Public Pension Reform and the Equity–Efficiency Trade-off*

Johan Gustafsson†

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Abstract

Alternative structures of public pension programs have distinct implications for the trade-offs that determine economic behavior over the life cycle. This paper studies these implications in terms of labor supply and economic inequality to characterize the equity–efficiency trade-off between a redistributive (Beveridgean) and an earnings-based (Bismarckian) benefit formula. The economy is modeled as a continuous-time overlapping generations model with endogenous labor supply, savings, and human capital formation. Individuals differ in ability, and they are free to choose how much to work at each period in time and when to enter and exit the labor market. Numerical simulations provide the qualitative insights that a redistributive pension system introduces opposite effects on the incentives to retire for high- and low-skilled individuals, which leads to an increased earnings inequality. This effect can, in turn, dominate the reduced pension inequality such that lifetime and population-wide income inequality increases. Ultimately, it appears that the equity–efficiency trade-off is difficult to characterize when accounting for endogenous labor supply on both the intensive and extensive margins.

Keywords: Equity-Efficiency, Inequality, Public Pension Policy

JEL Codes: H55, J22, I24

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†Umeå School of Business, Economics, and Statistics. E-mail: johan.a.gustafsson@umu.se

1 Introduction

It is axiomatic that policies that promote equity generate losses in efficiency. This equity–efficiency trade-off becomes relevant when considering structural reforms of public pension systems (see e.g., Cremer and Pestieau, 2011). For example, a benefit formula could be designed to prioritize intragenerational redistribution, which could be achieved by setting up a Beveridgean, basic-income system. As an alternative, the formula could be designed to supplement individual private savings, as with a Bismarckian, earnings-based system.[1] The first is expected to promote equity by compressing the distribution of pension income, and the second would promote efficiency following lower labor supply distortions (see e.g., Casamatta et al., 2000; Hachon, 2008; Krieger and Traub, 2013). The present study examines this equity–efficiency trade-off in a life-cycle perspective by analyzing how pension policy affects labor market entry and exit as well as the intensive margin labor supply.

In the context of pension reform, the equity–efficiency trade-off may be harder to characterize than is commonly assumed. Evidence suggests that countries with a Beveridgean pension system tend to display a higher degree of overall economic inequality than countries with a Bismarckian system (Pestieau, 1999; Casamatta et al., 2000; Galasso and Profeta, 2002). This observation can be explained by the voting preferences of low-income households who are expected to benefit financially from a Beveridgean structure (Ignacio Conde-Ruiz and Profeta, 2007).[2] An alternative explanation, which relies on reverse causality, can be obtained within a canonical life-cycle model with endogenous labor supply. Sommacal (2006) shows that when funds are allocated toward a Beveridgean pillar, the labor supply of low-skilled individuals drops disproportionally more than that of high-skilled individuals. In a competitive equilibrium, earnings inequality increases and it may offset the reduction in

[1] Emphasizing the egalitarian rationale for welfare programs, the World Bank recommends the first pillar of a pension system to be redistributive, while the second and third pillars should focus on life-cycle savings (World Bank, 1994). The contribution of public pensions to a redistributive tax-benefit system can be justified by societal preferences toward redistribution happening at retirement rather than at earlier stages of the life cycle (Cremer and Pestieau, 2011).

[2] The Beveridgean plan will make low-income individuals net-beneficiaries since their pension income will be partly subsidized by the contributions made by high-income individuals.

pension inequality such that lifetime inequality remains unchanged.

The present study argues that the structure of public pensions also has direct and indirect effects on the trade-offs determining labor market participation. In fact, when accounting for behavioral responses on the labor market entry and exit margins, I find that the increased earnings inequality induced by a Beveridgean system can dominate the reduced pension income inequality.

The key intuition is as follows. The properties of the contribution–benefit formula of the pension system will affect the rate at which individuals accumulate a sufficient capital buffer to finance their retirement. If the system is Beveridgean, the existence of a public pension system may allow low- (high-) income households to secure a sufficient retirement income with less (more) labor supplied relative to the amount supplied under a Bismarckian system of comparable size. This effect increases the difference between high- and low-income earners in labor market participation. I find that the realized increase in earnings inequality can dominate the reduced pension inequality, leading to an increase in overall lifetime inequality.

In addition to modifying incentives on the retirement margin, the effects of pension policy can affect labor market entry. Since the decision to retire ultimately ends the investment horizon over which the financial returns to human capital investments are realized, pension policy indirectly affects the incentives for labor market entry (e.g., Jacobs, 2009; Ludwig et al., 2012; Kindermann, 2015). If the pension system promotes an early retirement, the incentives for human capital investments early in life are weakened. This complementarity has at least two important implications for intragenerational inequality. First, if the indirect effects of pension policy on human capital formation differ across skill groups, the reform will affect earnings inequality through changes in labor market participation among individuals. Second, if human capital inequality increases (decreases), the policy will increase (decrease) wage inequality.

The main purpose of this paper is therefore to study the impact of public pension structures on labor supply and intragenerational redistribution in terms of income inequality.

The findings are highly relevant for the ongoing debates about whether public pension programs should be reformed, or even refinanced, to ensure their fiscal sustainability in times of population aging. Many developed economies, such as Sweden, Poland, and Italy, have introduced notional pension accounts as a main feature to the public pension system. Such a reform implies that individual contributions are appreciated by a government-determined, fictitious rate of return to mimic the capitalization of funds without abandoning the pay-as-you-go (PAYG) transfer mechanism (Könberg et al., 2006).[3] Parallel to this development, Beveridgean pension systems have been introduced in many developing countries (Kemmerling and Neugart, 2019), and they have largely remained untouched in developed countries such as Canada, The Netherlands, and Denmark.[4]

To account for both the labor market entry and exit and intensive margins of labor supply, this paper builds on the continuous-time OLG model presented by Jacobs (2009).[5] However, the analyses require some modifications to the original framework. First, the Jacobs model concerns the behavior of a representative agent, so it is not suitable for studying issues of intragenerational redistribution. Second, the original model treats retirement benefits as exogenous. I remedy these properties by adding skill heterogeneity and an endogenous pension system with both a Beveridgean and a Bismarckian pillar to account for the different behavioral incentives implicit to its design.

Since a Bismarckian system introduces a link between an individual's earnings history and realized pension benefits, any rational, foreseeing individual will acknowledge that the opportunity cost of leisure includes foregone pension income. To account for this lagged ef-

[3]Since the system remains PAYG, the notional defined contribution (NDC) reform can only partially remedy the solvency issue. Many scholars instead suggest a refunding of pension systems by capitalizing contributions on the capital market. However, this transition is difficult to make without violating the Pareto criterion, as one generation will be forced to carry the financial burden of contributing to both the PAYG and the fully funded systems but only benefit from the latter (e.g., Andersen et al. (2020)).

[4]This is particularly interesting since Sweden and Denmark are commonly placed in the same group of social democratic welfare states, yet they appear to display polarized views on the structure of social security (Andersen and Larsen, 2002).

[5]While several papers analyze implications of pension design and funding in two- or three-period Diamond model frameworks (e.g., Sommacal, 2006; Wen et al., 2015; Frassi et al., 2019), it is difficult to augment the discrete-time OLG models with additional periods to study the extensive margins of labor supply without compromising on tractability.

fect, the model is solved as a *delayed response* optimal control problem (see e.g., Kamien and Schwartz (2012)). This way, this paper introduces a method of solving for the consumption–saving and labor supply behavior of individuals integrated into an earnings-based pension system via the maximum principle. The first-order condition for optimal leisure is modified such that both the contemporaneous labor–leisure trade-off and the effect on future pension income, following a change in leisure intensity, are accounted for in the solution.

The predictions of the model are illustrated using numerical simulations. If individuals are not overly patient, a redistributive pension system can increase lifetime inequality, reflecting a larger difference in labor market participation. This is a remarkable finding since the model makes standard behavioral assumptions. More specifically, if individuals are impatient enough, and the public pension system is extensive enough, a reform from a flat-rate type pension system toward an earnings-based system can promote both efficiency, as labor supply incentives are strengthened among both high- and low-income individuals, and equity, since earnings inequality is reduced. This result suggests that it is not obvious that a reform toward more individualized, earnings-based pension systems will increase overall economic inequality relative to under a flat-rate scheme of comparable size.

In addition to these insights, the paper conforms to much of the established wisdom in a more general microfoundations environment. In particular, consistent with the findings in Kindermann (2015), if the NDC system is return-dominated by private savings, labor supply becomes more attractive later in life as the foregone compound interest by investing in the pension system decreases over the life cycle. The implicit tax treatment thus lowers the opportunity cost of tertiary education, and it promotes a delayed entry into the labor market. The introduction of an NDC system then promotes an increase in educational attainment, while a Beveridgean system has the opposite effect.

The rest of the paper is organized as follows. Section 2 reviews the literature. The model is introduced and solved in section 3. Section 4 contains the numerical simulations. Section 5 concludes.

2 Literature Review

This study is most closely related to the research on how compulsory public pensions modify incentives for economic behavior over the life cycle. Since this paper concerns outcomes in terms of labor supply and economic inequality, these properties are the primary focus of the literature review.

An important strand of research focuses on how public pension systems affect labor supply incentives (see e.g., Browning, 1975; French and Jones, 2012; Frassi et al., 2019). In particular, the contribution rate, net of the present value of incremental pension benefits realized from such transfer, effectively instruments an income tax (see e.g., Beckmann, 2000; Fenge and Werding, 2004; Goda et al., 2011). The magnitude of such implicit taxation is then determined by the relationship between individual contributions and benefit entitlements. If this link is strong, as with a Bismarckian system, the implicit tax rate is low. On the contrary, if the system is characterized by a large degree of within-cohort redistribution, as with a Beveridgean system, the implicit tax rate is high. Allocating funds from a Beveridgean pillar to a Bismarckian pillar is thus expected to increase efficiency in the economy at the cost of higher income inequality. This implies that the policymaker is faced with an equity–efficiency trade-off.

Cigno (2008) concludes in a two-period model that a Beveridgean pension system always distorts the intensive margin labor supply, as the implicit taxation reduces the cost of leisure. The same is not true for an actuarially fair Bismarckian pension system, where the returns to pension contributions are equivalent to capital interest. Thus, the system acts only as a form of mandatory saving. Consistent with the equity–efficiency trade-off, he finds that reducing the size of a Beveridgean system has positive effects on labor supply, but it promotes increased income inequality. Wen et al. (2015) show in a two-period Diamond model that reforming a pension structure from Beveridgean to Bismarckian substantially reduces labor supply disincentives. Focusing on the extensive margin response, Gruber and Wise (1999) find a systematic relationship between implicit taxation of social security systems and retirement behavior, which can explain early retirement behavior in the developed world.

Following the implicit income taxation induced by public pension systems, this study also builds on the insights of researchers who study how labor income taxation, in general, affects aggregate labor supply. In a highly influential paper, Prescott (2004) finds that differences in labor income taxation can explain much of the variations in aggregate labor supply between Europe and the United States. Jacobs (2009) and Wallenius (2013) are other notable contributions where the authors quantify the labor supply response to variations in labor income tax rates. While Prescott (2004) and Wallenius (2013) focus on adjustments on the retirement margin, Jacobs (2009) considers both entry and exit, as well as the intensive margin response. He finds the uncompensated labor supply elasticity to increase by almost 50% when accounting for decisions on education and retirement. This suggests that labor income taxation is much more distortionary when the labor market entry and exit margins are accounted for.

Sommacal (2006) studies the intragenerational redistributive effect of reforming a pension system from Bismarckian to Beveridgean. In a two-period Diamond-type model with endogenous intensive margin labor supply, when the labor market is competitive and where high- and low-skilled labor are perfect substitutes in production, he finds that an increase in the flat-rate pillar does not reduce economic inequality. The explanation is that as funds are allocated toward the Beveridgean pillar, the low-skilled reduce their labor supply more than the high-skilled. As a result, the reform increases earnings inequality, which offsets the direct effects of the reduced pension income inequality. This result is, however, not robust to the inclusion of a minimum wage, which suggests that labor market institutions play an important role in determining the redistributive outcome of pension system reform.

