G-1}\Big]^{\frac{\rho_G}{\rho_G-1}} \int_0^{\alpha^*} \psi(u)^{\frac{\rho_G}{1-\rho_G}} du, \\ \mathcal{I}(\alpha^*) &\equiv \int_0^{\alpha^*} \psi(u)^{\rho_G} k(u)^{\rho_G} du = \Big[\frac{r+\delta}{p_G} G^{\rho_G-1}\Big] \int_0^{\alpha^*} k(u) du. \end{aligned} \tag{A11}$$

Denote K^* the aggregate capital across the tasks that use it, so that $K^* \equiv \int_0^{\alpha^*} k(u)du$. The second of the expressions above therefore imply that:

$$\frac{r+\delta}{p_G} G^{\rho_G-1} = \frac{\mathcal{I}(\alpha^*)}{K^*} \tag{A12}$$

The first expression can therefore be rewritten as:

$$\mathcal{I}(\alpha^*) = \Big[\frac{\mathcal{I}(\alpha^*)}{K^*}\Big]^{\frac{\rho_G}{\rho_G-1}} \int_0^{\alpha^*} \psi(u)^{\frac{\rho_G}{1-\rho_G}} du, \tag{A13}$$

and we can then solve for $\mathcal{I}(\alpha^*)$ and we find:

$$\mathcal{I}(\alpha^*) = \Psi(a^*) K^{*\rho_G}, \tag{A14}$$

where we set

$$\Psi(a^*) \equiv \Big(\int_0^{a^*} \psi(u)^{\frac{\rho_G}{1-\rho_G}} du\Big)^{1-\rho_G}.$$

Similarly we can define:

$$\mathcal{J}(\alpha^*) \equiv \int_{\alpha^*}^{1} l(u)^{\rho_G} du, \tag{A15}$$

and we can solve for $\mathcal{J}(\alpha^*)$ in a similar fashion, and we find:

$$\mathcal{J}(\alpha^*) = (1-\alpha^*)^{1-\rho_G} L^{*\rho_G}, \tag{A16}$$

where $L^{*\rho_G} \equiv \int_{\alpha^*}^{1} l(u)du$ is the aggregate labour used to produce G. Putting all these expression together we find (assuming optimal choices for all tasks):

$$\begin{aligned} G(y(u)) &= \left(\int_0^1 y(u)^{\rho_G} du \right)^{\frac{1}{\rho_G}} \\ &= (1-a^*)^{\frac{1-\rho_G}{\rho_G}} \left[(1-a^*)^{\rho_G - 1} \Psi(a^*) K^{\rho_G} + L_L^{\rho_G} \right]^{\frac{1}{\rho_G}} \equiv G(K, L, a^*) \end{aligned} \tag{A17}$$

which is the expression given in Equation (14).

We can furthermore describe how the function $G(k,1,a)$, i.e. the production per unit of low skilled labour, behaves when a is determined optimally, so that $\psi(a^*) = \frac{r+\delta}{w_l}$. First we observe that the ratio of prices can be expressed in terms of the production function itself:

$$\frac{r+\delta}{w_l} = \frac{G'_k(k,1,a)}{G(k,1,a) - kG'_k(k,1,a)}, \tag{A18}$$

and the RHS of the expression is:

$$\frac{G'_k(k,1,a)}{G(k,1,a) - kG'_k(k,1,a)} = \Psi(a^*)(1-a^*)^{\rho_G - 1} k^{\rho_G - 1}, \tag{A19}$$

and therefore, given some value k, a^* solves :

$$\psi(a^*) = \Psi(a^*)(1-a^*)^{\rho_G - 1} k^{\rho_G - 1}. \tag{A20}$$

From the definition of a^* we can determine how a^* responds to k, we find:

$$\frac{da^*}{dk} = \frac{1}{k} \frac{\rho_G - 1}{\frac{\psi'(a^*)}{\psi(a^*)} - \frac{\Psi'(a^*)}{\Psi(a^*)} + \frac{\rho_G - 1}{1-a^*}}, \tag{A21}$$

which, given our assumptions concerning $\psi(a)$, $\Psi(a^*)$, ρ_G and a, is positive. Finally, the definition of a^* in equation (A20) allows us to write the output $G(k,1,a^*)$ as:

$$\begin{aligned} G(k,1,a^*) &= \left[\Psi(a^*) k^{\rho_G} + (1-a^*)^{1-\rho_G} \right]^{1/\rho_G} \\ &= \left[\Psi(a^*) k^{\rho_G} + \frac{\Psi(a^*)}{\psi(a^*)} k^{\rho_G - 1} \right]^{1/\rho_G} \\ &= k\Psi(a^*)^{1/\rho_G} \left[1 + \frac{1}{\psi(a^*)k} \right]^{1/\rho_G}. \end{aligned} \tag{A22}$$

For cases where k is large enough for the second term in the previous expression to reach a limiting value, output per head $G(k,1,a^*)$ is therefore almost proportional to k .