The structure and funding of public pensions will also directly and indirectly affect incentives for human capital investments. For example, enrolling in non-mandatory education increases labor income prospects, and this typically increases the rate of contributions to the public pension system.[6] However, it also delays the time when the individual starts to contribute. Caliendo and Findley (2019) examine the relationship between social security taxation and human capital formation under both Beveridgean and Bismarckian contribution–benefit formulas, keeping the retirement age fixed. Their model can replicate the negative correlation between public pension contribution rates and average educational attainment among OECD economies. This result is, however, not consistent with the findings in Kindermann (2015) who shows that NDC pension systems subsidize human capital formation. The intuition behind this finding is that the NDC system introduces a higher implicit marginal tax on younger workers because of the foregone compound interest when returns to the pension system underperform returns to private savings. Both Caliendo and Findley (2019) and Kindermann (2015) model labor market entry as endogenous while keeping the retirement margin fixed.

Docquier and Paddison (2003) and Le Garrec (2012) study the impact of retirement policies on human capital formation in endogenous growth OLG models. Generally, they find PAYG systems to crowd out both physical and human capital in the economy. Physical capital is crowded out by the forced savings of the pension system. This finding is well-established following the seminal work of Feldstein (1974). The imposed scarcity of physical capital, in turn, increases market interest, which makes human capital investments less attractive. Docquier and Paddison (2003) show that the more a pension system is redistributive, the larger are the disincentives for human capital investments. Krieger and Traub (2013) support this conclusion. They show in a two-period model with human capital formation that the more weight is put on the redistributive pillar of social security, the higher are the disincentives for human capital formation.

Hachon (2010) illustrates in a model with physical capital-driven growth and heterogeneous longevity that a Beveridgean system increases savings dispersion between high- and low-income individuals. In his model, more productive individuals compensate for the re-

[6]This is not necessarily the case for pension systems in which contributions are subject to an upper-limit.

duced replacement income by saving more privately. On the contrary, the higher replacement income faced by agents of below-average productivity will reduce private savings further. When acknowledging that life expectancy is positively correlated with income, the net effect on overall capital accumulation and economic growth is found to be positive.

It is well worth mentioning an alternative approach to linking public pension systems and intragenerational inequality as exemplified within the political economy literature. Pestieau (1999) studies the implications of the contribution–benefit formula on the optimal size of public pension systems. He describes a paradox in that Beveridgean systems are often smaller than Bismarckian pension systems, even though they should enjoy more support among low-income groups. Ignacio Conde-Ruiz and Profeta (2007) rationalize this phenomenon in a bidimensional voting model where the public votes on both the size and the design of public pensions. They propose an explanation that the design of pension systems is endogenous to the income distribution of the economy. In such a setting, an economy with a coalition of a large share of low- and high-income individuals are likely to support a small redistributive system, while economies with a large middle class prefer a more extensive Bismarckian system. Without denying the mechanism that supports this finding, this paper provides an explanation that uses reverse causality within a canonical life-cycle model. That is, a redistributive pension system may fail to reduce economic inequality once labor supply effects are accounted for.

3 The Model

3.1 Economic environment

Time is continuous and denoted t. Consider a steady-state OLG-type environment as described in Jacobs (2009) in which individuals of all ages are represented at each instant in time and identically replicate themselves. The economy consists of two types of rational individuals varying only in human capital ability and indexed i=1,2, where $i = 1$ $(= 2)$ corresponds to low (high) ability. Unless explicitly needed, this indexation will be suppressed to avoid notational clutter. Individuals are strict non-altruists and thus have no bequest motive. Birth-cohort size is kept constant such that population growth is zero. This ensures that the economy is stylized as dynamically efficient for any non-negative interest rate. For convenience, the population is normalized to unity. Λ is the population weight of low-ability individuals. The economy is small and open, which together with the steady-state assumption implies exogenous and fixed factor prices.

3.2 The individuals

I assume that the economic life starts upon completion of upper secondary school, at the model age of $t = 0$, and ends with certainty at $t = T$.[7] The economic life of any individual can be decomposed into three distinct phases: tertiary education $t \in [0, S)$, working life $t \in [S, R)$, and retirement $t \in [R, T]$. At each instant t, individuals are endowed with one unit of time. Individuals discount the future at a constant rate θ.

While enrolling in tertiary education, individuals devote all time available to productivity-enhancing training. This assumption is made for mathematical convenience, and it is consistent with the original model specification in Jacobs (2009). The human capital decision is subsequently reduced to a choice about labor market entry. During the working life, individuals allocate the time available between leisurely activities $l(t)$ and working $1 - l(t)$. For each unit of effective labor supply, individuals earn the wage rate w, net of a pension contribution rate τ. Retirement is assumed to be an absorbing state, which implies a permanent full-time withdrawal from the labor market. Retirement leisure is considered as a completely separate good compared to leisure time during the working life, and it depends on the time spent in retirement $(T - R)$. Individuals derive felicity from the consumption of non-durable goods $c(t)$ throughout their economic life. Any net-of-tax income not used for contemporaneous consumption flows into the individual's asset account $k(t)$, which grows

[7] While the original model by Jacobs (2009) assume individuals to enter the model at age 6, I follow the modeling assumption of Caliendo and Findley (2019) in this aspect.

at the constant risk-free interest rate r. Since students are natural borrowers, and debt is a commonly observed feature of the economic life, I allow for $k(t) < 0$ at any interior point in time.[8]

3.3 Human capital production

The production of human capital is stylized to the following functional form:

$$F_i(S_i) = A_i S_i^{\rho_i}, \tag{1}$$

where A is an overall propensity for converting time spent in tertiary education into labor productivity units, and ρ is an elasticity of productivity with respect to education. Let $\rho \in [0, 1)$ such that the production function exhibits diminishing marginal returns. In this specification, time spent in education has a pure multiplicative scaling effect on the efficiency profile. This specification allows for heterogeneity in productivity between individuals without affecting the intertemporal trade-off between labor and leisure.

3.4 Pension system

To make results applicable to the OECD in general, this paper considers a pension system that is stylized to resemble three key mechanisms (pillars) of most modern public pension systems following the recommendations of World Bank (1994). Two pillars are mandatory: the earnings-based, Bismarckian pillar, and the flat-rate, Beveridgean pillar. I assume the third pillar to be made up entirely of voluntary private savings on the capital market. The transfer mechanism is PAYG, implying that contributions are contemporaneously realized as pension benefits of the retired. This type of specification encompasses the general

[8] This assumption is also made in Caliendo and Findley (2019), and it is not problematic as long as the aggregate capital stock is positive. In a two-period Diamond model, it is natural to impose credit constraints to avoid solutions in which individuals finance debt in the first period by borrowing against future pension benefits. As will be discussed in the results section, the assumption of perfect credit markets limits the parameter range for θ.

contribution-transfer mechanism of many OECD economies (Bovenberg et al., 2012).

In this model, retirement benefits are specified as annuities. The annuity received by individual i at any time $t \in [R, T]$ is the weighted sum of the Bismarckian and the Beveridgean pillars, annuitized over the time spent in retirement:

$$b(t) = \frac{1}{T - R}[\kappa B^E(R)e^{\gamma(t-R)} + (1 - \kappa)B^C]. \tag{2}$$

In this specification, B^E is the Bismarckian pillar, and γ is the notional interest on the fraction of contributions allocated toward this pillar. B^C is the Beveridgean pillar, which does not appreciate in the absence of population growth. $\kappa \in [0, 1]$ governs the relative size of the two pillars, with $\kappa = 1$ implying a completely earnings-based benefit formula, and $\kappa = 0$ a completely redistributive system. κ thereby determines the correlation between individual earnings and pension entitlements.

The redistributive pillar is specified as follows:

$$B^C = \tau w H, \tag{3}$$

where $H = \Lambda F_1(S_1) \int_{S_1}^{R_1}(1 - l_1(t))dt + (1 - \Lambda)F_2(S_2) \int_{S_2}^{R_2}(1 - l_2(t))dt$.

Since the common benefit level is derived from aggregate contributions in the economy, it is treated as exogenous by the individuals.

The individuals do, however, rationalize how their labor supply decisions affect their pension benefits through the earnings-based component. The accumulation of individual pension entitlements following the labor supply choice of the individual is specified as follows:

$$\dot{B}^E = \frac{\partial B^E(t)}{\partial t} = (1 - l(t))wF(S)\tau + \gamma B^E(t). \tag{4}$$

Solving the differential equation, I obtain the following expression for the total amount of accumulated benefit entitlements through the Bismarckian pillar at retirement age:

$$B^E(R) = e^{\gamma R}\Bigg[B(0) + w\tau F(S)\int_S^R (1 - l(t))e^{-\gamma t}dt\Bigg]. \tag{5}$$

Assuming that all individuals start out with zero benefit entitlements, $B(0) = 0$, substituting Equations (3) and (5) into Equation (2) yields the following expression:

$$b(t) = \frac{w\tau\Big[\kappa F(S)e^{\gamma t}\int_S^R(1 - l(t))e^{-\gamma t}dt + (1 - \kappa)H\Big]}{T - R}. \tag{6}$$

Equation (6) is the fully specified retirement benefit system, where benefits are computed as a weighted average between the Bismarckian and Beveridgean pillars.

It is important to note that this structural representation abstracts from non-linear features of the contribution–benefit formula common to real-world systems.[9] While such properties undoubtedly play important roles in the performance of public pension systems, these rules vary substantially between countries. In addition, incorporating such non-linear features of the contribution–benefit formula in an optimal control framework introduces substantial modeling challenges (see e.g., Wang and Li (2017)).

3.5 The maximization problem

I consider a stylized model to obtain analytical expressions for optimal controls. Suppose that lifetime utility takes the following additive separable form:

$$\begin{aligned} V = \int_0^S \ln(c(t))e^{-\theta t}dt + \int_S^R [\ln(c(t)) + \beta\ln(l(t))]e^{-\theta t}dt + \\ \int_R^T \ln(c(t))e^{-\theta t}dt + \frac{\eta[T - R]^{1-\frac{1}{\phi}}}{1 - \frac{1}{\phi}}, \end{aligned} \tag{7}$$

[9]These include, among other, benefit penalties or subsidies related to eligibility age and upper or lower boundaries to contributions or realized benefits.

where β is the weight attached to leisure during the working life, η the weight on retirement leisure preferences, and ϕ a parameter that makes utility non-linear in time spent in retirement.[10] I make the assumption of log preferences over consumption and intensive margin leisure following common practice in quantitative macroeconomics.[11] Preferences for retirement leisure are instead specified in the more general CRRA-form since the log structure would not be defined for the possible corner solution of $R = T$.[12]

Individual wealth $k(t)$ at any time during tertiary education, $t \in [0, S)$ is:

$$k(t) = k(0) + \int_0^t [rk(s) - c(s)]ds. \tag{8}$$

For the working life, $t \in [S, R)$:

$$k(t) = k(S) + \int_S^t [(1 - l(s))wF(S)(1 - \tau) + rk(s) - c(s)]ds, \tag{9}$$

and finally for the retirement phase, $t \in [R, T]$:

$$k(t) = k(R) + \int_R^t [b(s) + rk(s) - c(s)]ds. \tag{10}$$

From Equations (8)-(10) it is possible to obtain the conventional expressions for the time

[10] This can be referred to as an elasticity of the retirement good.

[11] The assumption of log utilities follows the specification of the CRRA utility, such that the parameters of the intertemporal elasticity of substitution in consumption and leisure are equal to unity. This is reasonable given the empirical evidence reviewed by Thimme (2017) for consumption and Blundell and MaCurdy (1999) for leisure.

[12] Note that the retirement good is not discounted into present value. This is consistent with the model specification in Jacobs (2009). It is also difficult to determine which age should be used as a reference for discounting such a good, since it is a function of the years in retirement but not specified as a flow variable.

derivatives describing the saving dynamics over the life cycle:

$$\dot{k} = \begin{cases} rk(t) - c(t) & \text{for } t \in [0, S), \\ (1 - l(t))wF(S)(1-\tau) + rk(t) - c(t) & \text{for } t \in [S, R), \\ b(t) + rk(t) - c(t) & \text{for } t \in [R, T]. \end{cases} \tag{11}$$

The discrete changes in the state function as specified in Equation (11) allows for the specification of the optimal control problem as a multiple-stage control problem.[13]

3.6 A maximum principle solution

I begin by examining the optimal consumption and intensive margin leisure behavior conditional on labor market entry and exit. In a second stage I solve for optimal labor market entry and exit. The optimization problem of maximizing Equation (7) subject to (11) allows me to define the following Hamiltonian functions corresponding to the optimal control problems of each stage of the multiple-stage control problem. I introduce superscripts to distinguish the different life-cycle phases: 1 = tertiary education phase $t \in [0, S)$; 2 = working life phase $t \in [S, R)$; 3 = retirement phase $t \in [R, T]$. These Hamiltonians can be written in present value terms, discounted to time $t = 0$, as follows for phase 1:

$$\mathcal{H}^1(t) = \ln(c(t))e^{-\theta t} + \mu^1(t)[rk(t) - c(t)], \tag{12}$$

phase 2:

$$\mathcal{H}^2(t) = [\ln(c(t)) + \beta \ln(l(t))]e^{-\theta t} + \mu^2(t)[(1 - l(t))wAS^\rho(1-\tau) + rk(t) - c(t)], \tag{13}$$

[13] This is essentially a generalization of a two-stage control problem as outlined in Kamien and Schwartz (2012). A three-stage optimal control problem also is illustrated and solved as a generalized salvage value problem in Gustafsson (2021).

and finally phase 3:

$$\mathcal{H}^3(t) = \ln(c(t))e^{-\theta t} + \mu^3(t)[b_i(t) + rk(t) - c(t)]. \tag{14}$$

Note that the utility of time spent in retirement does not enter any of the Hamiltonian functions since it is not specified as a dynamic control variable. Instead, it will enter as an explicit argument when solving for optimal retirement age in section 3.6.6.

3.6.1 Phase 1: Non-mandatory education

Recall that the education phase is characterized by full-time engagement in training. The agent subsequently makes only the intertemporal choice on the consumption–savings margin. The condition characterizing optimal consumption at each instant becomes:

$$\frac{\partial \mathcal{H}^1(t)}{\partial c(t)} = \frac{e^{-\theta t}}{c(t)} - \mu^1(t) = 0. \tag{15}$$

The law of motion governing the marginal utility of wealth is derived as:

$$\dot{\mu}^1 = -\frac{\partial \mathcal{H}^1(t)}{\partial k(t)} = -r\mu^1(t). \tag{16}$$

Solving the differential equation yields the following expression for the marginal utility of wealth:

$$\mu^1(t) = \mu_0 e^{-rt}, \tag{17}$$

where μ_0 is an unknown constant to be determined. Substituting the expression in Equation (17) into Equation (15) and solving for consumption yields:

$$c^*(t) = \frac{e^{(r-\theta)t}}{\mu_0}. \tag{18}$$

Equation (18) implies that optimal consumption evolves according to the Euler equation. If $r > \theta$ ($r < \theta$), consumption increases (decreases) over the time domain, and if $r = \theta$, it remains constant.

3.6.2 Phase 2: Working life

In contrast to phase 1, phase 2 is characterized by decision making on both the consumption–savings margin and the labor–leisure margin. Analogous to the solution for phase 1, optimal consumption during the working life satisfies the following first-order condition:

$$\frac{\partial \mathcal{H}^2(t)}{\partial c(t)} = \frac{e^{-\theta t}}{c(t)} - \mu^2(t) = 0. \tag{19}$$

Following the maximum principle, the total effect on the utility of a small change in leisurely time should be zero. Conditional on that $\kappa > 0$, the total effect is partially realized contemporaneously through the conventional trade-off between leisure utility and foregone labor income. However, it is also realized through foregone pension income according to the Bismarckian pillar. This essentially constitutes a delayed response problem as described in (Kamien and Schwartz, 2012), albeit slightly modified. More specifically, the lagged response takes place over a continuum of future time periods as opposed to one future instant in time as conventionally specified. The inclusion of a Bismarckian pillar makes this consideration necessary, and it marks one of the distinct modifications I make to the model specification in Jacobs (2009), in which pension benefits are modeled as exogenous. As a result, I successfully integrate an endogenous pension system in an optimal-control environment.

The first-order condition must reflect that the leisure choice made at *any* instant $t \in [S, R)$ will affect the annuitized benefits at *every* instant $t \in [R, T]$. For clarity, I introduce the temporary time notation $u = t \in [R, T]$ since the benefits realized over the retirement phase are a function of the leisure activities during the working life phase. The first-order condition ensuring optimality of the intensive margin leisure trajectory subsequently becomes:[14]

$$\begin{aligned} &\frac{\partial \mathcal{H}^2(t)}{\partial l(t)} + \int_R^T \frac{\partial \mathcal{H}^3(u)}{\partial l(t)} du \\ &= \frac{\beta e^{-\theta t}}{l(t)} - \mu^2(t) w F(S)(1-\tau) + \int_R^T \mu^3(u) \frac{\partial b_i(u)}{\partial l(t)} du = 0, \end{aligned} \tag{20}$$

and the law of motion satisfies:

$$\dot{\mu}^2 = -\frac{\partial \mathcal{H}^2(t)}{\partial k(t)} = -r\mu^2(t). \tag{21}$$

Solving the differential equation results in the following expression:

$$\mu^2(t) = \mu^2(S) e^{-r(t-S)}, \tag{22}$$

which if substituted into Equation (19) yields the following expression for optimal consumption and leisure over the working life:

$$c^*(t) = \frac{e^{(r-\theta)t - rS}}{\mu^2(S)}. \tag{23}$$

The interpretation of Equation (23) is identical to the condition in Equation (18): namely that the dynamics of optimal consumption obey the conventional Euler equation. The explicit solution for the leisure profile will be derived after solving for the law of motion for the retirement phase and imposing the transversality conditions for the law of motion governing optimal savings behavior over the entire life cycle.

[14] I am able to obtain the same condition when using a Lagrangean function instead of the Hamiltonian. These calculations are available upon request.

3.6.3 Phase 3: Retirement

Since the retirement phase is characterized by the cessation of labor supply, the agent only makes decisions on the consumption–savings trade-off.

$$\frac{\partial \mathcal{H}^3(u)}{\partial c(u)} = \frac{e^{-\theta u}}{c(u)} - \mu^3(u) = 0 \tag{24}$$

Identical to the solutions in phase 1 and 2, the law of motion must satisfy the following condition:

$$\dot{\mu}^3 = -\frac{\partial \mathcal{H}^3(u)}{\partial k(u)} = -r\mu^3(u). \tag{25}$$

Solving the differential equation results in the following expression:

$$\mu^3(u) = \mu^3(R)e^{-r(u-R)}, \tag{26}$$

which, if substituted into Equation (24), yields the expression for optimal consumption during the retirement phase:

$$c^*(u) = \frac{e^{(r-\theta)u-rR}}{\mu^3(R)}. \tag{27}$$

Equation (27) together with Equations (23) and (18) ultimately implies that consumption evolves over the life cycle in a continuous and monotonic fashion consistent with the Euler equation.

3.6.4 Transversality condition 1: Continuity of co-state variable

Following the principle for optimality of a multiple-stage control problem, the following conditions ensure continuity of the co-state variable over the entire control domain:

$$\mu^1(S) = \mu^2(S) \tag{28}$$

$$\mu^2(R) = \mu^3(R) \tag{29}$$

Equations (28) and (29) ensure that optimal behavior over the life cycle is consistent with smoothing the marginal utility of wealth. Thus,

$$\mu^3(t) = \mu^3(R)e^{r(t-R)} = \mu^2(S)e^{-r(R-S)}e^{-r(t-R)} = \mu_0 e^{-rS}e^{-r(R-S)}e^{-r(t-R)} \tag{30}$$

Equation (30) simplifies to the following expression, which defines the law of motion governing optimal savings behavior over the entire life cycle:

$$\mu(t) = \mu_0 e^{-rt}. \tag{31}$$

Following the calculations outlined in the appendix, μ_0 takes on the following expression:

$$\mu_0 = \frac{\int_0^T e^{-\theta t}dt + \beta \int_S^R e^{-\theta t}dt}{w\{F(S)[(1-\tau)\int_S^R e^{-rt}dt + \frac{\kappa\tau}{T-R}\int_R^T e^{(\gamma-r)t}dt \int_S^R e^{-\gamma t}dt] - \frac{(1-\kappa)\tau H}{T-R}\int_R^T e^{-rt}dt\}} \tag{32}$$

3.6.5 Summary of optimal controls

Optimal consumption is characterized by the following condition over the entire life-cycle domain for each individual:

$$c^*(t) = \frac{e^{(r-\theta)t}}{\mu_0}. \tag{33}$$

Following the specification of utility as additive separable between consumption and leisure, the pension system does not affect optimal consumption behavior explicitly. Rather, its effect is only indirect by modifying the marginal utility of wealth.

The optimality condition for intensive margin leisure follows from Equation (20), which in its explicit form reads:

$$l^*(t) = \frac{\beta e^{-\theta t}}{\mu_0 A S^\rho[(1-\tau)e^{-rt} + \frac{\kappa\tau e^{-\gamma t}}{T-R}\int_R^T e^{(\gamma-r)t}dt]}. \tag{34}$$

Equation (34) informs that the opportunity cost of leisure consists of the contemporaneous loss of earnings and any future loss in terms of foregone pension income. The opportunity cost is reduced by the fraction of contributions that are allocated to the Beveridgean pillar $1-\kappa$, and any foregone compounded interest of contributing to an actuarially unfair Bismarckian pillar conditional on $\gamma < r$.

Some special cases of Equations (34) are of analytical interest with regard to the explicit effects of changes in the contribution rate. If $\kappa = 0$, the condition for optimal intensive margin leisure collapses to the following expression:

$$l^*(t) = \frac{\beta e^{(r-\theta)t}}{\mu_0 A S^\rho (1-\tau)}. \tag{35}$$

This implies that the contribution rate becomes a pure implicit labor income tax rate as the returns to contributions are not recognized by the individual following the flat benefit-design of the transfer mechanism. If instead considering $\kappa > 0$, i.e., that benefits bear some explicit relationship to contributions, and contributions earn market return, $\gamma = r$, the expression in Equation (34) simplifies to the following expression:

$$l^*(t) = \frac{\beta e^{(r-\theta)t}}{\mu_0 A S^\rho [(1-\tau) + \kappa\tau]}. \tag{36}$$

The expression in Equation (36) implies that in an actuarially fair system, the contributions to the earnings-based pillar, $\kappa\tau$, are rationalized as a perfect substitute for private savings. If the system is fully Bismarckian, i.e., $\kappa = 1$, the contemporaneous decrease in labor income from an increase in the contribution rate is perfectly offset by the increase in pension income. That is, an actuarially fair Bismarckian system does not introduce any direct disincentives for labor supply. Furthermore, for the actuarially fair pension system, the expression in Equation (32) for μ_0 simplifies to:

$$\mu_0 = \frac{\int_0^T e^{-\theta t} dt + \beta \int_S^R e^{-\theta t} dt}{w F(S) \int_S^R e^{-rt} dt}. \tag{37}$$

This implies that an increase in the contribution rate does not have any direct effect on the marginal utility of wealth, so there is no indirect effect on consumption–savings or the intensive margin labor–leisure trade-off.

3.6.6 Transversality condition 2: Optimal switching points

The final step is to determine the optimal timing for labor market entry and exit (i.e., $\{S^*, R^*\}$). Conditional on optimal controls, Equation (7) can be rewritten as:

$$\begin{aligned} V = &\int_0^S \{\mathcal{H}^1(t, c^*(t), k^*(t), \mu^1(t)) - \mu^1(t)\dot{k}\}dt \\ &+ \int_S^R \{\mathcal{H}^2(t, S, c^*(t), l^*(t), k^*(t), \mu^2(t)) - \mu^2(t)\dot{k}\}dt \\ &+ \int_R^T \{\mathcal{H}^3(u, S, R, c^*(u), l^*(t), k^*(u), \mu^3(u)) - \mu^3(u)\dot{k}\}du + \frac{\eta[T-R]^{1-\frac{1}{\phi}}}{1-\frac{1}{\phi}}. \end{aligned} \tag{38}$$

Integrating equation (38) by parts, and using Equations (28)-(29), and the initial and terminal conditions on individual wealth, $k(0) = k(T) = 0$, one obtains the following expression:

$$\begin{aligned} V = &\int_0^S \{\mathcal{H}^1(t, c^*(t), k^*(t), \mu^1(t)) - \dot{\mu}^1 k^*(t)\}dt \\ &+ \int_S^R \{\mathcal{H}^2(t, S, c^*(t), l^*(t), k^*(t), \mu^2(t)) - \dot{\mu}^2 k^*(t)\}dt \\ &+ \int_R^T \{\mathcal{H}^3(u, S, R, c^*(u), l^*(t), k^*(u), \mu^3(u)) - \dot{\mu}^3 k^*(u)\}du + \frac{\eta[T-R]^{1-\frac{1}{\phi}}}{1-\frac{1}{\phi}} \end{aligned} \tag{39}$$

The first switching point which corresponds to the age of labor market entry, S^*, must satisfy the following first-order condition:

$$\frac{\partial V}{\partial S^*} = \mathcal{H}^1(S^*) - \mathcal{H}^2(S^*) + \int_{S^*}^R \frac{\partial H^2}{\partial S^*}dt + \int_R^T \frac{\partial H^3}{\partial S^*}du = 0, \tag{40}$$

which in its explicit form reads:

$$\begin{aligned}\frac{\partial V}{\partial S^*} = -\beta \ln(l(S^*))e^{-\theta t} - \mu_0 w A\Bigg[&(1-l(S^*))S^{*\rho}(1-\tau)e^{-rS^*}\\ &-\rho S^{*\rho-1}(1-\tau)\int_{S^*}^{R}(1-l(t))e^{-rt}dt\\ -\frac{\kappa\tau}{T-R}\left(\rho S^{*\rho-1}\int_{S^*}^{R}(1-l(t))e^{-\gamma t}dt - S^{*\rho}(1-l(S^*))e^{-\gamma S^*}\right)&\int_{R}^{T}e^{(\gamma-r)t}dt\Bigg] = 0\end{aligned} \tag{41}$$

The condition characterized in Equation (41) can be interpreted as a modified Mincer condition for optimal human capital formation. In addition to the conventional Mincer trade-off between foregone initial labor income and wage prospects, the individual also accounts for the foregone utility associated with leisurely activities when employed. Compared to the condition derived in Jacobs (2009), Equation (41) also includes the effect of additional time spent in education on pension income via the Bismarckian pillar. On one hand, the individual forgoes labor income, and therefore pension contributions, for the time spent in education. On the other hand, additional time in education enhances productivity and thus raises labor and pension income prospects. This effect is only realized via the Bismarckian pillar as the individual does not directly account for contributions to the Beveridgean pillar. Assuming an actuarially fair system, $\gamma = r$ and $\kappa = 1$, the first-order condition for optimal education simplifies to the following expression:

$$\begin{aligned}\frac{\partial V}{\partial S^*} = -\beta \ln(l(S^*))e^{-\theta t} - \mu_0 w A\Bigg[&(1-l(S^*))S^{*\rho}e^{-rS^*}\\ &-\rho S^{*\rho-1}\int_{S^*}^{R}(1-l(t))e^{-rt}dt\Bigg] = 0.\end{aligned} \tag{42}$$

This expression suggests that the optimality condition determining labor market entry becomes independent of the contribution rate. The explanation for this result is that the actuarially fair system does not induce any leakage from the life-cycle labor income stream following perfect substitutability between saving privately and in the pension system. Thus, the returns to life-cycle human capital formation are unchanged.

The second switching point which corresponds to the age of retirement, R^*, must, in turn, satisfy the following first-order condition:

$$\frac{\partial V}{\partial R^*} = \mathcal{H}^2(R^*) - \mathcal{H}^3(R^*) + \int_{R^*}^{T} \frac{\partial \mathcal{H}^3}{\partial R^*} dt - \eta[T - R^*]^{-\frac{1}{\phi}} = 0, \tag{43}$$

which in its explicit form becomes:

$$\begin{aligned} \frac{\partial V}{\partial R^*} = \beta \ln(l(R^*))e^{-\theta R^*} - \eta[T - R^*]^{-\frac{1}{\phi}} + \mu_0 \Bigg[(1 - l(R^*))wAS^\rho(1-\tau)e^{-rR^*} \\ -b(R^*)e^{-rR^*} + \frac{w\tau}{(T-R^*)^2} \Bigg(\kappa AS^\rho \Big((T - R^*)(1 - l(R^*))e^{-\gamma R^*} \\ + \int_S^{R^*} (1 - l(t))e^{-\gamma t} dt \Big) \int_{R^*}^{T} e^{(\gamma - r)t} dt + (1-\kappa)H \int_{R^*}^{T} e^{-rt} dt \Bigg) \Bigg] = 0 \end{aligned} \tag{44}$$

The interpretation of the retirement condition is straightforward. The costs of entering retirement is the explicit loss of earnings, the implicit loss of prospects for pension income, and utility loss of foregone intensive margin leisure. This cost is partially offset by the withdrawal of pension benefits and the utility of retirement leisure. Similar to the condition for optimal education, the condition for optimal retirement provides an interesting insight when considering an actuarially fair system. Setting $\gamma = r$ and $\kappa = 1$, equation (44) simplifies to:

$$\frac{\partial V}{\partial R^*} = \beta \ln(l(R^*))e^{-\theta R^*} - \eta[T - R^*]^{-\frac{1}{\phi}} + \mu_0(1 - l(R^*))wAS^\rho e^{-rR^*} = 0. \tag{45}$$

Analogous to the entry condition, the exit condition becomes neutral to changes in the contribution rate.

This has a similar explanation in that the life-cycle returns to labor supply remain unchanged following an increase in the contribution rate, as contributions are effectively realized as pension income plus interest.

As a result, the cost of entering retirement in terms of foregone earnings is not affected by the forced contributions to the pension system. Ultimately, an increase in the contribution rate when the pension system is actuarially fair affects behavior only through the budget constraint. That is, to smooth consumption over the life cycle,[15] the individual compensates for the foregone instantaneous labor income when young by borrowing more and saving less throughout the working life.

4 Numerical Analysis

4.1 Parametrization

Since the fundamental analyses concern the implications of pension design, I calibrate the model to a benchmark scenario of self-financing agents. Such a scenario is then characterized by $\tau = 0$, which is conceptually identical to a pension system with only a third pillar of voluntary savings. I normalize the per unit of efficient labor wage to $w = 1$, and I consider an interest rate of $r = 3.5\%$. I assume that all individuals start their economic life upon completion of upper secondary education at the age of 18, and die with certainty at the age of 80, i.e., $T = 62$. I assume half of the population to be of low ability, $\Lambda = 0.5$ (e.g., Golosov et al. (2013)).

For the baseline simulation of self-financing individuals, I consider $\theta = r$ such that the consumption path is constant over the life cycle. I then vary θ to study the sensitivity of the results to the rate of time preferences. The assumption of log utilities in consumption and leisure is equivalent to assuming that the intertemporal elasticities of consumption and leisure both take the value of unity under a CRRA specification. This yields a compensated wage elasticity of the labor supply of 0.5.

Following Acemoglu (2002), Sommacal (2006), and Hachon (2010), I consider a target for the human capital (wage) premium $\left(\frac{F_2(S_2)}{F_1(S_1)}\right)$ close to 1.7. Aiming for the high-skilled

[15]To clarify, the individual has a preference for smoothing the marginal utility of wealth over the life cycle. However, since the utility is specified as additive separable between consumption and leisure, this directly translates to a preference for consumption smoothing.

individual to spend around five years in tertiary education (to earn a Master's degree), and the low-skilled individual to spend less than one year in tertiary education, the following set of human capital-related parameters were used: $A_1 = 1$, $A_2 = 1.55$, $\rho_1 = 0.01$, and $\rho_2 = 0.055$.

The remaining behavioral parameters were determined such that average retirement occurs around the age of 65, and the average hours of work/week are between 30 and 35. This assumes that the individual has 17 waking hours/day and 5 days/week to allocate between labor supply and leisure activities during their working life.[16] I consider these targets as representative of many OECD economies (OECD, 2021). Following Duval (2004), Jacobs (2009), and Gruber and Wise (2019), I set $\phi = 0.99$. Then, by setting $\beta = 1.9$ and $\eta = 1.1$, the specified targets are met. These parameters assume that the relative weight of intensive-margin leisure and retirement leisure preferences $\left(\frac{\beta}{\eta}\right)$ takes the value of 1.72, which is slightly lower than the value of 1.86 in Jacobs (2009).

4.2 Numerical algorithm

When $\kappa < 1$, realized pension benefits become a function of the aggregate efficient labor supply in the economy. This implies that individuals' behaviors are interdependent, and should be solved for jointly. Therefore, I employ the following iterative process to obtain numerical results:

1. Guess the value of the aggregate efficient labor supply, H_{guess}.
2. Given the value for H_{guess}, solve the optimal control problems of the individuals to obtain their optimal efficient labor supply behavior.
3. Aggregate the efficient labor supply of the individuals, $H_{feedback}$.
4. Replace H_{guess} with $H_{feedback}$ and iterate until $(H_{feedback} - H_{guess})^2 < 0.0001$.

[16]This is consistent with the time endowment of 12 hours/day, assuming all days being available, in Goulder et al. (2019)

When the iteration process has converged according to the criterion in step 4, I consider the model to be in equilibrium. That is, no individual benefits from making any adjustments on their margins of decision.

4.3 Results

The numerical exercises illustrate the different outcomes for labor supply, savings, and inequality. For this purpose, I focus on three key specifications of the public pension structure in Equation (6): an actuarially unfair Bismarckian NDC system, ($\kappa = 1$, $\gamma = 0$), an actuarially fair Bismarckian NDC system ($\kappa = 1$, $\gamma = r$), and a Beveridgean system ($\kappa = 0$). This allows me to illustrate the polar cases of the contribution–benefit formula as determined by κ. It also allows me to emulate the incentive structure imposed by a fully funded system in which contributions are capitalized, as the actuarially fair NDC scheme is defined to yield capital returns. These scenarios, in turn, encompass the qualitative insights related to any intermediate value of κ and γ. The individual labor supply and private savings profiles under the different pension scenarios are illustrated in Figures (1)-(3). The inequality measures are illustrated in Figures (4)-(6).

4.3.1 Labor Supply

I begin by comparing the effects of an increase in the contribution rate on the individuals' life-cycle labor supply behavior for the three public pension scenarios. I consider three values for the contribution rate: $\tau = 0$, $\tau = 7.5\%$, and $\tau = 15\%$. This yields some general results. As long as the interest rate is higher than the discount rate, optimal labor supply decreases over the life cycle (Equations (34)-(36)). On the aggregate, this can be interpreted as decreasing labor market participation over the life cycle.[17].

Under an actuarially fair NDC system, individuals understand that any contributions to the public pension system are perfect substitutes for bank deposits. That is, consistent with

[17]This is partially consistent with OECD data, as the age-participation profile is generally found to increase until age 25-29, remain stable until age 50-54, and then decline (OECD, 2021).

the analytical insights obtained from Equation (37), contributions are implicitly capitalized from the point of view of the individual. This is illustrated in Figure (1). If the notional return falls short of the market interest rate, however, as with the return-dominated NDC system, returns to contributions will underperform risk-free returns on the capital market. This opportunity cost is larger for younger workers, following a compound interest effect. As such, the implicit tax treatment of a return-dominated NDC system is asymmetric over the life cycle. There is a higher implicit marginal tax on younger workers relative to older workers. This mechanism explains why an increase in the contribution rate increases optimal labor supply intensity among older individuals, as illustrated in Figure (2). This is particularly noticeable when comparing the effects of tax treatment between systems when $\theta = r = 0.035$. For both types of individuals enrolled in the actuarially fair NDC system, the labor supply is constant over the life cycle, while it increases for individuals in the actuarially unfair NDC system. The magnitude of the asymmetric tax treatment is, in turn, determined by the difference between the notional and market interest rate, as expressed in Equation (34). This comports with the result in Kindermann (2015). The larger the difference, the greater is the implicit tax imposed on younger individuals because of the foregone compound interest.

Regarding entry and exit responses, participation in the actuarially unfair NDC system introduces two effects. Retirement becomes cheaper as the net payoff to labor supply decreases. As a result, both types of individuals retire earlier. The decision to retire earlier, in turn, shortens the horizon over which returns to human capital accrue. Hence, it constitutes a disincentive for spending more time in education. However, this effect is found to be dominated by the reduced opportunity cost of education expressed in terms of foregone labor income as the contribution rate increases. The net effect is that both types of individuals spend more time in education. This finding contests the results of Caliendo and Findley (2019) that an increase in the contribution rate leads to lower educational attainment in the economy for both Beveridgean and Bismarckian specifications of the public pension system. For an actuarially fair NDC system, the life-cycle returns to labor supply are not modified, as any contributions perfectly substitute private savings. Ultimately the introduction of public pensions does not change the incentives for human capital formation or retirement timing, as analytically derived in Equations (42) and (45). Thus, both margins become neutral to changes in the contribution rate. The results in Kindermann (2015) that an actuarially unfair NDC pension system promotes more time spent in education are thus robust to the inclusion of endogenous retirement. However, the implicit education subsidy imposed by the foregone compound interest is found to be partially offset by the compound returns to human capital following an earlier retirement.

The effects on labor supply incentives for individuals enrolling in a Beveridgean pension system are illustrated in Figure (3). Since individual-specific benefits are not linked to their contributions, the contribution rate perfectly instruments a labor income tax (Equation [35]). As such, contributions are not recognized as an alternative savings technology, implying that the implicit tax treatment is symmetric over the life cycle. As preferences are specified as log, the income and substitution effect perfectly offset each other such that the contribution rate has no direct effect on the intensive margin labor supply. However, individuals face a modification of the active—retired trade-off through the taxation of earnings and changes in replacement income. First, as the contribution rate imposes a tax only on earnings and not on retirement utility, an increase in the contribution rate lowers the cost of retiring. Second, as a result of a redistributive benefit formula, the low-skilled individual will experience an increase in replacement income. This further lowers the cost of retirement as the low-skilled household can achieve a sufficient retirement buffer with less labor supplied. The opposite effect applies to the high-skilled individual who will face a lower replacement income relative to under a Bismarckian system. Ultimately, the labor supply of the low-skilled individual drops disproportionally more than the labor supply of the high-skilled individual. Ultimately, this finding is consistent with the result in Sommacal (2006) that a redistributive pension system increases earnings inequality through an increased dispersion in labor supply. Since individuals decrease their total labor supply, the pension system also implicitly introduces a disincentive for human capital formation, and this crowds out human capital in the economy as both high- and low-skilled individuals decide to enter the labor market earlier.

4.3.2 Savings

Studying the savings behavior gives an interesting insight with regard to the crowding-out effects induced by the forced savings mechanism. In general, both individuals accumulate some debt during their time in tertiary education to finance consumption in the absence of labor income. Comparing Figures (1) and (2), an actuarially fair NDC system crowds out more private savings relative to an actuarially unfair NDC system, since the agents face weaker incentives to reduce private savings when contributions are allocated toward a return-dominated technology. For more patient individuals (i.e., $\theta = 0.015$ or 0.025), the contribution rate of the actuarially fair system is not extensive enough to crowd out private savings entirely, leaving the aggregate capital stock positive at any time. However, if individuals grow moderately impatient, and the forced contribution rate is high enough, private savings are entirely crowded out and individuals borrow against future annuities. Since the pension fund is notional and thus illiquid, such behavior is not allowed in any real-life context. If individuals were instead credit rationed and unable to borrow against entitled pension income, it may be that the actuarially fair system would also distort the labor supply of the individuals.

As illustrated in Figure (3), the Beveridgean system also crowds out private savings as the contribution rate tightens the budget constraints. However, since this pillar is not rationalized as an alternative savings mechanism, but as leakage from the labor payoff stream, it does not crowd out as much private savings as either of the NDC system specifications. In addition, as less time is spent in education, the individuals do not accumulate as much private debt and return to solvency earlier in their lifetime. Finally, as individuals retire earlier, they also reach a peak in private savings earlier in their lifetime.

These results are only partially consistent with the findings by Hachon (2010) that a

Beveridgean system increases savings dispersion between low- and high-income households and that overall capital accumulation is larger than under a Bismarckian system. For the subjective discount rates considered in this analysis, the Beveridgean pension system crowds out less private savings when compared to the Bismarckian system, despite the absence of heterogeneous longevity. This can be explained in that individuals enrolled in a Bismarckian system rationalize the contribution rate as an alternative, albeit inferior, savings mechanism. If individuals are patient enough ($\theta = 0.015$), the Beveridgean system crowds out less private savings among high-income individuals relative to the Bismarckian system, but more among low-income individuals. Since patient individuals retire earlier, they finance a larger share of lifetime consumption from pension income and begin claiming benefits earlier in the life cycle. As a result, more productive individuals compensate for their lower replacement income under a Beveridgean system by saving more privately. The opposite holds for less productive individuals who, following a more prosperous replacement income, save less privately. As individuals grow less patient, however, the replacement income effect is discounted at a higher rate. As a result, for $\theta = 0.025$ and $\theta = 0.035$, the Bismarckian system crowds out more capital for both high- and low-income individuals, implying that the implicit tax effect dominates the replacement income effect for the high-income individual.

4.3.3 Inequality

Since each ability type is represented by one individual, I construct the index for inequality in terms of the ratio of income types, where income is expressed in present value terms discounted at market interest. The index for Earnings inequality (EI) is:

$$\mathrm{EI} = \frac{(1-\Lambda)F_2(S_2)\int_{S_2}^{R^2}(1-l_2(t))e^{-rt}dt}{\Lambda F_1(S_1)\int_{S_1}^{R^1}(1-l_1(t))e^{-rt}dt}, \tag{46}$$

and for life-time inequality (LI):

$$\text{LI} = \frac{(1-\Lambda)[(1-\tau)F_2(S_2)\int_{S_2}^{R^2}(1-l_2(t))e^{-rt}dt + \int_{R_2}^{T} b_2 e^{-rt}dt]}{\Lambda[(1-\tau)F_1(S_1)\int_{S_1}^{R^1}(1-l_1(t))e^{-rt}dt + \int_{R_1}^{T} b_1 e^{-rt}dt]}. \tag{47}$$

Omitting the discount term in Equations (46) and (47) yields measures of intergenerational inequality at any time t. Earnings inequality among the working population (WI) at any time can then be expressed as:

$$\text{WI} = \frac{(1-\Lambda)F_2(S_2)\int_{S_2}^{R^2}(1-l_2(t))dt}{\Lambda F_1(S_1)\int_{S_1}^{R^1}(1-l_1(t))dt}, \tag{48}$$

and population-wide income inequality (PWI) as:

$$\text{PWI} = \frac{(1-\Lambda)[(1-\tau)F_2(S_2)\int_{S_2}^{R^2}(1-l_2(t))dt + \int_{R_2}^{T} b_2 dt]}{\Lambda[(1-\tau)F_1(S_1)\int_{S_1}^{R^1}(1-l_1(t))dt + \int_{R_1}^{T} b_1 dt]}. \tag{49}$$

In addition, I consider a measure of human capital inequality as the human capital premium, and retirement age dispersion as $RI = R_2 - R_1$. The effects of increasing the contribution rate on human capital and retirement age dispersion, conditional on pension design, are illustrated in Figure (4). Figure (5) illustrates the effect on earnings and lifetime inequality (intragenerational inequality). Finally, Figure (6) illustrates the working population and population-wide inequality (intergenerational inequality).

Comparing the human capital inequality of self-financing individuals in Figure (4) (i.e., for $\tau = 0$) and varying the rate of time preferences reveal that an increased impatience among individuals increases the difference in educational attainment. Indeed, impatient individuals need to work longer to finance the debt accumulated when young. As individuals retire later, the financial returns to the human capital increase. Since the high-ability individuals enjoy a higher marginal return to human capital formation, the increase in the time spent in tertiary education is disproportionately larger relative to the increase among low-ability individuals.

As previously discussed, the Bismarckian and Beveridgean systems have opposite total effects on human capital formation. Both systems impose a disincentive for human capital formation by promoting early retirement, conditional on that $\gamma < r$. For the actuarially unfair NDC system, labor supply becomes more lucrative when a person is old as the opportunity cost of foregone investment opportunities grows smaller over the life cycle. As a result, it becomes more costly—in terms of foregone earnings—to retire earlier for both individuals. Given the wage premium of the high-skilled individual, the decrease in retirement age is smaller when compared to that of the low-skilled individual. This implies that the disincentives for human capital formation following an increase in the contribution rate are larger for the low-skilled individual. Since the Bismarckian system also lowers the opportunity cost of foregone labor earnings when young, an opposite effect arises following the conventional Mincer trade-off. This effect ultimately promotes more time invested in education, and is found to dominate the disincentives induced by lower lifetime returns to human capital. The total effect is that high-skilled individuals increase their educational attainment more than those in low-income households.

While the implicit labor income taxation of the Beveridgean system reduces the opportunity cost of education in terms of foregone labor income, this effect is dominated by the reduced lifetime financial returns to human capital. Since the human capital production function is concave in the duration of education, and $\rho_2 > \rho_1$, a proportional reduction in the educational attainment of both individuals will reduce human capital inequality. When comparing the different pension systems, the Beveridgean (Bismarckian) system ultimately reduces (increases) human capital inequality. As evident from Figure (4), however, the change in human capital inequality following an increase in the contribution rate is small irrespective of the design of the pension system.

Under a Beveridgean system and irrespective of the degree of impatience among individuals, the difference in retirement age between individual types is larger compared to that under a Bismarckian scenario. As a result, earnings inequality increases, as illustrated in Figure (5). This is consistent with the finding in Sommacal (2006). Since less-patient individuals spend more time working, the increased inequality in these scenarios becomes larger. In fact, only for the scenario with $\theta = 0.015$ does the redistributive pension system achieve lower lifetime inequality in the economy, while the exponential increase in earnings inequality overturns the decrease in pension inequality for the other scenarios, except for very low values of the contribution rate. Ultimately, the increased dispersion in hours worked over the life cycle clearly dominates the reduced human capital inequality induced by the Beveridgean system.

It is important to note that since the measurement of lifetime inequality includes discounting, the weight attached to the dispersion in pension income in the measure will be reduced. By omitting the discount factor, and thereby giving earnings and pension income the same weight in the inequality measure, the results can instead be interpreted in terms of intergenerational inequality given that the population is kept constant.

As is illustrated in Figure (6), the redistributive effect of a Beveridgean scheme is more convincing in terms of population-wide income inequality following higher (lower) replacement income for low- (high-) skilled individuals. As θ varies across the scenarios, the redistribution through the flat-rate benefit formula can reduce population-wide inequality for the scenarios with low degrees of impatience (i.e., $\theta = 0.015$ or 0.025) for the range of contribution rates considered. However, when the degree of impatience is increased to $\theta = 0.035$, the exponential increase in earnings inequality as measured across the working population dominates the reduced inequality among pensioners when the contribution rate is higher than 16.5 %. This follows from the delayed exit from the labor market as individuals need to repay the debt that financed their consumption when they were young. This has been shown to increase human capital inequality in scenarios with more patient individuals.

5 Concluding remarks

This paper studies the implications of the structural design of public pension systems for labor supply behavior and redistribution in a continuous-time OLG model. The model includes the entry and exit and the intensive margin of labor supply to capture a rich set of adjustable margins for the individual. The baseline model follows Jacobs (2009), which allows for the modeling of labor market entry and exit. This paper augments this framework in two ways. It uses heterogeneous agents in terms of earnings ability, and it endogenizes retirement benefits by using a stylized combined earnings-based/redistributive pension system which is at best actuarially fair. The model is then solved as a multi-stage delayed-response control problem to account for the inclusion of schooling, working life, and retirement, and following the link between labor supply and realized pension benefits. This novel approach provides a highly analytical framework for studying pension policies.

The main results are obtained through numerical simulations. Scenarios with a redistributive pension system display higher earnings inequality than scenarios with earnings-based pension systems. If the increase in earnings inequality is comprehensive enough, a redistributive pension system may in fact increase lifetime inequality. Therefore, the reform of public pensions from flat-benefit accounts to earnings-based individual accounts might not harm economic equality. A return-dominated notional defined-contribution system introduces an asymmetric tax treatment of labor supply over the life cycle. The opportunity cost of foregone investment returns—given the forced savings mechanism of pension contributions—grows smaller as the individual approaches retirement. This, in turn, lowers the opportunity cost of education when young, and it promotes a delayed entry into the labor market following more time spent in tertiary education.

There are many avenues for future research. First, it is important to acknowledge the limits of a stylized representation of public pensions when drawing policy conclusions. Instead of focusing on the precise institutional features of any one pension system, this paper aims to make a general illustration of the incentives implicit in a common feature of modern public pension systems. Naturally, by abstracting from various non-linearities in the contribution–benefit formula and specific eligibility rules, the implicit taxation induced by the contribution rate is simplified. Therefore, it is important to conduct further research in more precise institutional contexts to draw extensive conclusions about the redistributive impact of the structure of particular public pension systems.

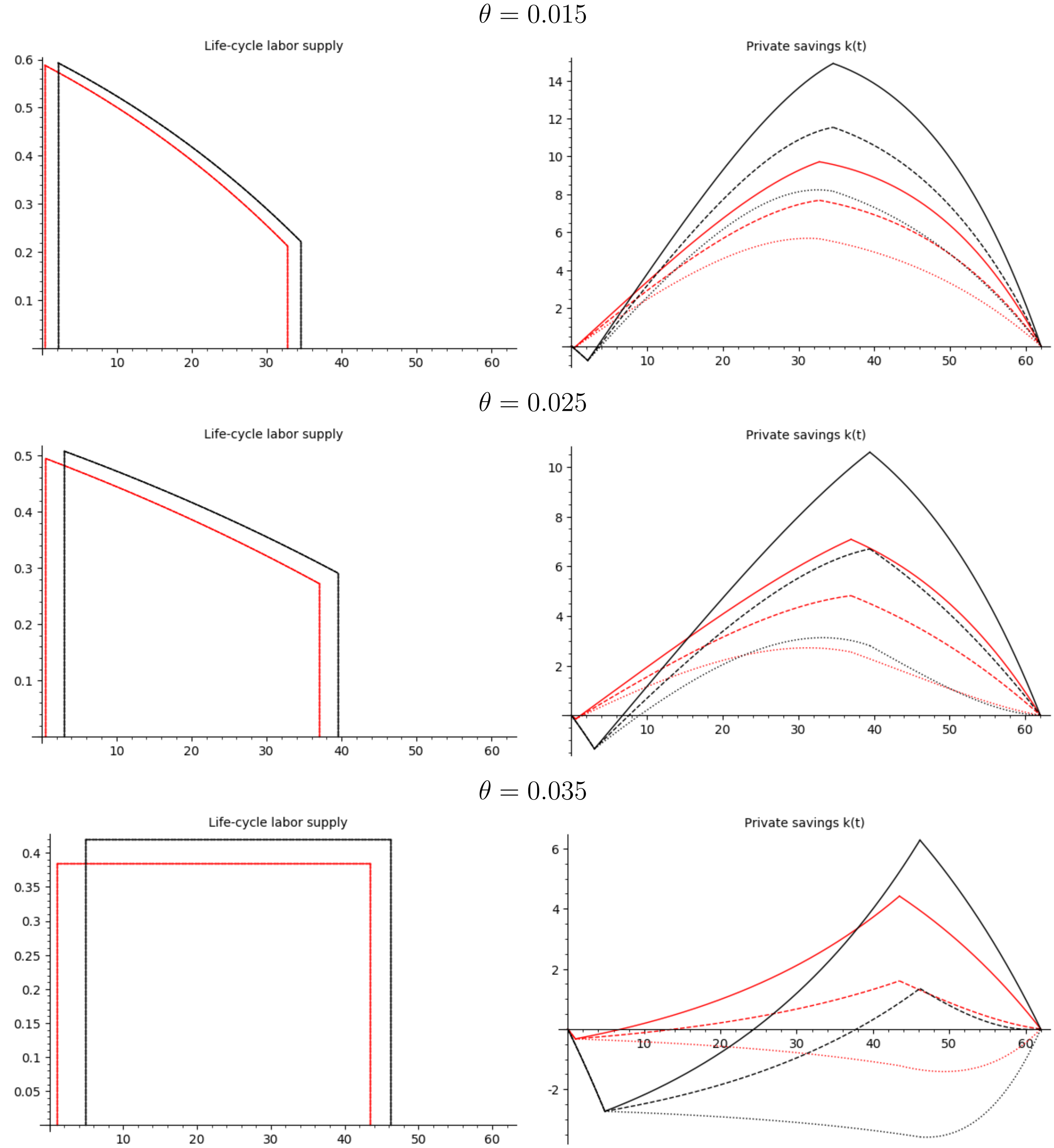
Second, the model in this paper assumes fixed factor prices to study the microfoundations. A natural next stage is to extend the analysis to general equilibrium and observe its predictions regarding the dynamics of economic aggregates over time and the transition paths to new steady state(s) following pension reform. This would be necessary for extending the analysis beyond steady state comparison and model the dynamic outcomes of various pension reforms on physical and human capital stocks and their implications for public finances over time.

Third, the assumption of perfect credit markets means that impatient individuals may borrow against their future retirement income if the pension system is extensive. For the analysis to encompass every possible combination of parameters within reasonable ranges without violating institutional rules and regulations, the inclusion of credit rationing is necessary. I recognize that integrating horizontal constraints in optimal control theory without compromising on analytical tractability is a demanding task, but it should, if successful, constitute a substantial contribution.

Lastly, this paper considers rational individuals who perfectly foresee future events and fully comprehend how their behavior interacts with features of the pension system. Evidence from behavioral economics suggests that this does not necessarily represent actual behavior. A consideration for future studies is to integrate various forms of behavioral failures such as time-inconsistency and bounded rationality. Another option would be to consider an age-dependent discount factor to capture an increasing awareness of realized pension wealth as the individuals grow older. As concluded by De Nardi and Fella (2017), the perception of replacement effects may also vary with wealth. If individuals are wealthy enough, incentives implicit in the pension system may not be large enough to influence labor supply behavior over the life cycle. Likewise, if individuals live on a hand-to-mouth basis, they may only rationalize the instantaneous take-home pay when making their labor supply decisions. I leave these suggestions for future studies.

Figures

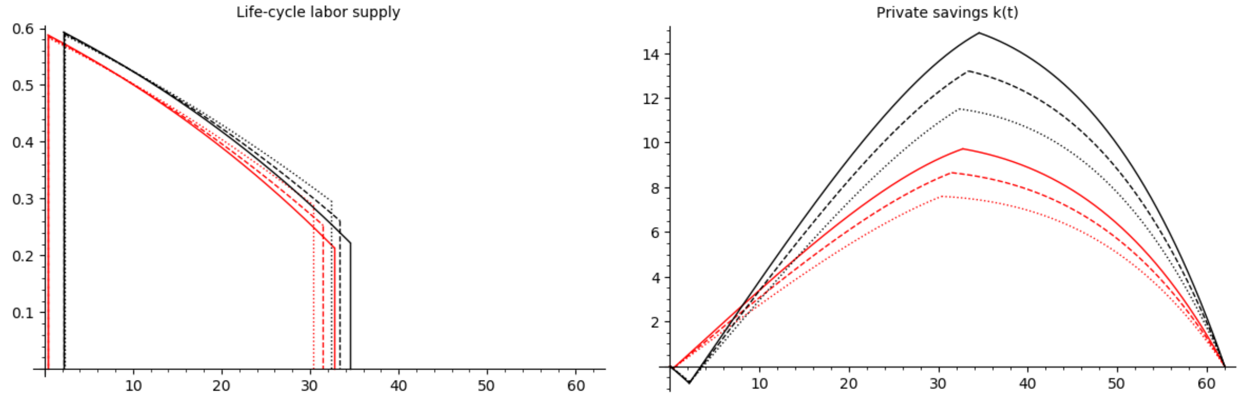
Figure 1: Actuarially fair "Bismarckian" system



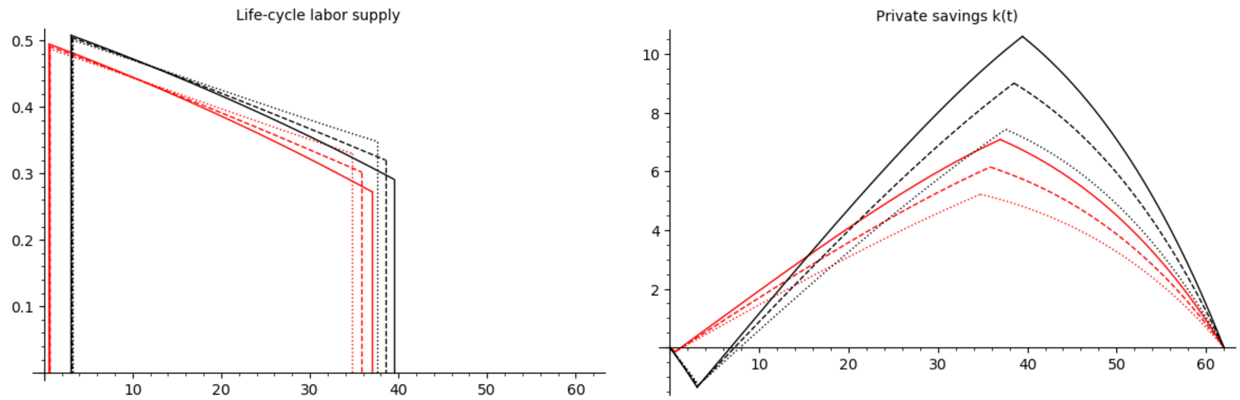
Note: LHS: The y-axis represents labor supply, and the x-axis model age. RHS: The y-axis represents private savings, and the x-axis model age. Black lines correspond to the high-skilled individual, red to low-skilled individuals. Solid lines correspond to $\tau = 0$, dashed to $\tau = 0.075$, and dotted to $\tau = 0.15$.

Figure 2: Actuarially unfair "Bismarckian" system

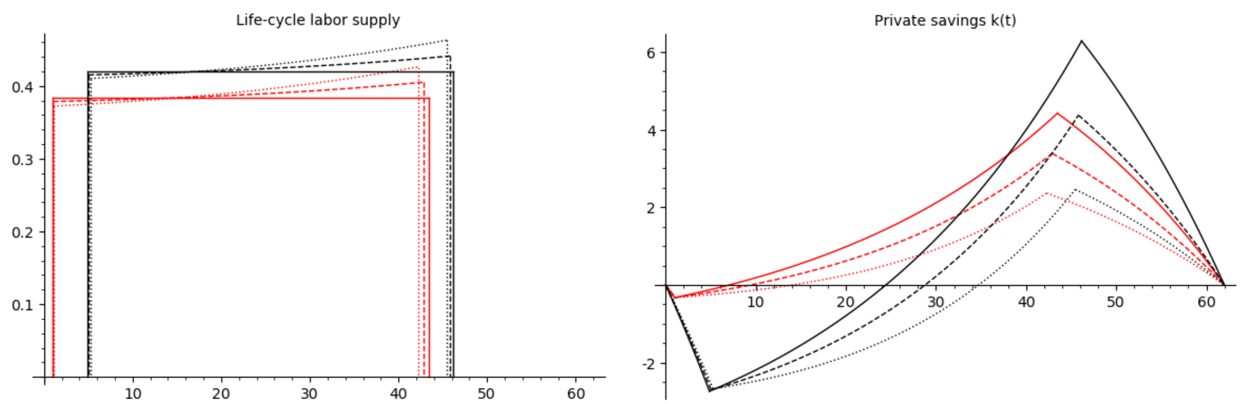
$\theta = 0.015$



$\theta = 0.025$



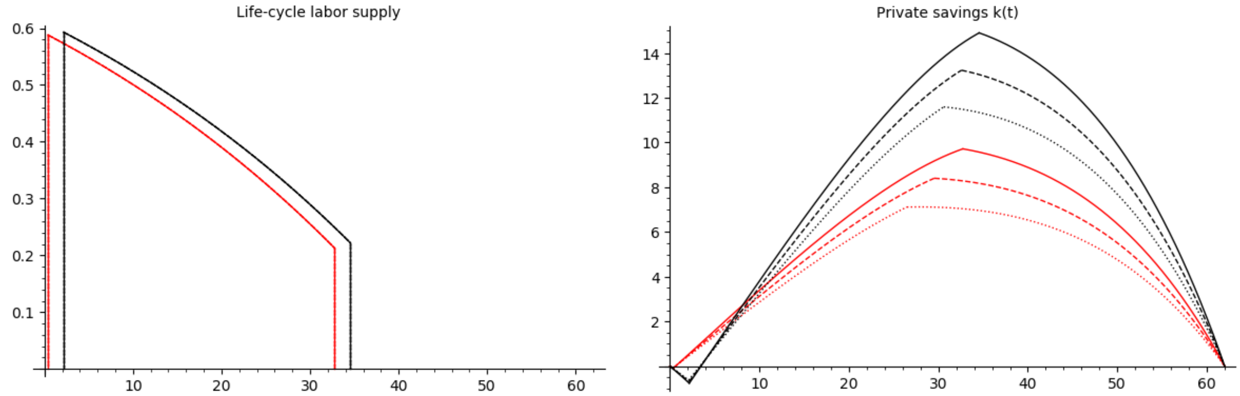
$\theta = 0.035$



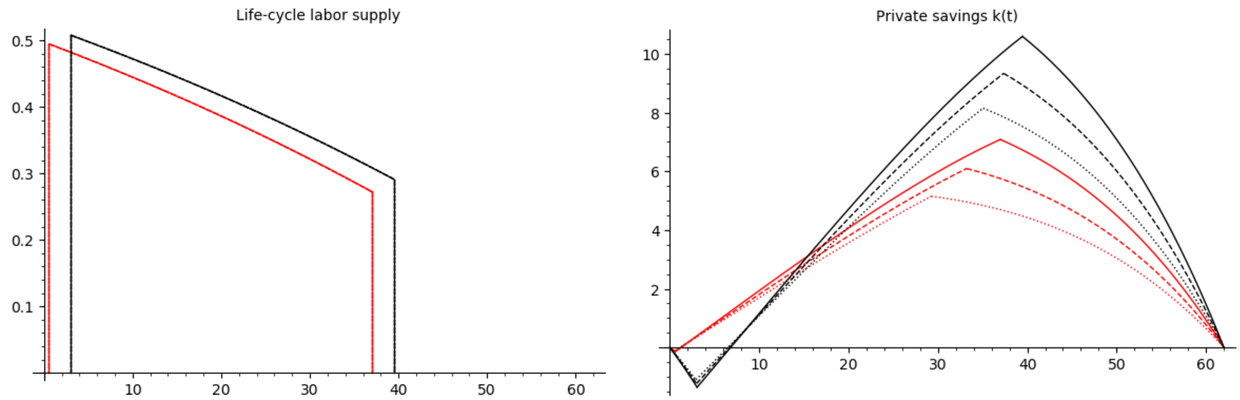
Note: LHS: The y-axis represents labor supply, and the x-axis model age. RHS: The y-axis represents private savings, and the x-axis model age. Black lines correspond to the high-skilled individual, red to low-skilled individuals. Solid lines correspond to $\tau = 0$, dashed to $\tau = 0.075$, and dotted to $\tau = 0.15$.

Figure 3: "Beveridgean" PAYG system

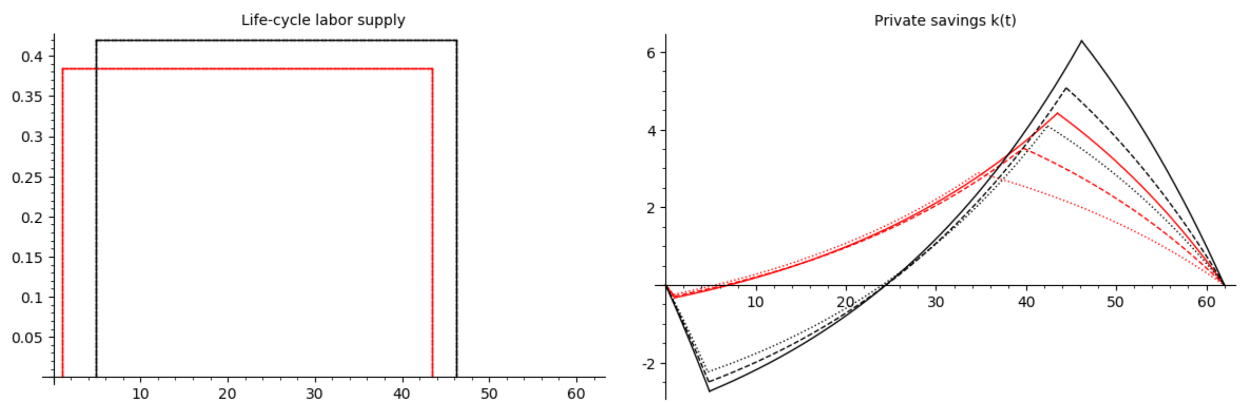
$\theta = 0.015$



$\theta = 0.025$

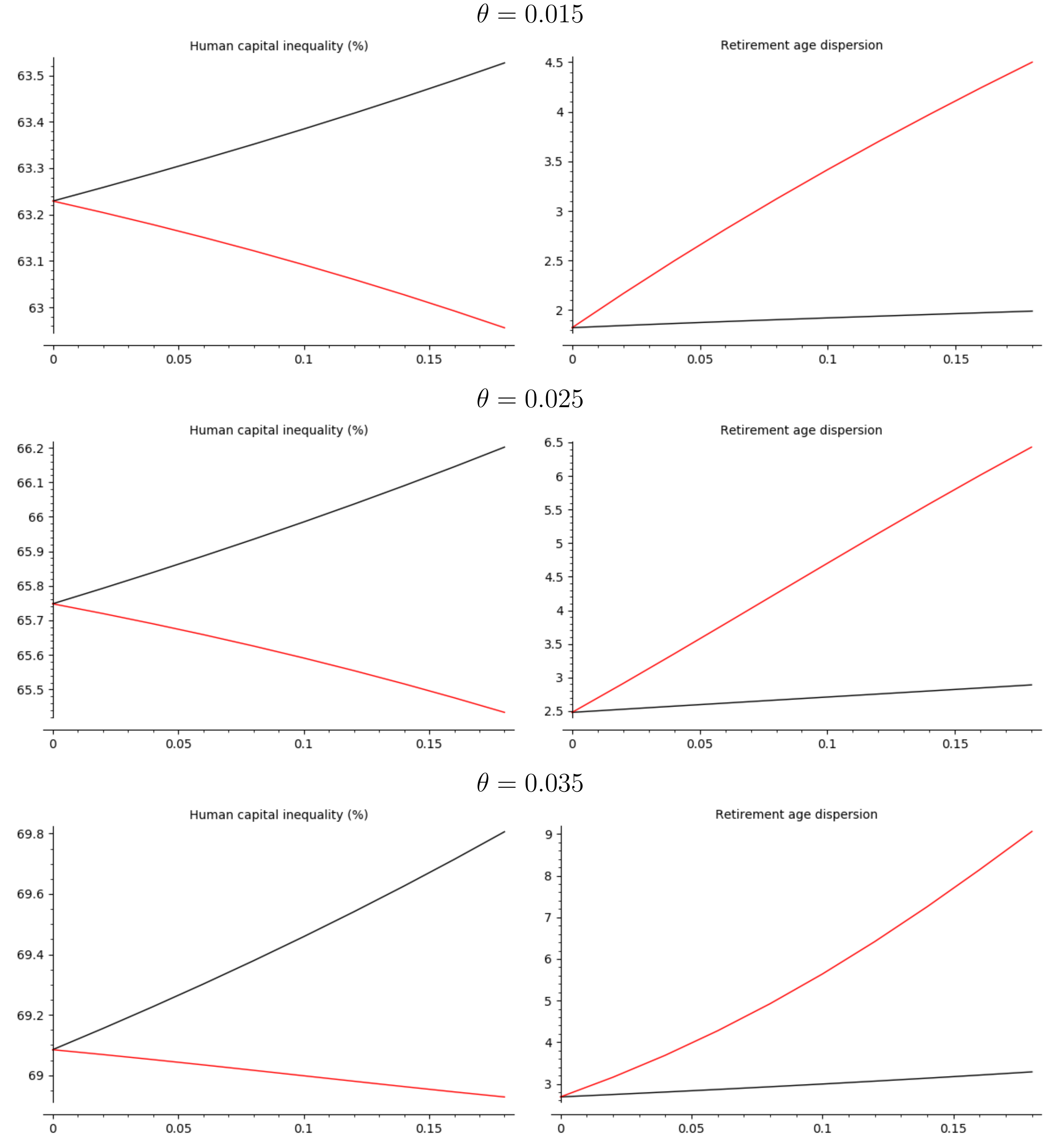


$\theta = 0.035$



Note: LHS: The y-axis represents labor supply, and the x-axis model age. RHS: The y-axis represents private savings, and the x-axis model age. Black lines correspond to the high-skilled individual, red to low-skilled individuals. Solid lines correspond to $\tau = 0$, dashed to $\tau = 0.075$, and dotted to $\tau = 0.15$.

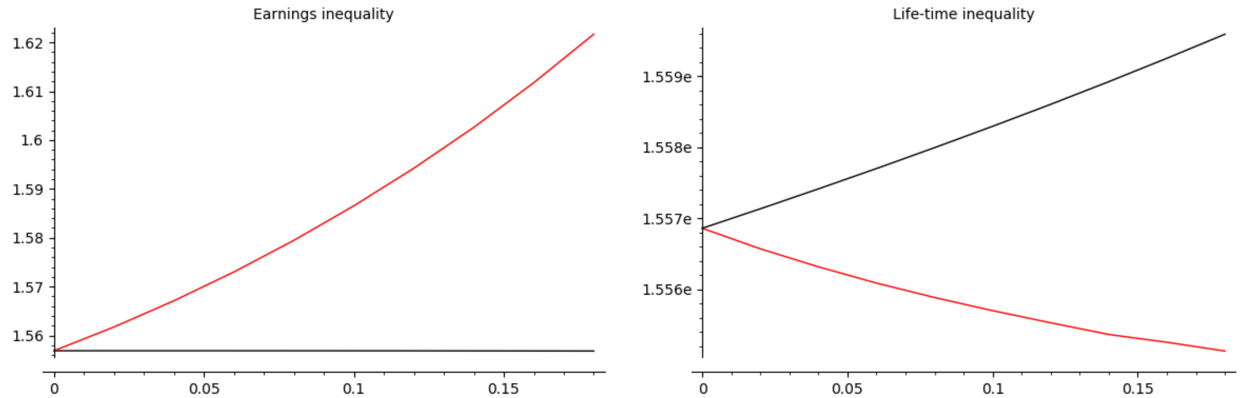
Figure 4: Bismarckian PAYG vs Beveridgean PAYG - Human Capital Inequality and Retirement Age Dispersion



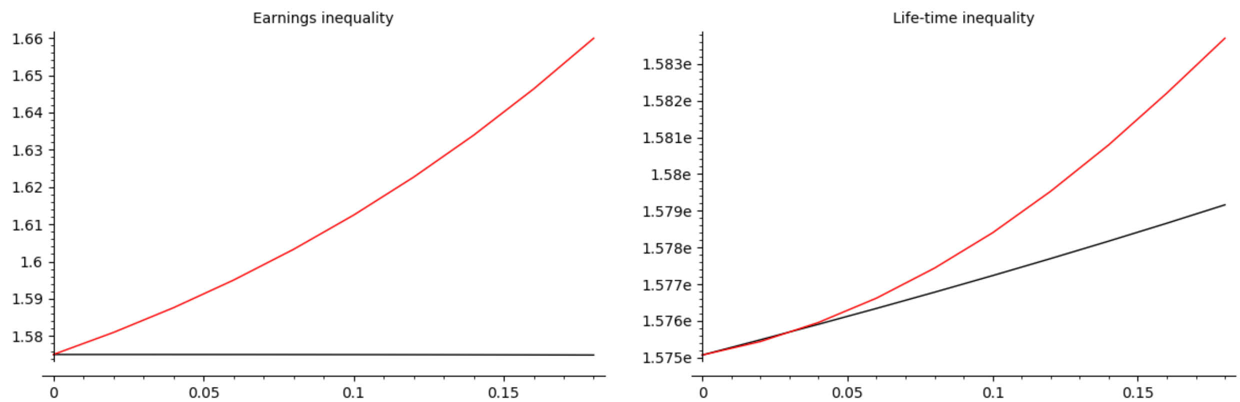
Note: Black lines = Bismarckian system. Red lines = Beveridgean system. The y-axis represents human capital inequality and retirement age dispersion respectively, and the x-axis the contribution rate.

Figure 5: Bismarckian PAYG vs Beveridgean PAYG - Intragenerational Inequality

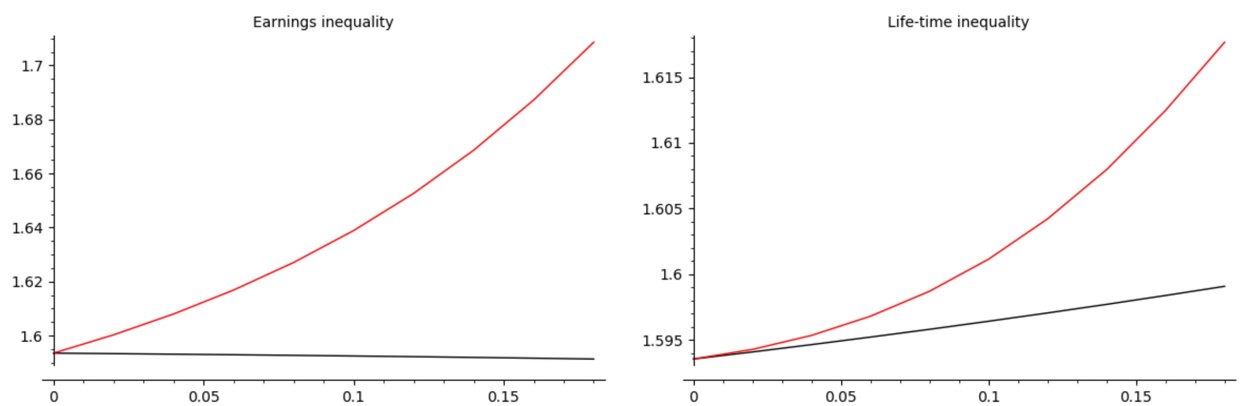
$\theta = 0.015$



$\theta = 0.025$



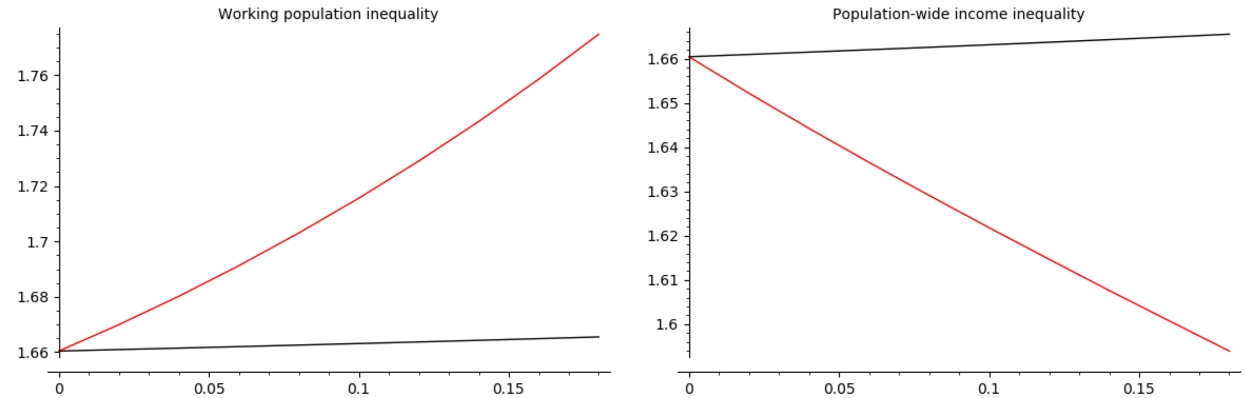
$\theta = 0.035$



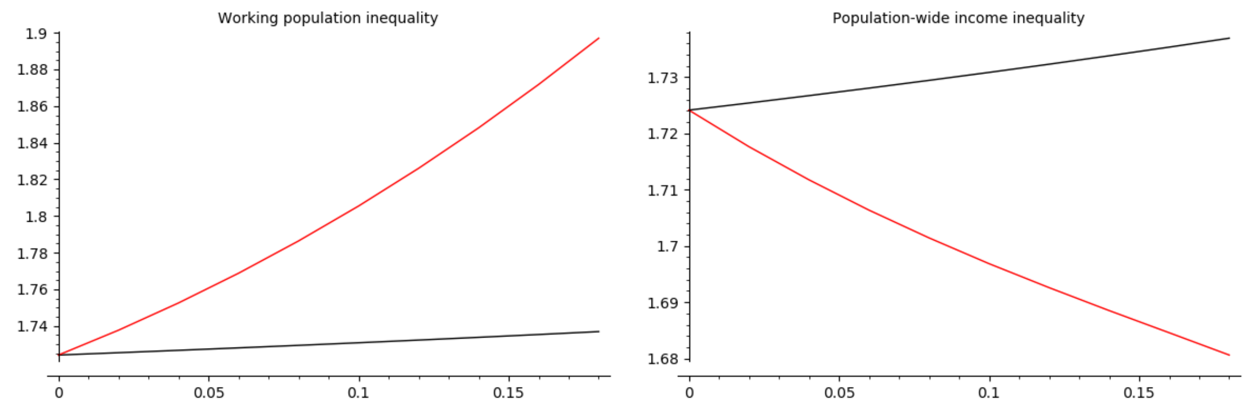
Note: Black lines = Bismarckian system. Red lines = Beveridgean system. The y-axis represents the inequality measures, and the x-axis the contribution rate.

Figure 6: Bismarckian PAYG vs Beveridgean PAYG - Intergenerational Inequality

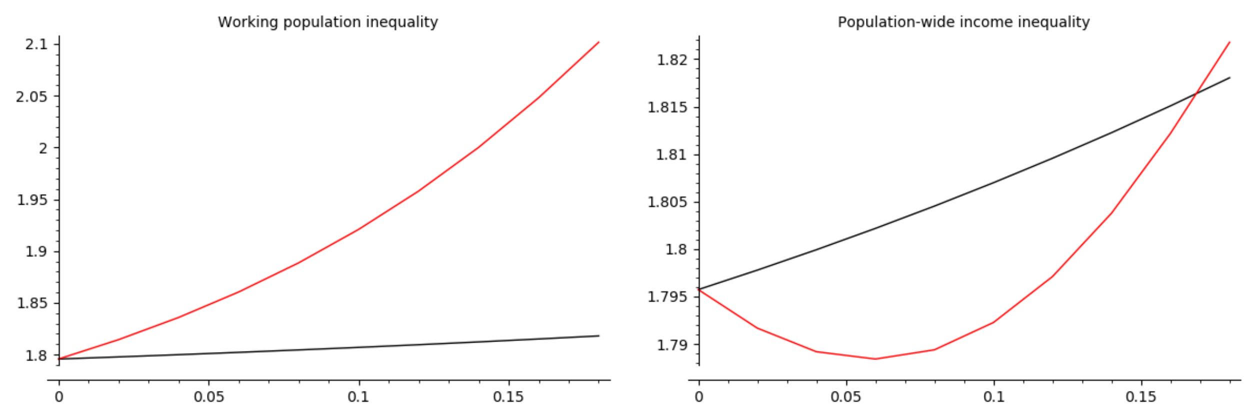
$\theta = 0.015$



$\theta = 0.025$



$\theta = 0.035$



Note: Black lines = Bismarckian system. Red lines = Beveridgean system. The y-axis represents capital, and the x-axis the contribution rate.

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Appendix. Solving for the marginal utility of wealth

The first order conditions for consumption and intensive margin leisure can be expressed as follows:

$$\frac{e^{-\theta t}}{c(t)} = \mu_0 e^{-rt} \tag{50}$$

$$\frac{\beta e^{-\theta t}}{l(t)} = \mu_0 w F(S)(1-\tau)e^{-rt} + \frac{\mu_0 \kappa w F(S)\tau e^{-\gamma t}}{T-R}\int_R^T e^{(\gamma-r)t}dt. \tag{51}$$

Multiplying both sides of Equations (50) and (51) with their control arguments and integrating the expressions over their respective control domain, one obtains the following expressions:

$$\int_0^T e^{-\theta t}dt = \mu_0 \int_0^T c(t)e^{-rt}dt, \tag{52}$$

$$\beta \int_S^R e^{-\theta t}dt = \mu_0 w F(S)(1-\tau)\int_S^R l(t)e^{-rt}dt + \frac{\mu_0 \kappa w F(S)\tau}{T-R}\int_R^T e^{(\gamma-r)t}dt \int_S^R l(t)e^{-\gamma t}dt. \tag{53}$$

Adding Equations (52) and (53) yields:

$$\begin{aligned}\int_0^T e^{-\theta t}dt + \beta \int_S^R e^{-\theta t}dt = \mu_0 \Big\{ & w F(S)(1-\tau)\int_S^R l(t)e^{-rt}dt \\ & + \frac{\kappa w F(S)\tau}{T-R}\int_R^T e^{(\gamma-r)t}dt \int_S^R l(t)e^{-\gamma t}dt + \int_0^T c(t)e^{-rt}dt \Big\}.\end{aligned} \tag{54}$$

For the purpose of further mathematical operations, Equation (54) can be rewritten as:

$$\begin{aligned}\int_0^T e^{-\theta t}dt + \beta \int_S^R e^{-\theta t}dt = \mu_0 \Big\{ & w F(S)(1-\tau)\int_S^R l(t)e^{-rt}dt \\ & + \frac{\kappa w F(S)\tau}{T-R}\int_R^T e^{(\gamma-r)t}dt \int_S^R l(t)e^{-\gamma t}dt + \int_0^T c(t)e^{-rt}dt \\ & - w F(S)(1-\tau)\int_S^R e^{-rt}dt + w F(S)(1-\tau)\int_S^R e^{-rt}dt \\ & - \frac{\kappa w F(S)\tau}{T-R}\int_R^T e^{(\gamma-r)t}dt \int_S^R e^{-\gamma t}dt + \frac{\kappa w F(S)\tau}{T-R}\int_R^T e^{(\gamma-r)t}dt \int_S^R e^{-\gamma t}dt \Big\}.\end{aligned} \tag{55}$$

This expression in turn simplifies to:

$$\begin{aligned}\int_0^T e^{-\theta t}dt + \beta \int_S^R e^{-\theta t}dt = \mu_0 \Big\{ & w F(S)(1-\tau)\int_S^R (l(t)-1)e^{-rt}dt \\ & + \frac{\kappa w F(S)\tau}{T-R}\int_R^T e^{(\gamma-r)t}dt \int_S^R (l(t)-1)e^{-\gamma t}dt + \int_0^T c(t)e^{-rt}dt \\ & + w F(S)(1-\tau)\int_S^R e^{-rt}dt + \frac{\kappa w F(S)\tau}{T-R}\int_R^T e^{(\gamma-r)t}dt \int_S^R e^{-\gamma t}dt \Big\}.\end{aligned} \tag{56}$$

Given the life-cycle budget constraint:

$$wF(S)(1-\tau)\int_S^R (1-l(t))e^{-rt}dt + \int_R^T b(t)e^{-rt}dt - \int_0^T c(t)e^{-rt}dt, \tag{57}$$

which upon substitution of the expression for the benefit annuity in Equation (6) can be written as,

$$\begin{aligned} wF(S)(1-\tau)\int_S^R (1-l(t))e^{-rt}dt + \frac{w\tau}{T-R}\Big[\kappa F(S)\int_S^R (1-l(t))e^{-\gamma t}dt \int_R^T e^{(\gamma-r)t}dt \\ +(1-\kappa)H\int_R^T e^{-rt}dt\Big] - \int_0^T c(t)e^{-rt}dt, \end{aligned} \tag{58}$$

the first three additive terms within the curly brackets on the RHS of Equation (58) are equal to the negative of the pension income corresponding to the Beveridgean pillar. Equation (58) thus simplifies to the following expression:

$$\begin{aligned} \int_0^T e^{-\theta t}dt + \beta\int_S^R e^{-\theta t}dt = \mu_0 w\Big\{F(S)\Big[(1-\tau)\int_S^R e^{-rt}dt \\ +\frac{\kappa\tau}{T-R}\int_R^T e^{(\gamma-r)t}dt\int_S^R e^{-\gamma t}dt\Big] - \frac{(1-\kappa)\tau H}{T-R}\int_R^T e^{-rt}dt\Big\} \end{aligned} \tag{59}$$

Solving Equation (59) for μ_0 yields the final expression:

$$\mu_0 = \frac{\int_0^T e^{-\theta t}dt + \beta\int_S^R e^{-\theta t}dt}{w\{F(S)[(1-\tau)\int_S^R e^{-rt}dt + \frac{\kappa\tau}{T-R}\int_R^T e^{(\gamma-r)t}dt\int_S^R e^{-\gamma t}dt] - \frac{(1-\kappa)\tau H}{T-R}\int_R^T e^{-rt}dt\}}. \tag{60}$$